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Model of Behaviour and Response of Corrugated Sandwich Plate based on Structural Smearing and Classical Laminate Theory

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Abstract The present work, describes an analytical model, for handling the effective characteristics of structural anisotropic plate, with special attention giving to sandwich plate with corrugated cores. The in-plane extensional shear rigidities and the out of plane bending and twisting rigidities are evaluated using the classical laminate assumption and structural smearing. The in-plane extensional and bending material stiffness of crossply(0/90) laminas of the integrated thermal protection system are evaluated using the classical laminate theory and thein-plane extensional and bending material stiffness of laminas other than 0/90 are evaluated using the synergy in the CLT and axis rotation. The in-plane extensional and bending stiffness of the integrated thermal protection system in the problem coordinate axis are evaluated using the synergy in classical laminate theory and structural smearing.. An Algorithm of solution was developed and an excel program was written for the analytical response of the sandwich construction. The accuracy of the proposed model was verified by comparing results with models from other researchers, who used the finite elements methods to validate their results. A complete model is described and can be adjusted to accommodate all form of corrugation in any material. It is recommended that the proposed model be used to predict the elastic behaviour of sandwich plates with corrugated cores, particularly Integral Thermal Protection System (ITPS), bridge decks and grillages

Keywords Anisotropic plate, Integrated Thermal Protection System and structural smearing.

1. Introduction

Corrugated Sandwich Panels Are Composite Structural Systems, Composed of Plates at Top and Bottom and Corrugated Web Cores, Which May Be of The Same or Different Materials.

A Greater Use of Sandwich Panels Can Open New Opportunities for Entrepreneurship and Job Creation. They Are Used in Construction, Transportation, and Industrial Sectors for Applications Such as Bridge Decks, Aircraft Muffin Wings, Sound Insulation Systems, And Among Others.

They Possess Lower Bending Deflection, High Stiffness to Weight Ratio, High Crush Resistance, High Critical Buckling Loads, High Natural Frequencies and Greater Transverse Load Carrying Capacity, Compared to Monolithic Structures of Equal Weight.

Available literature on sandwich plates shows that, its usage dates back to the 19th century, and its studies broadly covers: determination of elastic constants, maximum load carrying capacity and energy absorption.

The analysis and design of sandwich panels are associated with numerous challenges as a result of the composite nature of its configuration.

The elastic theories for plate analysis are associated with rigorous mathematics. The plastic theories of analysis lead to over reinforcement of reinforced concrete section and larger section in steel, timber, plastic and nonferrous concrete. Numerical modeling of the corrugated sandwich panels is usually expensive, owing to the complicated geometry. Several homogenization methods have been evolved to homogenize this obviously heterogeneous construction.

A review of various homogenization methods is presented in Igor *et al* (2015), Aleksender, *et al* (2015), Arthur *et al* (2012), Abbes *et al* (2010), Talbei *et al* (2009), Buannic *et al* (2003), Naoki *et al* (1995) among others. In the same vein, various studies have been dedicated to the development of an alternative approaches, the most popular being the equivalent plate method, obtained by relaxing some of the stringent requirements of the homogenization method. Some of the authoritative works on equivalent plate models include Huimin *et al* (2019), Jian *et al* (2018), Young Jo *et al* (2015), Bartolozzi *et al* (2013,2014), Zheng *et al* (2014), Wang *et al* (2011), Biancolini *et al* (2005), Brassoulis *et al* (1986) among others. Martinez *et al* (2007) were the first authors to develop an equivalent plate model for composite corrugated-core sandwich panels using micromechanics approach. They idealized the composite corrugated sandwich plate as an equivalent orthotropic thick plate continuum and employed the strain energy approach to evaluated the extensional, bending coupling stiffness matrices as well as shear stiffness term for the equivalent plate. Higher order shear deformation plate theory was used to determine the maximum deflection and stresses in the plate and found good agreement with the finite element analysis. Rajesh (2014) in doctoral studies, extended the work of Martinez (2014) by considering the effects of various geometric parameters on the global bending response of composites sandwich plates with corrugated cores. The sandwich plates were made of unidirectional corrugated core with repeated unit cell. He evaluated the global bending response by first calculating the stiffness matrix for an equivalent homogenous plate and then implemented the minimum potential energy technique. Rajesh systematically varied the geometric parameters of the unit cell to determine their effects on the global bending response, which include global deflection, bending moments and shear force distributions.

To address the complexity of these methods, and associated high requirements of computation time and money, the need for a simplified solution is recognized.

In this study, a model of behavior and response of corrugated sandwich plate based on the classical laminate theory, axis rotation and structural smearing is presented. The present analytical model for simply supported sandwich plate subjected to transverse static load in the absence of temperature gradient and support sinkage is corroborated by both the finite element results and results from other researchers.

2. Governing Equations

The modeling process was based on the calculation of rigidities in material and problem coordinate axis, development of an equivalent plate model of homogenization based on the classical laminate theory and structural smearing to determine the in-plane extensional shear stiffnesses, the out of plane bending and twisting stiffnesses

2.1 Equivalent Model for the Sandwich Construction

To evaluate the overall stiffness of the sandwich plate with corrugated core, the stiffness matrices of each laminate in the laminate has to be translated to the common reference axis before the summation is done. For symmetric orthotropic laminated composite, this occurs at the neutral axis.

2.2 Determination of the Neutral Axis of the Sandwich Plate

The centroid of the unit cell in figure 1 given as:

$$h_s = \frac{A_{BP}h_{BP}+A_W h_W+ A_{TP}h_{TP}}{A_{BP}+ A_W+ A_{TP}} \quad (1)$$

Where A_{TP}, A_{BP}, $A_W$are the area of the top plate, the area of the bottom plate and the area of the web respectively while

Z_{TP}- h_s : is the soffit of the top surface from the centroid y-axis

Z_{bP} - h_s : is the soffit of the bottom surface from the centroid y-axis

Z_w - h_s : is the soffit of the web surface from the centroid y- axis

h_s : is the position from the bottom plate to the centroid y-axis and are defined in equations 2,3, and 3 respectively.

$$Z_{BP} = \frac{t_{tp}}{2} \quad (2)$$

$$Z_{TP} = \frac{t_{Tp}}{2} \quad +t_{BP}+ \ d_c \quad (3)$$

$Z_w = \frac{d_c}{2} + t_{BP}$ (4)

h= $t_{TP} + t_{bP} + d_c$ (5)

When the thickness of the bottom plate is equal to the thickness of the top plate

i.et_{BT} = t_{TP} = t

Then centroid of the unit cell from the bottom of the unit cell of Fig 1 becomes

$h_s = t/2 + d_c/2$ (6)

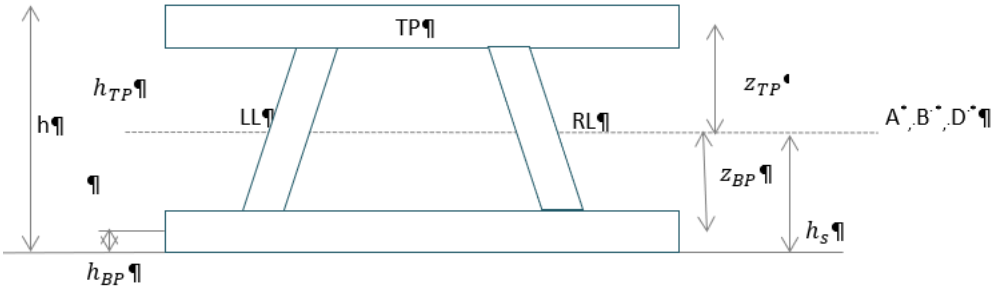

Figure 1: A Unit Cell of the Corrugated Core for Determination of Neutral Axis.

2.3 Constitutive Equation for the Equivalent Model

For the sandwich plate shown in Fig 1 the overall stiffness of each lamina in the laminate, is translated to the common reference axis, using the parallel axis theorem for axis transformation. All the stiffness from the lamina coordinate axis is translated to the problem coordinate axis $(A^*_{ij}, B^*_{ij}, D^*_{ij})$.

2.3.1 Stiffness Model of Sandwich Plate with Corrugated Core

This section presents the derivation of ply stiffnesses for a sandwich component at a distance z from the problem axis.; derivation of the in-plane material stiffness by horizontal rotation into the problem axis for face plate laminate inclined in the horizontal plane; derivation of the out of plane material b ending and twisting stiffness by horizontal rotation into the problem axis for face plate laminate inclined in the horizontal plane ; derivation of ply stiffnesses vertical rotation into the problem axis for core laminates, inclined to the horizontal plane ; derivation of ply stiffnesses for core laminates about the middle height using parallel axis translation**;** derivation of reduced stiffness matrix of core laminate; structural smearing of the sandwich plate with corrugated**.**

2.3.1.1 Derivation of Ply Stiffnesses for a Sandwich Component Ply at A Distance z From the Problem Axis

From the classical laminate theory, the in-plane extensional shear stiffness, the in-extensional shear and bending coupling stiffness and the out of plane bending and twisting stiffness of the sandwich construction can be evaluated using equations 7, 8 and 9 respectively.

$|A| = \sum_{k=1}^{n} \left|\bar{Q}_{x-y}\right|_k (z_k - z_{k-1})$ (7)

$|B| = \sum_{k=1}^{n} \left|\bar{Q}_{x-y}\right|_k (z_k^2 - z_{k-1}^2)/2$ (8)

$|D| = \sum_{k=1}^{n} \left|\bar{Q}_{x-y}\right|_k (z_k^3 - z_{k-1}^3)/3$ (9)

2.3.1.2 Derivation of the In-plane Material Stiffness by horizontal rotation into the problem axis for face plate laminate inclined in the horizontal plane.

The in-plane extension shear stiffness by horizontal rotation into the problem axis for face plate laminate inclined in the horizontal plane for the unit cell shown in Figure 2 is evaluated using equation 7.

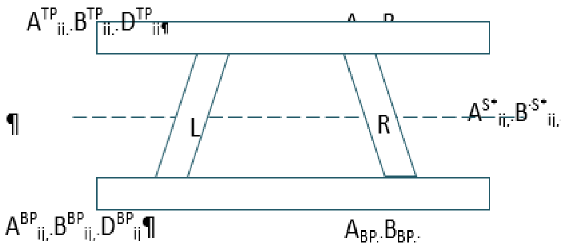

Figure 2: Unit Cell of Equivalent Model

Wheret_k is the thickness of kth ply and n is the total number of plies in the laminate

2.3.1.3 Derivation of The Out of Plane Material Bending and Twisting Stiffness by Horizontal Rotation into The Problem Axis for Face Plate Laminate Inclined in The Horizontal Plane.

The out of plane bending and twisting stiffness by horizontal rotation into the problem axis for face plate laminate inclined in the horizontal plane for the unit cell shown in Figure 2.is evaluated using equation 8.

2.3.1.4 Derivation of Ply Stiffnesses by Vertical Rotation into The Problem Axis for Core Laminates, Inclined to The Horizontal Plane.

The in-plane extensional shear stiffness, the in-plane extensional shear and bending coupling stiffnesses and the out of plane material bending and twisting stiffness by vertical rotation into the problem axis for core laminates, inclined to the horizontal plane are given in equations 10,11 and 12 respectively.

$$\left|A_{ij}{}^{"}\right|_c = \sum_{k=1}^{n}\left[\bar{Q}_{ij}\right]_k t_k \quad (10)$$

$$\left|B_{ij}{}^{"}\right|_c = \sum_{k=1}^{n}\left[\bar{Q}_{ij}\right]_k \left(t_k \check{Z}^2\right) \quad (11)$$

$$\left|D_{ij}{}^{"}\right|_{c_{ij}} = \sum_{k=1}^{n}\bar{Q}_{ij}\left(\frac{t_k{}^3}{12} + t_k\check{Z}^2\right) \quad (12)$$

Derivation of Ply Stiffnesses for Core Laminates about the middle height using parallel axis translation.

From Figure 3, Rotating a small element ds about the $x' - axis$, originally in the material $y'$and z' axis. to y'' and z'' axis and then translating to the reference axis for the cross-section selected and then integrating along the width core s_c, will yield equations for the membrane stiffness, in-plane, and bending coupling stiffness as well as bending stiffness of the inclined core Sedel (2006).

That is:

$$\left|\bar{A}_{ij}\right|_c = S_c|A''|_c \quad (13)$$

$$\left|\bar{B}_{ij}\right|_c = S_c|B''|_c \quad (14)$$

$$\left|\bar{D}_{ij}\right|_c = S_c|D''|_c + sin^2\theta\,|A''|_c \quad (15)$$

Where $\left[\underline{A}_{ij}\right]_c$, $\left[\underline{B}_{ij}\right]_c$ and $\left[\underline{D}_{ij}\right]_c$ are the stiffness matrix of the core about the mid-height of the sandwich construction. And $[A'']_c, [B'']_c, [D'']_c$ are the core laminate stiffness matrix per unit width of inclined core and are found using the same formulation given in equation 7, 8 and 9.

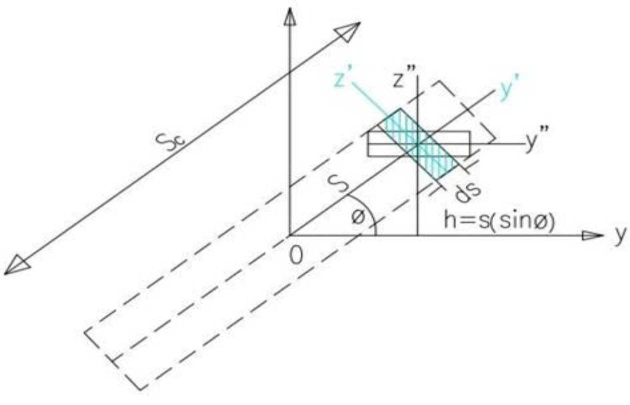

Figure 3: Small section of inclined core about the mid-height

2.4 Structural Smearing of The Sandwich Plate with Corrugated Cores

For composite sandwich plate shown in Figure 4. The core spacing is 2p and the width in the longitudinal direction is a and that in the transverse direction is b.

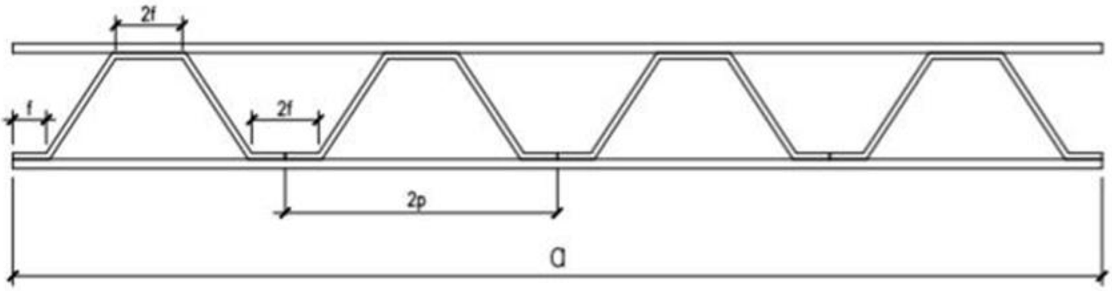

Figure 4: Sandwich plate with corrugated core under combined load

Using the structural smearing analogy, the equivalent in-plane extensional stiffness of the sandwich plate with corrugated cores shown in Figure 4 will be, the sum of the In-plane Extensional Stiffness of the individual face plates and the corrugated cores. This means that the A matrix for the sandwich plate with corrugated cores, which is the In-plane extensional stiffness per unit width is giving by the sum of the corresponding terms of the face plates and the cores considered separately;

$$\left[A^{S*}{}_{IJ}\right]_{equi..} = \left[A_{ij}\right]_{Faces} + \left[A_{ij}\right]_{corr.cores..} \quad (16)$$

With the subscript ij denoting the ij element of the A matrix, also the corrugated cores in-plane extensional stiffness can be defined as:

$$\left[A_{ij}\right]_{corr.cores..} = n_c\left[\underline{A}_{ij}\right]_{single\ ores} \quad (17)$$

Where $\left[A^{S*}{}_{IJ}\right]_{equi..}$, $\left[A_{ij}\right]_{Faces}$, $\left[A_{ij}\right]_{corr.cores..}$, $\left[\underline{A}_{ij}\right]_{single\ cores}$ and n_c are the equivalent in-plane extensional shear stiffness matrix of the sandwich plate with corrugated core, the equivalent in-plane extensional shear stiffness matrix of face plates of the sandwich construction, the equivalent in-plane extensional shear stiffness matrix of the corrugated core, in-plane extensional shear stiffness matrix of single core laminate in the problem coordinate axis and the number of corrugated cores in the sandwich construction.

Determining the number of corrugated cores involves some approximation. If there are corrugated cores right at the edge shown in Figure 4, the number of corrugated cores is given by

$$n_c = int\lfloor \frac{a}{2p} \rfloor + 1 \quad (18)$$

Where $int\lfloor ... \rfloor$ denotes the integer that is obtained when the quantity in the brackets is rounded down to the nearest integer. If the corrugated spacing is small, the second term in the right-hand side of the equation 18 can be neglected and the number of corrugated cores approximated by

$$n_c = \frac{a}{2p} \quad (19)$$

Now theA_{ij} terms for single corrugation can be estimated by averaging the corresponding in-plane extensional shear stiffness over the width (a) in the longitudinal direction.

Where (a) is the width in the longitudinal, For the case of A_{11} this thus gives

$$\left[A_{ij}\right]_{single\ corr.core.} = \frac{\left[\underline{A}_{ij}\right]_{corrugated\ cores}}{a} \quad (20)$$

If there are no corrugated cores at the edge of the top and bottom plate in either side of the plate width (a), equation 19 must be reduced by the total amount of overhanging.

For sufficiently large (a) and / or small 2p equation 19 can be used

Note that equation 18 typically is a rational number as the division a/2p is an integer only for judiciously chosen values of (a) and 2p. but for the purpose of stiffness evaluation, applying the rational number gotten from equation without rounding off is a consideration approximation.

Substituting equations 17 ,18 and 20 into equations equation 16 and noting that a one-dimensional corrugate core has negligible contribution to the stiffnesses other than the one panel to its axis own axis, Thus the A matrix models are as follows.

$$[A^{S*}{}_{11}]_{equi..} \approx [A_{11}]_{Faces} + \frac{[A_{11}]_{corr.cores..}}{2p} \tag{21}$$

$$[A^{S*}{}_{12}]_{equi..} \approx [A_{12}]_{Faces} \tag{22}$$

$$[A^{S*}{}_{66J}]_{equi..} \approx [A_{66}]_{Faces} \tag{23}$$

Following the same analogy in the transverse direction, the A matrix model yields

$$[A^{S*}{}_{22}]_{equi..} \approx [A_{22}]_{Faces} + \frac{[A_{22}]_{corr.cores..}}{2f} \tag{24}$$

Similarly, from the structural smearing analogy, the equivalent out of plane bending and twisting stiffness of the sandwich plate with corrugated cores shown in Figure 4 will be, the sum of the out of plane bending and twisting stiffness of the individual face plates and the corrugated cores. This means that the D matrix for the sandwich plate with corrugated cores, which is the per unit width is giving by the sum of the corresponding terms of the face plates and the cores considered separately;

$$[D^{S*}{}_{IJ}]_{equi..} = [D_{ij}]_{Faces} + [D_{ij}]_{corr.cores..} \tag{25}$$

$$[D_{ij}]_{corr.cores..} = n_c[\underline{D}_{ij}]_{single\ cores} \tag{26}$$

Where $[D^{S*}{}_{IJ}]_{equi..}$, $[D_{ij}]_{Faces}$, $[D_{ij}]_{corr.cores..}$, $[\underline{D}_{ij}]_{cores}$ and n_c are the equivalent out of plane bending and twisting stiffness matrix of the sandwich plate with corrugated core, the equivalent out of plane bending and twisting stiffness matrix of face plates of the sandwich construction, the equivalent out of plane bending and twisting stiffness matrix of the corrugated core, out of plane bending and twisting stiffness matrix of single core laminate in the problem coordinate axis and the number of corrugated cores in the sandwich construction.

The bending stiffness D_{11} for a single corrugated core can be determined by smearing its contribution over the entire width (a):

$$[D_{11}]_{single\ corr.core.} = \frac{[\underline{D}_{11}]_{corrugated\ cores}}{a} \tag{27}$$

While there no contributions to the D_{12} and D_{66} terms because the bending stiffness contribution from the corrugated core is negligible in these directions, the contribution to D_{22} follows the same analogy in the longitudinal direction:

Thus, the D matrix models are as follows.

$$[D^{S*}{}_{11}]_{equi..} \approx [D_{11}]_{Faces} + n_c\frac{[\underline{D}_{11}]_{corrugated\ cores}}{a} \tag{28}$$

$$[D^{S*}{}_{12}]_{equi..} \approx [D_{12}]_{Faces} \tag{29}$$

$$[D^{S*}{}_{66J}]_{equi..} \approx [D_{66}]_{Faces} \tag{30}$$

$$[D^{S*}{}_{22}]_{equi..} \approx [D_{22}]_{Faces} + \frac{[\underline{D}_{22}]_{corr.cores..}}{2f} \tag{31}$$

2.5 Equivalent Model for the Sandwich Construction

To evaluate the overall stiffness of the sandwich plate with corrugated core in Figure 4, the stiffness matrices of each laminate in the laminate has to be translated to the common reference axis before the summation is done. For symmetric orthotropic laminated composite, this occurs at the neutral axis.

2.5.1 Equivalent In-plane Extensional Stiffness

Recalling equation 20 for the sandwich plate with corrugated core in Figure 4 for a single core represented in Figure 2 under consideration, equations 21 to 24 becomes

$$[A^{S*}{}_{11}]_{equi..} \approx [A_{11}]_{Faces} + n\frac{[\underline{A}_{11}]_{corr.cores..}}{a} \tag{32}$$

$$[A^{S*}{}_{12}]_{equi..} \approx [A_{12}]_{Faces} \tag{33}$$

$$[A^{S*}{}_{66J}]_{equi..} \approx [A_{66}]_{Faces} \tag{34}$$

$$[A^{S*}{}_{22}]_{equi..} \approx [A_{22}]_{Faces} + \frac{[\underline{A}_{22}]_{corr.cores..}}{2f} \tag{35}$$

Where n is the number of laminas in the laminate

Substituting equation 13 into equation 17 and then in turn into equations 32 to 35 yields the equivalent in-plane extensional shear stiffness plate model for the sandwich construction are as follows;

$$[A^{S*}{}_{11}]_{equi..} \approx [A_{11}]_{Faces} + n\frac{s_c[A''{}_{IJ}]_{corr.cores}}{a} \tag{36}$$

$$[A^{S*}{}_{12}]_{equi..} \approx \quad [A_{66}]_{Faces} \tag{37}$$

$$\left[A^{S*}{}_{66J}\right]_{equi..} \approx \quad [A_{66}]_{Faces} \tag{38}$$

$$[A^{S*}{}_{22}]_{equi..} \approx \quad [A_{22}]_{Faces} + \frac{s_c[A''{}_{IJ}]_{corr.cores}}{2f} \tag{39}$$

2.5.2 Equivalent Bending and Twisting Stiffness

Recalling equation 20 for the sandwich plate with corrugated in Figure 4 for a single core represented in Figure 2 under consideration, equations 28 to 31 becomes

$$[D^{S*}{}_{11}]_{equi..} \approx \quad [D_{11}]_{Faces} + \frac{[D_{11}]_{corrugated\ cores}}{a} \tag{40}$$

$$[D^{S*}{}_{12}]_{equi..} \approx \quad [D_{12}]_{Faces} \tag{41}$$

$$\left[D^{S*}{}_{66J}\right]_{equi..} \approx \quad [D_{66}]_{Faces} \tag{42}$$

$$[D^{S*}{}_{22}]_{equi..} \approx \quad [D_{22}]_{Faces} + \frac{[D_{22}]_{corr.cores..}}{2f} \tag{43}$$

Substituting equation 15 into equation 26 and then in turn into equations 40 to 43 yields the equivalent bending and twisting plate model for the sandwich construction are as follows;

$$[D^{S*}{}_{11}]_{equi..} \approx \quad [D_{11}]_{Faces} + \frac{\left(\frac{s_c^3}{12}\varphi[A''{}_{IJ}] + s_c[D''{}_{IJ}]\right)_{corr.cires}}{a} \tag{44}$$

$$[D^{S*}{}_{12}]_{equi..} \approx \quad [D_{12}]_{Faces} \tag{45}$$

$$\left[D^{S*}{}_{66J}\right]_{equi..} \approx \quad [D_{66}]_{Faces} \tag{46}$$

$$[D^{S*}{}_{22}]_{equi..} \approx \quad [D_{22}]_{Faces} + \frac{\left(\frac{s_c^3}{12}\varphi[A''{}_{IJ}] + s_c[D''{}_{IJ}]\right)_{corr.cires}}{2f} \tag{47}$$

Equations 36 to 39 and 44 to 47. are used to evaluate the equivalent plate model for the in-plane extensional shear stiffnesses and the out of bending and twisting stiffness in the problem coordinate system for the unit cell in Fig 2.

The left-hand side of equations 36 to 39 and 44 to 47.can be represented as follows;

$$\begin{bmatrix} A_{11}^* & A_{12}^* & A_{16}^* \\ A_{12}^* & A_{22}^* & A'_{26} \\ A_{16}^* & A_{26}^* & A_{66}^* \end{bmatrix} \tag{48}$$

$$\begin{bmatrix} D_{11}^* & D_{12}^* & D_{16}^* \\ D_{12}^* & D_{22}^* & D_{26}^* \\ D_{16}^* & D_{26}^* & D_{66}^* \end{bmatrix} \tag{49}$$

For symmetric lamina about its mid plane, the extension shear coupling stiffness A_{16} A_{26} and the bending twisting coupling D_{16} D_{26} in equations 48 and 49 are zero for a balanced laminate.

Thus equations 48 and 49 become

$$\begin{bmatrix} A_{11}^{S*} & A_{12}^{S*} & 0 \\ A_{12}^{S*} & A_{22}^{S*} & 0 \\ 0 & 0 & A_{66}^* \end{bmatrix} \tag{50}$$

$$\begin{bmatrix} D_{11}^{S*} & D_{12}^{S*} & 0 \\ D_{12}^{S*} & D_{22}^{S*} & 0 \\ 0 & 0 & D_{66}^* \end{bmatrix} \tag{51}$$

3. Presentation of The Equivalent Plate Model

The in-plane extensional shear stiffness and bending and twisting stiffness can be evaluated as follows:

3.1. The In-plane extensional shear stiffness

Based on the classical laminate theory and structural smearing, terms in equation 50 can be defined for the given sandwich plate with corrugated cores using equations 36 and 39 respectively as follows;

The in-plane normal extensional stiffness in the longitudinal direction is:

$$[A^{S*}{}_{11}]_{equi..} = [A_{11}]_{FTP} + [A_{11}]_{FBP} + \frac{n}{a}s_c[A''{}_{11}]_{corr.cores} \tag{52}$$

The in-plane normal extensional stiffness in the transverse direction is:

$[A^{S*}{}_{22}]_{equi..} = [A_{22}]_{FTP} + [A_{22}]_{FBP} + \frac{1}{2f} s_c [A''{}_{22}]_{corr.cores}$ (53)

The in-plane extensional shear stiffness in the diagonal direction

$[A^{S*}{}_{12}]_{equi..} = [A_{12}]_{FTP} + [A_{12}]_{FBP}$ (54)

And the in-plane extensional shear stiffness in the torsional direction

$[A^{S*}{}_{66}]_{equi..} = [A_{66}]_{FTP} + [A_{66}]_{FBP}$ (55)

$where\ S_c$ in equations 54 and 55 is the width of the inclined web of the laminated construction and is equal to $\frac{d_c}{sinsin\ \varphi}$;

3.2. The bending and twisting stiffness

Terms in equation 51 for the given sandwich plate can be defined using equations 44 to 47 follows.

The normal bending stiffness in the longitudinal direction is:

$[D^{S*}{}_{11}]_{equi..} = [D_{11}]_{FTP} + [D_{11}]_{FBP} + \frac{1}{a}\left(\frac{s_c^3}{12}\varphi[A''{}_{11}] + S_c[D''{}_{11}]\right)_{corr.cires}$ (56)

The normal bending stiffness in the transverse direction is:

$[D^{S*}{}_{22}]_{equi..} = [D_{22}]_{FTP} + [D_{22}]_{FBP} + \frac{1}{2f}\left(\frac{s_c^3}{12}\varphi[A''{}_{22}] + S_c[D''{}_{22}]\right)_{corr.cires}$ (57)

The normal bending coupling stiffness

$[D^{S*}{}_{12}]_{equi..} = [D_{12}]_{FTP} + [D_{12}]_{FBP}$ (58)

and the bending twisting stiffness

$[D^{S*}{}_{66}]_{equi..} = [D_{66}]_{FTP} + [D_{66}]_{FBP}$ (59)

Based on the classical laminate theory and structural smearing, terms in equation 26 can be defined for the given sandwich plate with corrugated cores using equations 9 and 12 respectively as follows;

3.3 Implementation of the Classical Laminate Theory, Axis Rotation and Structural Smearing

The following algorithm was developed for the solution of the in-plane extensional shear and out of plane bending stiffness of the sandwich plate with corrugated cores, using the proposed model.

1. Evaluate the material stiffness of each ply. Using equation A in Appendix A
2. Evaluate the transformed stiffness matrix $\bar{Q}_{11}, \bar{Q}_{22}, \bar{Q}_{12}$ and $\bar{Q}_{66}$, using equations B, C, D and E respectively for ply with orientation θ to problem axis
3. Substitute values of $\bar{Q}_{11}$, $\bar{Q}_{22}$, $\bar{Q}_{12}$ and $\bar{Q}_{66}$from equations B, C, D and E into equations 7 in turn, with the calculated thickness of constituent plies to obtain the respective in-plane extensional stiffness for composite laminas.
4. Values from equations 7 for the respective in-plane material stiffness and s_c are then substituted into equations 52, 53, 54 and 55 respectively to get results for the in plane extensional stiffness of the given sandwich construction.
5. Values from equations B, C, D and E, are substituted into equations 9 in turn, with the calculated $t_{k\,,}\ and\ z^2$, of constituent plies to evaluate the respective bending and twisting stiffness in each lamina
6. Values from equations 9 for the respective bending and twisting material stiffness and s_c are then substituted into equations 56,57, 58 and 59 respectively to get results for the bending and twisting stiffness of the given sandwich construction.

4. Numerical Examples

The proposed models are now compared to results from literature and finite element models to determine their effectiveness.

4.1 In-plane and out of plane stiffnesses

The first example is taken from Martinez et al (2012), who used micromechanics method of homogenization an implemented higher order shear deformation plate theory to determine the in-plane and out of plane stiffnesses of an Integrated Thermal Protection System ITPS. with the following parameters, $2p = 160mm, d = 80mm, t_t = 1mm, t_b = 1mm, t_w = 1mm, a = 650mm, b = 650mm$ of graphite/ epoxy composite with $E_1 = 138GPa$, $E_2 = 9GPa, v_{12} = 0.3, G_{12} = 6.9GPa$. with four lamina in each component and stacking sequence of$(0/90)_s$ .with angle of inclination of web ∅=75^{0}shown in Figure 5.

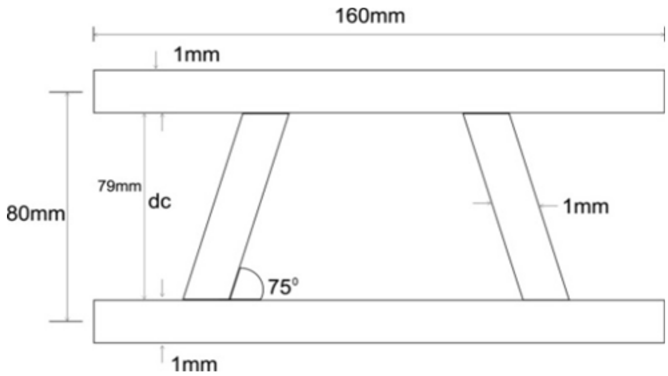

Figure 5: Unit Cell for ITPS Panel to be Sought for Membrane and Bending Stiffness

Table 1 lists the comparison of in-plane extensional shear stiffness values obtained from the proposed model, Martinez et al (2012), and finite element method of Martinez (2012). As seen in the Table, the in-plane-extensional stiffness A_{11}, A_{22}, A_{12} and A_{66}, for the analytical model shows a deviation range of between 0.0 % to 5.8 %, when compared to Martinez model and a deviation range of between 0.0 % to 4.5 %, when compared to the finite element results. This indicates excellent conformity between the FEM, Martinez et al 2012 and the present method.

Table 1: Results for the In-plane extensional Stiffnesses Coefficient [A] of an Integrated Thermal Protection System for the EPM, Martinez and FEM for Web Angle of 90^0

Membrane Stiffness	A_{11} N/m	A_{22} N/m	A_{12} N/m	A_{66} N/m
Analytical	2.10E+08	1.48 E+08	5.43 E+06	1.38 E+07
Martinez	2.23 E+08	1.48 E+08	5.43 E+06	1.43 E+07
FE	2.20 E+08	1.48E+06	5.43 E+08	1.41 E+07
% diff. (Martinez)	5.8 %	0.0 %	0.0 %	3.5 %
% difference (FE)	4.5 %	0.0 %	0.0%	2.1 %

Table 2 shows the comparison of bending and twisting stiffness values obtained from the proposed model, Martinez et al (2012), and Finite Element Method of Martinez (2012). In this Table, the bending and twisting stiffnesses D_{11}, D_{22} D_{12} and D_{66} of the proposed method shows a deviation range of between -2.2 % to 4.1 % when compared to Martinez et al (2012) and a deviation range of between -1.4 % to 3.8 %, when compared to the Finite Element Method. This shows good agreement between the FEM, Martinez et al (2012) and the present model.

Table 2: Results for the bending and twisting Stiffnesses [D], Coefficient for an ITPS Sandwich Panel Using the EPM for Web Angle of 75^0at a = 0.65 and b = 0.65

Bending Stiffness	D_{11} Nm	D_{12}Nm	D_{22} Nm	D_{66} Nm
Analytical	2.82E+05	8422	2.28 E+05	21,407
Martinez	2.76E+05	8790	2.37E+05	22,327
FE	2.78E+05	8690	2.37E+05	22,200
% diff. (Martinez)	-2.2 %	4.1 %	3.8 %	4.1%
% difference (FE)	-1.4 %	3.1 %	3.8%	3.6%

4.2 Equivalent properties of sandwich plate.

The second example is taken from Martinez et al (2012) and Rajesh (2014) to sought the equivalent properties of an Integrated Thermal Protection System ITPS. (Martinez et al 2007) with the following geometric and material parameters $\emptyset = 90^0,\ 2p = 160mm, d = 80mm, t_t = 1mm, t_b = 1mm, t_w = 1mm, a = 650mm, b = 650mm$ of graphite/ epoxy composite with $E_1 = 138GPa\ , E_2 = 9GPa, v_{12} = 0.3, G_{12} = 6.9GPa.$ with four lamina in each component and stacking sequence of$(0/90)_s$ in Figure 5.The former used micromechanics method of homogenization an implemented higher order shear deformation plate theory and the later used minimum potential energy .This problem is solved here using the present method.

Table 3 and 4 lists the comparison of in-plane extensional shear stiffness and the out of plane bending and twisting stiffness values obtained from the proposed model, Martinez et al (2012), Rajesh (2014) and Finite Element Method of Martinez (2012). As seen in Table 3, the in-plane-extensional stiffness A_{11} A_{22} A_{12} and A_{66}, for the analytical model shows a deviation range of 0.0 % to - 5.3 % for Rajesh, 0.0 % to -3.8 % for Martinez and 0.1 % to - 4.7 % for the finite element method (2012).In Table 4 the bending and twisting stiffness $D_{22,}$ $D_{12,}$ and $D_{66,}$ for the analytical model shows a deviation range of between 3.1 % to 4.0 % when compared to Rajesh, 0.4% to 5.4 % when compared to Martinez and a deviation range of between 0.2% to 4.5%, when compared to the Finite Element Method (2012), which is also a good agreement between the proposed model and other methods except for the D_{11} where the percentage difference is -14.0%, -13.0 % and -13.0 % for Rajesh, Martinez and FEM respectively.

Table 3: Results for the In-plane extensional Stiffnesses Coefficient [A] of an Integrated Thermal Protection System for the EPM, Martinez and FEM for Web Angle of 90^0

Stiffness	Analytical	Rajesh	Martinez	FEM	Diff.Raj.	Dif.Mar.	Dif.FEM.
$A_{11}\,N/m$	2.325 E+10^8	2.208 E+08	2.24 E+10^8	2.22 E+10^8	-5.3%	-3.8%	-4.7%
$A_{22}\,N/m$	1.478 E+10^8	1.478 E+08	1.48 E+10^8	1.48 E+10^8	0%	0.1%	0.1%
$A_{12}\,N/m$	5.432 E+10^6	5.431 E+06	5.43 E+10^6	5.43 E+10^6	0%	0%	0%
$A_{66}\,N/m$	1.38 E+10^7	1.38 E+07	1.43 E+10^7	1.41 E+10^7	0%	3.5%	3.5%

Table 4: Results for the bending and twisting Stiffnesses [D], Coefficient for an ITPS Sandwich Panel Using the EPM for Web Angle of 90^0at a = 0.64 and b = 0.64

Stiffness	Analytical	Rajesh	Martinez	FEM	Diff.Raj.	Dif.Mar	Dif.FEM
$D_{11}Nm$	3.132E+10^5	2.745 E+05	2.76 E+10^5	2.77 E+10^5	-14%	-13.1%	-13.1%
$D_{22}Nm$	2.283 E+10^5	2.37E05	2.37E+10^5	2.67 E+10^5	3.1%	4.1%	3.1%
$D_{12}Nm$	8422	8700	8788	8691	3.2%	0.4%	0.2%
$D_{66}Nm$	21,110	22,000	22,327	22,100	4%	5.4%	4.5%

4. Conclusion

The complexity of models of composite structural systems, resulting from homogenization approaches, based on the various micromechanics techniques, shear deformable theory, minimum potential energy and Euler Bernoulli solutions are recognized. The aim of this study is to develop a simplified reliable model for analysis and design of composite sandwich plates with corrugated cores.The proposed model is based on the combination of the classical laminate theory of homogenization, axis rotation and structural smearing on the basis of the analytical results from the proposed model in comparison with the Finite element method and result from other researchers, the following conclusions and recommendation were arrived at. The proposed model presents a considerably simpler alternative for the analysis of the behaviour and response of sandwich plate with corrugated cores compared to the existing models. The proposed model reliably reproduced the in-plane extensional shear and out of plane bending and twisting stiffnesses of the corrugated sandwich plate with striking accuracy.

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Appendix A

$Q_{11} = \frac{E_1}{1-v_{12}v_{21}} \qquad Q_{22} = \frac{E_2}{1-v_{12}v_{21}} \qquad Q_{12} = \frac{E_2 v_{12}}{1-v_{12}v_{21}} Q_{66} = G_{12}$ A

$\bar{Q}_{11} = ¢_1 + ¢_2 \cos 2\theta + ¢_3 \cos 4\theta$ B

$\bar{Q}_{12} = ¢_4 - ¢_3 \cos 4\theta$ C

$\bar{Q}_{22} = ¢_1 - ¢_2 \cos 2\theta + ¢_3 \cos 4\theta$ D

$\bar{Q}_{66} = ¢_5 - ¢_3 \cos 4\theta$ E

Where the ¢'s are the linear combination of material stiffness defined in equation F

$$¢_1 = \frac{(3\ Q_{11} + 3\ Q_{22} + 2\ Q_{12} + 4\ Q_{66})}{8}$$

$¢_2 = \frac{(Q_{11} - Q_{22})}{8}$

$¢_3 = \frac{(Q_{11}+ Q_{22}-2 Q_{12}-4 Q_{66})}{8}$ F

$¢_4 = \frac{(Q_{11}+ Q_{22}+6 Q_{12}-4 Q_{66})}{8}$.

$¢_5 = \frac{(Q_{11}+ Q_{22}-2 Q_{12}+4 Q_{66})}{8}$.

$where\ S_c$ in equations 3.76 and 3.79 is the width of the inclined web of the laminated construction and is equal to $\frac{d_c}{sinsin\ \varphi}$;