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Research Article

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Effect of Real Gases of the Van Der Waals Type on Shock Waves

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Abstract In this article we studied the influence of the nature of the real gas of the type of Van Der Waals on the shock wave. The resolution of the polynomial of degree 4 by FORTRAN by applying the algorithm of Newton. The equations that govern our problem and adimensionalized them, allowed us to see that the number pro and He have an influence on the wave. These numbers play on the value of the Mach number of the formation of the shock and its intensity of the shock increases with the number of mach. But also, the more the additional number pro increases the more the appearance of the shock is done with increasingly high number of mach. We note the same result for the additional number he.

Keywords shock wave, compressible flow, Mach number, Van Der Waals gas, effect of real gases

1. Introduction

The shock wave is a large overpressure that travels through the fluid at a speed close to sound. It is an irreversible process, because there is dissipation of energy. This dissipation shows the difference between shock and sound waves. The importance of understanding these phenomena is demonstrated by the number of pilots who have lost their lives trying to break the sound barrier (destruction of the aircraft by vibrations, incomprehensible loss of lift, among others).

To understand these phenomena, studies have been carried out. We can cite the work of Draine and Mc Kee [3] on the variation of temperature due to the shock wave. Landau and Liftshtz (1959) [4], studied the attached shock, the detached shock. For the study of the shock wave in MHD [9], we can cite the work of Roberge and Draine (1990). The influence of shock on chemistry has been studied by Flower et al [6]. Mc Kee and Hollenbach (1980) studied interstellar shocks [8]. Despite everything [4, 2, 5, 7, 1], these studies, shock waves remain mysterious objects.

To fill the lack of information on the wave, more realistic models that will be able to reproduce reality as closely as possible should be taken into consideration. But the resolution becomes more and more complex. We have seen that shock waves arise in several situations such as in aerodynamics, oceanic flows, entering the astrophysical atmosphere, detonation flow in a pipe (supersonic propulsion). The shock wave generates pressure losses resulting in a loss of performance of the jet engine. It also plays on the lift and the drag in these interactions with the boundary layers. The shock wave varies the temperature, generates tremors, etc. This shows us the importance of better understanding and controlling this phenomenon which has an energy stake; economical, safe, comfortable and technical.

Therefore, we studied the shock wave by including the effect of real gases, contained in the Van Der Waals equation of state. This equation takes into account the volume of the atoms and their interactions, unlike the ideal gas. For the first time, the effect of real gases of the Van Der Waals type is taken into account in the study of the shock wave.

2. Mathematical Model

Shock wave analysis is based on the concept of a fixed pressure wave. The upstream and downstream states of the shock wave are denoted by the subscripts 1 and 2 respectively.

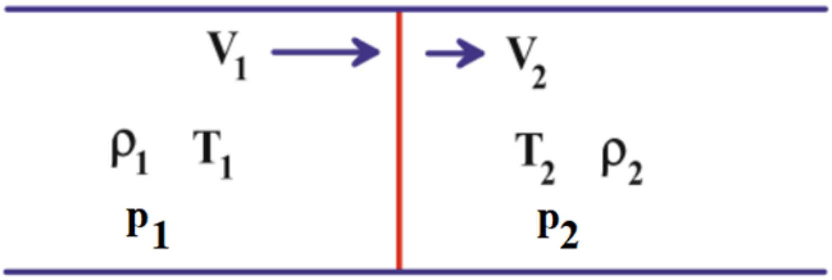

Figure 1: physical model

For the modeling of the problem, we make the following assumptions:

-steady flow

-the viscous and gravity terms are negligible compared to the pressure and convective terms.

-The thickness of a shock wave Δx is so small (approximately on the order of a micron) that it is assumed to have no cross-section change (in a variable-section pipe), hence $A_1 \simeq A_2$ and the equation of continuity is written

$\rho_1 v_1 = \rho_2 v_2 = cte$

the momentum equation is:

$p_1 - p_2 = \rho_2 v_2^2 - \rho_1 v_1^2$

The energy equation:

$e_1 + \frac{p_1}{\rho_1} + \frac{v_1}{2} - q_1 = e_2 + \frac{p_2}{\rho_2} - q_2$

$e = c_v T - a\rho$

the equation of state;

$p = \frac{rt\rho}{1-b\rho} - a\rho^2$

he term $a\rho^2$ is the internal pressure modeling the interaction between atoms (molecules).

b is the molar covolume which represents the volume of the atoms.

In the rest of the study, these terms will be grouped in ad dimensional numbers when transforming the equation into a unitary equation. Assuming the upstream conditions (p_1 , v_1 , ρ_1 , T_1 , e_1) and q∗ = q_2 −q_1 are known, the preceding equations present 5 algebraic relations with unknowns (p_2 , v_2 , ρ_2, T_2 , e_2). use the existence of at least two solutions. The good one among them is according to the second law of thermodynamics, which requires $s_2 > s_1$.

Rearranging and eliminating ρ_2, e_2 , p_2 and T_2 in the equations, on the velocity jump relationship between pre-shock and post-shock flow:

$$-\left(\frac{c_v}{R}+1\right) v_1^2 \left(\frac{v_2}{v_1}\right)^4 + \left[\frac{c_v}{R}\left(1+\frac{R}{c_v}+b\rho_1\right) v_1^2 + \frac{p_1}{\rho_1}\left(1+\frac{c_v}{R}\right)\right]\left(\frac{v_2}{v_1}\right)^3 - \left[\left(\frac{c_v}{R}b\rho_1+\frac{1}{2}\right) v_1^2 - ab\frac{c_v}{R}\rho_1^2 - a\rho_1\left(-\frac{c_v}{R}+1\right) + \left(\frac{c_v}{R}+1\right)\frac{p_1}{\rho_1} + q*\right]\left(\frac{v_2}{v_1}\right)^2 - \left(-\frac{c_v}{R}a\rho_1 + a\rho_1\right)\left(\frac{v_2}{v_1}\right) - \frac{c_v}{R}ab\rho_1^2 = 0$$

Which we have rewritten as:

$x^4 + \alpha x^3 + \beta x^2 + \gamma x + \omega = 0$

With:

$x = \frac{v_2}{v_1}$

$\alpha = -\frac{1+g+pro}{g+1} - K\left(\frac{1}{1-pro} - \frac{H}{2}\right)$

$\beta = \frac{2pro+gk\times h(pro+g-1)}{2(g+1)} + \frac{k}{1-pro} - \frac{hk}{2} + \frac{qg}{v_1^2(g+1)}$

$\gamma = \frac{hk(g-1)}{2(g+1)}$

$\omega = \frac{hkpro}{2(g+1)}$

$pro = \rho_1 b$, the term pro compares the mass volume with respect to the mass covolume of the gas

$he = \frac{a\rho_1}{v_1^2}$, the term he compares the internal pressure with respect to the dynamic pressure

$Ge = \frac{R}{c_v}$, Ge compares the ratio of the gas constant and the calorific coefficient at constant volume

$K = \frac{RT_0}{V_0^2}$, The parameter K compares the pressure of the ideal gas with compared with the dynamic pressure

Based on Van der waals model studies, we found that the addimensional numbers pro,he have relatively low values

$10^{-5} \le pro \le 1$; $10^{-6} \le HE \le 10^{-2}$ et finalement $10^{-3} \le Ge \le 10^{3}$

3. Numerical Method

To solve the equation numerically, we applied Newton's algorithm which is quadratically convergent. Calculating an approximate value of a root of the equation $f(x) = 0$ by Newton's method amounts to confusing the function with its tangent $y = f'(x_0)(x - x_0) + f(x_0)$; and find its point of intersection with the abscissa axis. Hence we have:

$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$

If there are multiple roots, the convergence test $(x_n - x_{n-1})$ does not allow access to them. Reason why we have two convergence tests. The first is given by $x_n - x_{n-1}$ and the second by the product $f(x_{n-1}) \times f(x_n)$ if it is negative, we applies Newton's method to the equation.

When the parameters a and b are zero, the Van der Waals equation degenerates into the perfect gas equation. To validate our work, we chose the pro and he parameters as null; this amounts to taking a and b null.

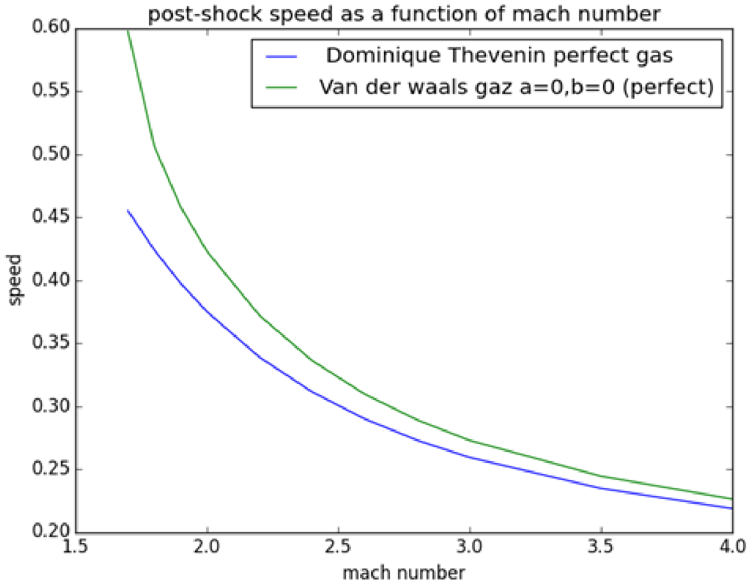

Figure 2: Ideal Gas Evolution

Dominique Thevenin [10] presents supersonic flow and shock tables for $\gamma = 1.4$. From this table, we collected and plotted the evolution of the post-shock speed as a function of the Mach number. That we compared with our study for $ge = 0.4$ we see that we have the same evolution. The difference is due to the error of the method and this error converges towards zero.

4. Results and Discussion

The results obtained are presented for the shock wave with the Van der Waals equation of state. The various tests presented in this study concern the influence of the nature of the gas on the wave. We varied the Mach number from 1.1 to 3 to determine the speed and pressure downstream of the shock wave. The study relates to six gases defined by a combination of the numbers pro and he.

Table 1: Study Gases

gaz	Pro	he	Ge
ideal gas(air)	0	0	0.4
gaz 1	0	0.005	0.005
gaz 2	0.05	0	0.005
gaz 3	0.05	0.005	0.005
gaz 4	0.07	0.006	0.005
gaz 5	0.07	0.006	0.01
gaz 6	0.1	0.005	0.005

Figure 2 represents the additional velocities and pressures, for four gases, as a function of the Mach number. We notice a decrease identical to that of the ideal gases already available in the literature. When the gases have the same number pro, but different numbers he, the two curves overlap almost perfectly in figure 2 and figure 3. On the other hand, when the gases have the same number he and different numbers of pro; the curves are shifted (figure 2.). We notice that for a Mach number close to 1.8 there is a decoupling between the intensity of the wave and the nature of the gas (the curves intersect at this value). When the number pro increases, the shock wave forms for a larger Mach number figure 3 and figure 4.

Larger pro numbers (relative to the volume of the molecules) delay the formation of the shock depending on the Mach numbers. They have a greater intensity up to a certain value of the Mach number. Then their intensities fall below that of the smaller pro numbers (figure 2). The he number (relating to the internal pressure) has a weak influence on the formation and intensity of the shock (ideal gas and gas 1). The number pro strongly influences the formation, intensity and evolution of the shock according to the Mach number.

And for the cases where there is a delay on the appearance of the shock (figure 4) from 1.3 to 3. In the extreme cases, we needed a Mach number of the order of 2 for the wave of shock is formed.

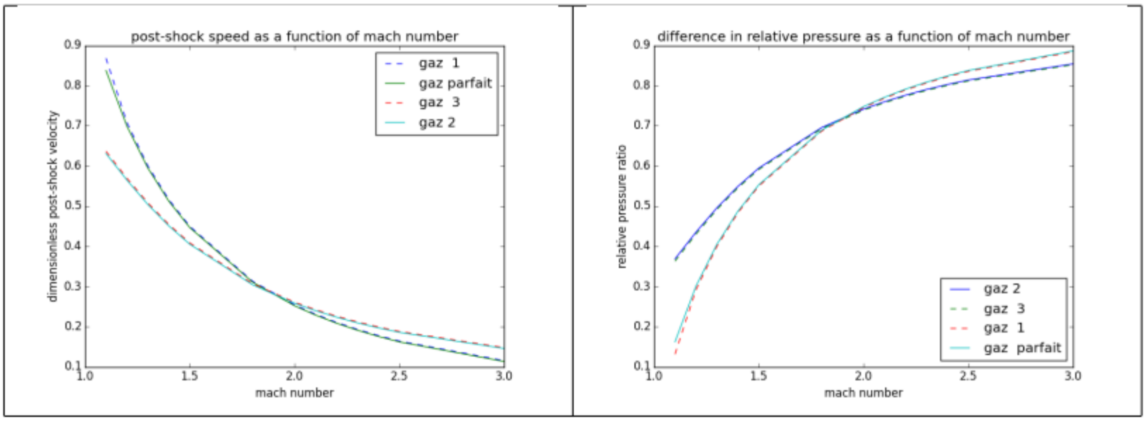

Figure 3: Evolution of the intensity of the shock

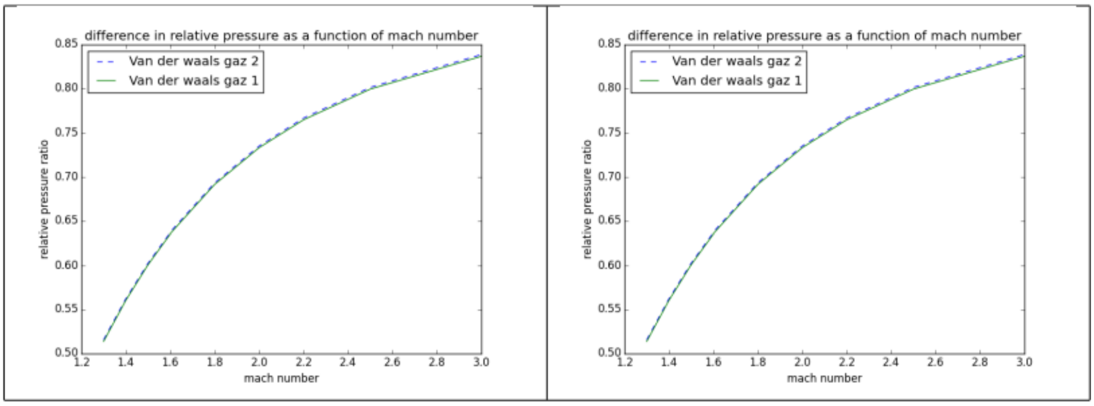

Figure 4: The influence of the variation of pro

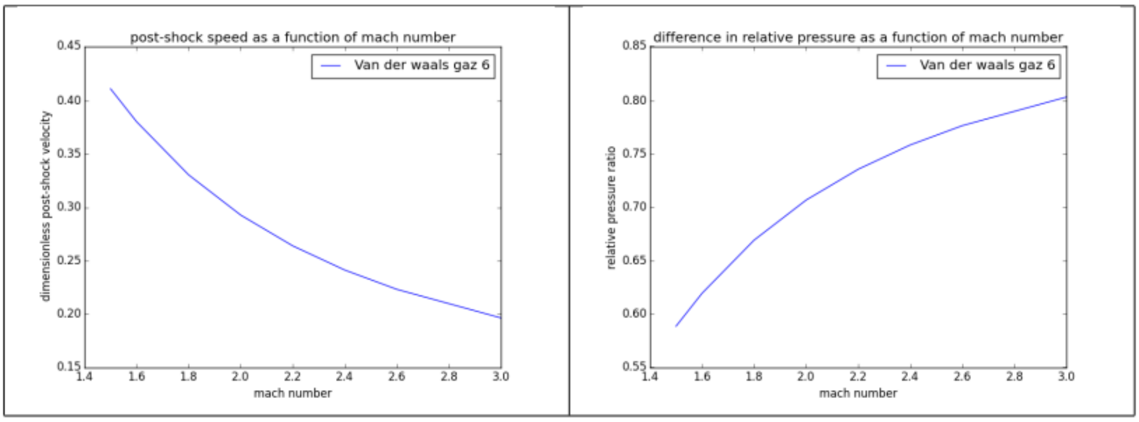

Figure 5: Normal shock wave

The effect of the real gases influences the nature of the shock. This limits the ideal gas model for its study. According to the value of the parameters pro (b and a respectively), the wave is only formed for a larger Mach number than usual, $Ma = 1.3$ and $Ma = 1.5$ (figure 3 and 4.) but with a high intensity.

The greater the effect of the real gases, the more the formation of the wave moves towards increasingly large Mach numbers (figures 2,3 and 4). And during training, the intensity of the shock increases with the values of the parameters pro and he (a and b of the Van der Waals equation). The nature of the shock wave strongly depends on the parameter pro which will be able to explained by the fact that the formation of the wave depends on the compressibility of the fluid.

5. Conclusion

In this study, we studied the influence of the nature of the gas on the shock wave. The effect of real gases is modeled by the Van der Waals equation of state. The equation obtained was solved numerically by Newton's algorithm. In this study, we varied the Mach number and the pro and he numbers. We have seen that the number pro relating to the volume of the molecules has a strong effect on the nature of the shock wave, unlike the number he relating to the internal pressure. The coupling of the effect of real gases on the shock wave, shows us that the ideal gas model is limited for the study of the shock wave. Looking at the study we can say that the effect of real gases delays the formation of the shock wave. Unlike ideal gas; its intensity is greater. This is in agreement with the fact that the intensity of the shock increases with the Mach number. Our model, allowed us to find that the effect of the real gases modifies the flow in a significant way.

6. Nomenclature

$\alpha, \beta, \gamma, \omega$: the coefficients of the quartic equation

ρ_1 : upstream density (Kg/m^3)

ρ_2: downstream density ($\mathrm{Kg/m^3}$)

A: the straight section (m^2)

q∗: heat exchanged with the outside (J)

c_v: heat capacity at constant volume (J/(Kg.K)

e: internal energy (J/Kg)

he: the ratio of internal pressure to dynamic pressure

p_1: upstream pressure (Pa)

p_2: downstream pressure (Pa)

pro: ratio of mass volume to mass covolume of gas

s: entropy(J/K)

v_1: upstream speed (m/s)

v_2 : downstream speed (m/s)

x : the dimensionless downstream velocity

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