# Optimizing the Allocation of Seats for Trains through the Self-Organizing Map and Symbiotic Organisms Search 

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#### Abstract

The revenue generated from both passenger and cargo transportation is the most important source of income for the train operators. The passenger transportation is a critical mission of the train industry for undertaking the public transportation. The suitable allocation of seats for a train is a crucial element that can affect the revenue from the passenger transportation since the allocation of seats directly relates to the ride rate of the train, thus influencing the profit coming from the operation of a train. However, the ride rate is affected by many factors, e.g. the train type, train schedule, stops, and the demand of passengers between two stops, etc. Hence, the problem of allocating seats for trains becomes a very complex optimization problem that is difficult to resolved by the traditional tools. However, few managers of trains or scholars have conducted sufficient research although the supervisor of trains did know that the allocation of trains' seats is an extremely significant issue in their operation and management. In this study, the Self-Organizing Map (SOM) neural network and Symbiotic Organisms Search (SOS) optimization algorithm are used to develop an optimization procedure to resolve the problem regarding the allocation of a train's seats. The effectiveness and efficiency of the proposed approach is verified by conducting a case study for trains operated by the Taiwan Railway Administration (TRA) of the Ministry of Transport in the Republic of China. The experimental results show that the procedure proposed in this study can yield the superior allocation of seats according to the different characteristics of passengers' demands, thereby bringing the higher operating profits and reputation for the managers of trains.


Keywords Allocation of seats, Self-Organizing Map (SOM), Symbiotic Organisms Search (SOS), Taiwan Railway Administration

## Introduction

The transportation capability of trains is influenced by lots of factors, e.g. the total number of available trains, the formation of train types, the number of tracks in an operating range, the lengths of blocks, the block signaling methods, etc. The managers of trains will be faced with a great loss as well as their reputation in both passenger and cargo transportation when the transportation capability of trains cannot meet the demands of passengers and freight forwarders. In this situation, passengers must frequently compete for tickets especially for the festivals and popular destinations. Therefore, the manager of trains must attempt to optimize the allocation for the trains' seats such that the revenue from passenger transportation can be maximized as well as reducing the loss of reputation. The allocation of trains' seats can directly affect the occupancy rate of trains, that is influenced by many elements such as train types, days and time of operation, stops, passengers' demand between stops, etc., thus yielding different operating profits for trains. Therefore, the problem of optimizing the allocation of seats has become a difficult problem that cannot be easily resolved by the traditional tools, e.g. linear programming. Even though the managers of trains realized that the seat allocation of a train is a critical topic in their operation and management, relatively few practitioners or researchers regarding trains' operation have conducted in-depth and sufficient
study. For example, [1] investigates the problem for simultaneously optimizing the seat allocation and determining the overbooking levels for passengers of two different classes in an airplane by considering the airline seat allocation between high and low fares, as well as reflecting the situations with or without stochastic passengers' cancellations. In addition, three elements, that have not been explored in the previous studies, including (1) the cost of lost sales, (2) the overbooking phenomenon, and (3) embedding the clarification of the concept of spill rate into the spill rate of passengers and flight rate. Based on to the analytical outcome, the results provided by this research are highly closer accord with the airline practice actually. [2] considers the demands for the classes are stochastically dependent to resolve the allocations of airline seats between two nested fare classes. They relax the assumption of statistical independence between demands in the well-known simple seat allocation formula of Littlewood. A much weaker monotonic association assumption is required in their proposed method. The problem of full passenger spillage and passenger upgrades from the discount class is used to validate their method, and adequate results are obtained. [3] proposes an approach for determining the optimal allocation of airline seats among itineraries in a network when the demands are stochastic through assigning some seats exclusively to each single- or multi-leg itinerary as in the fixed assignment, as well as allocating the remaining seats to groups of itineraries that is similar to the bucket control. Their study no only formulates the flexible assignment method, but also develops the rules for determining the optimal assignment in the situation with a single-fare-class network and further prove the superiority of their proposed approach for resolving problems with either fixed assignment or bucket control. The demonstration of their proposed approach is made by illustrating a numerical simulation to show the differences in the expected revenue among three approaches for controlling the seat inventory in the demand scenarios of four types. [4] considers an optimization problem to determine the most appropriate booking policies when an airplane has multiple fare classes, which utilize the same seating pool together on one leg of an airline flight and the seats are booked in a nested fashion. In addition, the lower fare classes are booked before the classes with higher fares. They characterize a fixed-limit booking policy that can maximize the expected revenue through using a simple set of conditions on the subdifferential of the expected revenue function. Both the discrete and continuous demand cases can be appropriate to their conditions. Furthermore, they simplify these conditions into a set that is related to the probability distributions of demand for the different fare classes corresponding to their respective fares. Notably, a solution can be guaranteed to obtain in the latter conditions while the demand's joint probability distribution is continuous. Next, the optimality of the fixed-limit policy over all admissible policies can be proved by characterizing the problem into a series of monotone optimal stopping problems. The authors also compare their acquired optimal solutions to the approximate solutions provided in the study of [5] based on the expected marginal seat revenue (EMSR) method. [6] proposes a multiobjective model to plan the optimal allocation of train seats on an intercity rail line which serves passengers with multiple pairs of origindestination. Maximizing the total passenger revenue for the train operator, as well as minimizing the total discomfort level of passengers are simultaneously considered in their study. They apply the fuzzy mathematical programming to generate a plan for the best-compromise allocation of train seats under a given set of traveler's demands, train's capacity, and stop-schedules. According to the plan, the allocations of reserved and non-reserved seats at each origin station for all subsequent destination stations for each running train that is operated during a specified operating period can be determined. A Taiwan high-speed rail system under construction is taken as an empirical case study to demonstrate the effectiveness of their propose model. In addition, their model can be applied to any combination with various setting of traveler's demands and stop-schedules with different capacities of train seats. [7] proposes a semi-dynamic pricing and seat allocation model to improve the shortcoming of static pricing approach such that the tickets are offered at a limited number of price levels determined in advanced, as well as the ticket price is only allowed to switch over time monotonically, i.e. in an increasing order. They intend to maximize the revenue by determining the number of seats which can be sold at different discount fares. The flexibility of a certain degree for pricing is allowed in their study while compromising the maximization of the potential revenue yielded by the dynamic pricing approach. Based on the experimental results of numerical examples, their proposed approach can generate near-optimal revenue for the cases that allow dynamic and reverse price changes. Furthermore, the heuristic method can perform well in general. [8] proposes a model for allocating airline seats by exploring the problem that can provide multiple fare classes, as well as can replenish for the lower fare classes while the number of the pre-allocated seats exceeds the demands for classes with the higher fare and the lower fare classes will be opened again with a certain discount of ticket prices before the airplane departures. Otherwise, the discount of ticket prices will not be allowed and the replenishment is prohibited from the viewpoint of revenue management when the demands of classes with high fares are large enough. In their study, the cancellation of customers' reservations is allowed to make their proposed model formulation be much closer to the airlines practice. The revenue with the replenishment policy can be expected to be greater than the one obtained without the replenishment or with a simple optimal booking policy under some conditions. In addition, the analytical properties of the revenue function and optimal policy are confirmed by illustrating several numerical examples in their research. [9] studies a model to investigate a case where two flights between two cities in a day
are provided, as well as the booking requests for the class of each fare are arrived randomly. The authors consider three types of booking requests including (1) the type only for the first flight, (2) the type only for the second flight, and (3) the flexible type suitable for taking either flight. The airline must decide whether to accept the booking when the customer's request has come, as well as determine which flight to accommodate the customer when the booking of the third type is accepted. They apply four monotone switching curves to reveal the structure of optimal booking policies. In addition, their basic model is also extended to discuss a case with multiple-flight. The derivation of their model, as well as the dynamics of the proposed optimal booking policies are demonstrated by using a numerical example. [10] investigates the revenue management problem of a single-leg airline where the arrivals follow the Poisson distribution in continuous time. Instead of using the traditional Hamilton-JacobiBellman equation, they apply a probabilistic approach, that does not rely on the smoothness of the value function, to build the value function as well as study its properties through using a continuous-time discrete-event dynamic programming operator. The analysis of the differentiability for the value function can be achieved through their proposed approach. In addition, they prove that the differentiability might break down when the arrival intensities are discontinuous, thus researchers should have much more caution when using the arguments based on the differentiability of the value function and the Hamilton-Jacobi-Bellman equation. [11] develops two stochastic programming models to formulate the single-stage and multi-stage decision making problem by considering passengers' choice behavior. In their study, the seats are allocated for maximizing the passenger rail revenue through determining the optimal allocation quantity of seats for each cabin class in each train service. In addition, they resolve the easier equivalent deterministic mathematical programs that are transformed from the stochastic models. A variety of seat allocation polices derived from the optimal solutions for the seat allocation models are also provided. Their proposed policies are validated by making several simulation tests, and appropriate results are obtained. [12] apply the equivalence charging transformation [13] and marginal revenue transformation [14] to obtain an equivalent readily-solved independent demand model through transforming the joint seat allocation and overbooking problem for fare families fare structures. In their study, the dynamic programming (DP) model with high dimensions is converted into its equivalent DP model of low dimensions, that can be directly resolved and implemented by the existing revenue management and inventory systems. In addition, the factors that were frequently ignored in the previous studies, e.g. the demand level, the effects of mixing classes, the cost from the booking class specific refund and from the rates of booking class specific cancellation, are considered into their proposed DP model. According to the simulation results by comparing the solution for the joint seat-allocation and overbooking problem with current industry practice and document, a significant revenue gain of $1 \%-3 \%$ can be obtained. [15] proposes an optimization method to resolve the train seat inventory control problem that considers the multiple trains and multiple levels of seats simultaneously with the aim of maximizing the total revenue of rail industry through formulating an integer linear programming model. They employ the MATLAB with CPLEX solver to yield the approximate optimal solutions, and two examples including a simple railway corridor and Wuhan-Guangzhou high-speed railway corridor are used to verify the effectiveness and performance of their proposed approach. In addition, the impact on the revenue while changing the model parameters is also analyzed by conducting the sensitivity analysis experiments. [16] considers a problem that maintains a uniform load on carriages by the systematic distribution of passengers with flexible tickets to explore the possibility of minimizing the boarding/alighting time. The flexible tickets might be season or anytime tickets that cannot provide seat information when passengers reserve seats. Furthermore, some other information, e.g. the passenger final destination, uniform load of luggage areas, and group travelers, is considered in their proposed model. The performance of the proposed method is evaluated by designing a discrete event simulation. Three algorithms with various test scenarios are also compared, and the experimental results indicate that their proposed method is superior based on minimizing the boarding/alighting time and increasing the success rate of acquiring group of seats for group of passengers. [17] proposes a nonlinear programming model for simultaneously optimizing the pricing and seat allocation in high-speed rail (HSR) networks, which are the complementary strategies for the revenue management ( RM ) in the railway industry and are typically considered independently in most previous studies. In their proposed method, the multistage and discriminatory pricing strategies are also applied to attract more passengers thus improving total revenue. A solution algorithm for large-scale joint optimization of the HSR pricing and seat allocation is designed in their study by employing the Davidon-Fletcher-Powell approach. Furthermore, the uncertain impacts regarding the model inputs on the outputs is analyzed by using the sensitivity analysis. A trade study is made to illustrate the advantages of their proposed joint RM strategy by comparing to different RM strategies. The demonstration results of a large-scale instance in the real HSR network proves that their proposed model and solution algorithm can indeed yield useful decision support for the daily operation and management of railway companies. [18] develops a probabilistic nonlinear programming model for a high-speed railway passenger service network where there are multi trains that have different stop schedule plans, as well as the train composition is flexible to relaxe the assumption of fixed capacity in the classical revenue management. The authors apply the ILGO CPLEX to solve an equivalent linear programming, that is transformed from their
proposed model to accelerate the solving process. In addition, both of stochastic demand and passenger choice behavior are considered at the same time when making decisions regarding the seat inventory control and train composition. From the numerical experimental results, the policy under flexible train composition is proven to be better than that under fixed train composition. Furthermore, the demand intensity, fare classes as well as elasticity of demand can significantly affect the policy through making the sensitive analysis. Their proposed model can assist the railway operator to form the decision-making basis for both of the discount sales and ticket allocation under flexible train composition. [19] converts the train seat scheduling problem into a parallel machine scheduling problem that is further redefined considering the scheduling perspective by considering the issues of negative customer satisfaction and revenue management due to the improper scheduling of passengers or dwell time of trains. They then develop a mathematical model and a heuristic algorithm for their mentioned problems. Based on their demonstration, their proposed algorithm can provide a feasible plan for scheduling in a reasonable time scale by simultaneously considering the dwell times and a proper scheduling plan. The comparison obtains adequate outcomes and proves the superiority of their approach. [20] investigates the problem of train overcrowding control in ticket allocation of high-speed railway (HSR) trains. The authors first build an optimization model of ticket allocation aiming to maximize the revenue where multiple trains and multiple stops are considered. Then, they estimate the number of passengers under the risk coefficient represented by the probability, thus obtaining the total number of passengers that arrive at any station on each train based on the concepts of the travel extension and risk coefficients. In addition, a constraint for preventing the number of passengers on the train from exceeding the capacity of a train is also introduced in their ticket allocation optimization model. The particle swarm optimization (PSO) algorithm is finally utilized to resolve the mathematical model. Based on numerical verifications of China HSR, the practical feasibility of their research has been illustrated by controlling the number of passengers on the train so as not to exceed the capacity of trains. Hence, the balance between revenue maximization and passenger riding experiences can be achieved at the same time. The comparison with those methods of ticket allocation for only maximizing the revenue in the literatures, their study can provide a new way of solving the problem of train overcrowding for the railway operation department, thus effectively improving the safety of train operation.
From the above literature review, various kind of heuristic optimization algorithms had been successfully applied to resolve problems of allocating seats for a train. However, the day and time regarding the operation of a certain train, that is a crucial factor influencing the distributions of passenger's demands thus affecting the policy for allocating seats of a train, had not been considered thoroughly. Therefore, this study aims to proposed a systematic approach based on the self-organizing map (SOM) and symbiotic organisms search (SOS) to simultaneously maximize the revenue of a train's operation, as well as minimizing the reputation's loss due to the discontent with passengers' demands of seats. The problem of seat allocation is first constructed and formulated as a mathematical model. Next, the SOM technique is utilized to divide the daily demands of passengers during a week into several clusters according to the characteristics of demands for each train's operation. The SOS algorithm is finally applied to resolve the constructed optimization model for the passengers' demands grouped in each cluster for a train's operation. The remaining sections are organized as follows. Section 2 presents the optimization model of seat allocation considered in our study. The clustering and optimization methods including the SOM and SOS are briefly introduced in Section 3. Section 4 depicts our proposed approach for resolving the optimization problem regarding a train's seats. In Section 5, the effectiveness and efficiency of the proposed approach is verified by making a case study on the trains operated by the Taiwan Railway Administration (TRA) of the Ministry of Transport in the Republic of China. Finally, the conclusions are provided in Section 6.

## Methodologies

## Self-Organizing Map

The self-organizing map (SOM) [21] is an unsupervised and competitive learning neural network that can map the data with a higher-dimensional space into the transferred data having a lower-dimensional space (typically one or two dimensions). The output result of SOM is capable of preserving the critical topological relationships existed in the original input data, named a feature map. Typically, the SOM consists of two layers: input layer and Kohonen layer as shown in Figure 1. Notably, the input layer is fully connected to the Kohonen layer that has two dimensions, and each neuron in the Kohonen layer does not connect to each other. In the Kohonen layer, a neuron represents a cluster, whose weight vector stands for an exemplar in the original input patterns associated with only this cluster. In the self-organizing process of SOM, a winner is chosen by selecting a neuron that can match most closely to the input pattern by evaluating the distance between its weight vector and the input data. According to the activation zone for each neuron, the winner along with its neighboring neurons can be determined as well as can update their corresponding weights. By following the architecture and algorithm for implementing the SOM neural network, the original input data then can be clustered into a certain number of groups in the Kohonen layer.

Assuming that a problem has the input patterns with a set of $x=\left(x_{1}, x_{2}, \ldots, x_{i}, \ldots, x_{n}\right)$ of continuous values in $n$ dimensions, and there are $m$ clustered neurons are distributed in the feature map. Next, the weight vector corresponding to the neuron $j$ in the Kohonen layer is depicted by $w=\left(w_{1 j}, w_{2 j}, \ldots, w_{i j}, \ldots, w_{n j}\right)$, as well as the $h_{j \prime j}$ (where $j^{\prime}$ and $j$ are the subscripts of neurons in the Kohonen layer) represents the neighborhood function that is utilized to control the process of relaxing. The competitive and weight adjustment processes of SOM in the training steps can be summarized as follows [22-23]:
Step 0: Initialize the weights $x_{j}$, neighborhood functions $h_{j^{\prime} j}$, radius for the topological neighborhood $R$, and learning rate $\alpha$.
Step 1: Execute Steps 2 to 8 if the stopping criteria have not been fulfilled.
Step 2: Implement Steps 3-5 for each input vector $x$.
Step 3: For each cluster neuron $j$, compute the distance between the neuron and input vector:

$$
D(j)=\left\|x-w_{j}\right\|
$$

Step 4: Find index $c$ that can minimize $D(c)$ in all cluster neurons.
Step 5: For all neurons $j$ that lies within the topological neighborhood of the radius $R$ centered on neuron $c$, update the weights associated with neuron $j$ :
$w_{j}(t+1)=w_{j}(t)+\alpha h_{c j}(t)\left[x-w_{j}(t)\right]$
where, $t$ is an index for discrete-time.
Step 6: Update the learning rate $\alpha$ and neighborhood function $h_{j^{\prime} j}$.
Step 7: Reduce the radius for the topological neighborhood $R$ based on the time pre-determined.
Step 8: Test the stopping criteria.
Notably, the learning rate $\alpha$ and radius for the topological neighborhood $R$ will decrease in the clustering process of SOM. Specifically, the neighborhood function $h_{j^{\prime} j}$ that decreases monotonically in time is a smoothing kernel function defined over the lattice. There are general two types of $h_{j^{\prime} j}$ can be chosen [23]. The first choice of neighborhood function is simpler that is a neighborhood set of array points around the winner $c$ defined as
$h_{c j}(t)= \begin{cases}1 & \text { if neuron } j \text { lies within a radius } R \text { of the winning neuron } c \\ 0 & \text { otherwise }\end{cases}$
The second generally applied function is the smoother Gaussian neighborhood function that centers on the winning neuron $c$ defined as follows
$h_{c j}(t)=\exp \left(-\frac{\left\|x_{c}-x_{j}\right\|^{2}}{2 \sigma^{2}(t)}\right)$
where $x_{c}$ and $x_{j}$ are the vectors corresponding to neurons $c$ and $j$, respectively, in the Kohonen layer; the parameter $\sigma(\mathrm{t})$, that is used to define the width of the kernel, is a monotonically decreasing function of time. In addition, the exact shape of the neighborhoods does not sensitively influence the performance of SOM actually. For the purpose of implementing efficiently, the rectangular and hexagonal neighborhoods are suggested in [23].


Input layer Kohonen layer
Figure 1: Topology of a typical SOM with two layers.

## Symbiotic Organisms Search

[24] proposes the symbiotic organisms search (SOS), that is a robust and powerful metaheuristic algorithm, to resolve the numerical optimization and engineering design problems. The SOS algorithm simulates the reliancebased relationship, known as symbiosis, to illustrate that the organisms rarely live solitarily due to their reliance on other species for the purpose of sustenance and even survival. Similar to the other population-based algorithms, e.g. artificial bee colony (ABC), particle swarm optimization (PSO) and genetic algorithm (GA), the SOS
iteratively utilizes a population of candidate solutions in the process for seeking the optimal global solution through gradually moving to the promising areas in the search space. In SOS, the initial population is called the ecosystem, that is usually generated randomly in the search space, thus forming a group of organisms. Each organism is then associated with a certain fitness value to reflect the degree of adaptation when this candidate solution solves the objective function of the optimization problem. In addition, the next generation in SOS is regulated by emulating the biological interaction between two organisms in the ecosystem. Three phases, including the mutualism, commensalism, and parasitism phases, are introduced in SOS to mirror the biological interaction model in the real world, and each phase has an interaction with its own character. In the mutualism phase, the interaction can benefit each other. However, the commensalism phase only benefits one side and does not impact the other side. Finally, the interaction will benefit one side and actively harm the other side in the parasitism phase. Notably, each organism can randomly interact with the other organism in all phases regarding the biological interaction model. The interaction process repeats until the termination criteria can be met. The SOS algorithm can be outlined as Figure 2 and the tree phases in SOS are further described as follows:

1. Mutualism phase

The $i$ th member in an ecosystem is represented by $x_{i}$ that can randomly select another organism (member) $x_{j}$ from this ecosystem to interact. According to the aim for increasing the mutual survival advantage in the ecosystem, the organisms $x_{i}$ and $x_{j}$ engage with each other in their mutualistic relationship. Therefore, the new candidate solutions for $x_{i}$ and $x_{j}$ then can be obtainted according to the mutualistic symbiosis between these two organisms, and are modeled by
$x_{i_{-} \text {new }}=x_{i}+\operatorname{rand}(0,1) \times\left(x_{\text {best }}-\right.$ Vector $\left._{\text {mutual }} \times B F_{1}\right)$
$x_{j_{-} \text {new }}=x_{j}+\operatorname{rand}(0,1) \times\left(x_{\text {best }}-\right.$ Vector $\left._{\text {mutual }} \times B F_{2}\right)$
Vector $_{\text {mutual }}=\frac{x_{i}+x_{j}}{2}$
where $\operatorname{rand}(0,1)$ is a vector consisting of random numbers. Furthermore, the benefit factors $\left(B F_{1}\right.$ and $\left.B F_{2}\right)$, are randomly determined as either 1 or 2 , to reflect that some mutualism relationships might give a larger beneficial advantage for one organism than another organism in the nature world. The levels of benefits for organisms are represented by the benefit factors. Next, the Vector $_{\text {mutual }}$ shown in Equation (5) represents the relationship characteristic between organisms $x_{i}$ and $x_{j}$. Therefore, the later parts in Equations (3) and (4), i.e. $\left(x_{\text {best }}-\right.$ Vector $\left._{\text {mutual }} \times B F_{1}\right)$ and $\left(x_{\text {best }}-\right.$ Vector $\left._{\text {mutual }} \times B F_{2}\right)$, describe the mutualistic effort for aiming the goal of increasing their survival advantages. In addition, the highest degree of adaptation among all organisms to the nature is represented by the vector $x_{\text {best }}$, that serves as the target point for increasing the fitness for both organisms $x_{i}$ and $x_{j}$. Finally, an organism can be updated only when its new fitness is better than its vale of fitness in the previous execution iteration.
2. Commensalism phase

To simulate the mutualism phase, an organism $x_{j}$ is randomly selected from the ecosystem to interact with $x_{i}$. The organism $x_{i}$ attempts to benefit during the interaction process, but the organism $x_{j}$ itself neither benefits nor suffers from the relationship in this circumstance. Hence, the new candidate solution for $x_{i}$ can be calculated based on the commensal symbiosis between organisms $x_{i}$ and $x_{j}$, and is formulated by
$x_{i \_n e w}=x_{i}+\operatorname{rand}(-1,1) \times\left(x_{\text {best }}-x_{j}\right)$
where $\operatorname{rand}(-1,1)$ is a random number between -1 and 1 . The part $\left(x_{\text {best }}-x_{j}\right)$ is used to reflect the beneficial advantage obtained from $x_{j}$ to assist $x_{i}$ to increase its survival advantage in an ecosystem to the organism based on the largest degree of adaptation in the current organism, represented by the $x_{\text {best }}$.
3. Parasitism phase

In the SOS algorithm, an artificial parasite, named Vectror $_{\text {Parasite }}$, is created for providing a role that is similar to the anopheles mosquito for an organism $x_{j}$ in the plasmodium parasite. By duplicating an organism $x_{i}$, and modifying the randomly selected dimensions using a random number, the Vectror $_{\text {Parasite }}$ in the search space can be created. Next, an organism $x_{j}$ is randomly selected from the ecosystem and treats as a host for the parasite vector. Then, the Vectror $_{\text {Parasite }}$ tries to replace $x_{j}$ in the ecosystem. The Vectror ${ }_{\text {Parasite }}$ can kill organism $x_{j}$ and substitute for its position in the ecosystem if the fitness value regarding the Vectror $_{\text {Parasite }}$ is superior. However, the Vectror Parasite will have immunity from the parasite and no longer survive in the ecosystem when the $x_{j}$ has a better fitness value.


Figure 2: SOS flowchart.

## Model of Allocating Seats

The mathematical model for simultaneously optimizing the total revenue from the passenger transportation and the loss of reputation for a certain train operation can be formulated as:
Maximize
$T R=\sum_{i=1}^{t s p n-1} \sum_{j=i+1}^{t s p n} \min \left(S_{i, j}, D_{i, j}\right) \times P_{i, j}-R C \times \sum_{i=1}^{t s p n-1} \sum_{j=i+1}^{t s p n} \max \left(D_{i, j}-S_{i, j}, 0\right) \times P_{i, j}$
Subject to
$\sum_{j=i}^{t s p n} S_{i, j} \leq \sum_{k=1}^{i} S_{k, i}$ for $i=1,2,3, \ldots, t s p n$
$S_{i, j} \leq \sum_{k=1}^{i} S_{k, i}$ for $i=1,2,3, \ldots, t s p n, j=i+1, i+2, \ldots, t s p n$
$S_{i, j} \in \mathbb{Z}_{0}^{+}$for $i=1,2, \ldots$, tspn; $j=i+1, i+2, \ldots$, tspn total number of stops for a certain train operation; $S_{i, j}$ represents the number of allocated seats between the $i$ th and $j$ th $(i<j)$ stops in its running region for a certain train operation. $D_{i, j}$ is the seat demand of passengers between the $i$ th and $j$ th $(i<j)$ stops for a certain train operation; $P_{i j}$, represents the ticket price between the $i$ th and $j$ th $(i<j)$ stops for a certain train operation; $R C$ is the penalty coefficient for losing the reputation.
In Equation (7) of the above mathematical model, the front half before the minus sign represents the revenue from providing seats to fulfill the passengers' demands and the second half behind the minus sign denotes the loss of reputation due to the allocated seats cannot satisfy seat demands of passengers. Next, Equation (8) states that the total number of allocated seats departing from a certain stop cannot exceed the total number of seats allocated by the previously stops and released in this stop in a certain train operation as shown in Table 1 (illustrated by the 4th stop). Notably, $S_{i, i}$ s (for $i=2,3,4, \ldots, t s p n$ ), i.e., the allocated seats for the same station of departure and destination, are zero, and $S_{1,1}$ represents the total number of available seats in a certain train operation. Equation (9) indicates that the number of allocated seats departing from each stop cannot exceed the total number of seats allocated by the previously stops and released in this stop. Finally, the number of allocated seats between two stops cannot be negative as expressed in Eq. (10).

Table 1: The relationship of allocated and released seats

| $S_{i j}$ |  | $j$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| $i$ | 1 | - | $S_{12}$ | $S_{13}$ | $S_{14}$ | ... |  |  |
|  | 2 | - |  | $S_{23}$ | $S_{24}$ | ... |  |  |
|  | 3 | - |  |  | $S_{34}$ |  |  |  |
|  | 4 | - |  | - |  | $S_{45}$ | $S_{46}$ | $S_{47}$ |
| Total number of seats allocated by the 1 st , 2nd and 3 rd stops as the departure stations and released in the 4th stop |  |  |  |  | Total number of seats allocated by the 4th stop with the destination stations of the 5th, 6th and 7th stops |  |  |  |

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## Proposed Approach

This study proposes a systematic approach for allocating seats of trains by using the SOM neural network and SOS optimization algorithm. Figure 3 briefly depicts the proposed methodology that are explained in more detail as follows:

1. Data Collection

The data regarding the trains' operation and passengers' demands are first collected. The collected information of the trains' operation includes (1) the departure and destination, (2) the stops, (3) the total number of available seats, (4) ticket prices between any paired stops. The data of passenger's demands in each train's operation are also collected. Notably, the real demands of passengers' travelling between any combination of stops cannot be known since passengers who transfer to the other trains cannot be checked accurately. Therefore, the passengers' demands of any paired stops must be estimated based on the booking records that including both of the successful booking, i.e. get reserved seats, and the unsuccessful booking, i.e. fail to get reserved seats, gathered from the booking web system.

## 2. Data Clustering

For each train run, the demand patterns of passengers during the usual working days might differ from those of the weekend. Therefore, the allocation of seats for the weekday must be different to the policy for allocating seats in the weekend for a train's operation. Furthermore, the stopping stations for different train runs are not the same. Hence, the passengers' demands for different trains' operations might have various characteristics. Thus, the data regarding the demands of passengers between stops for all trains in a full week are classified into groups though using the SOM neural network. The input data for the clustering approach for the train no. $t$ can be represented by $\left(D_{1,1}^{t}, D_{1,2}^{t}, \ldots, D_{1,1}^{t}, \ldots, D_{n-1, n}^{t}\right)$
where $D_{i j}^{t}$ is the demand of passengers between the $i$ th and $j$ th stops for the train no. $t$, and $n$ is the total number of stops in the operation range of train no. $t$.

## 3. Optimization of Seat Allocation

Symbiotic organisms search (SOS) algorithm is then applied to resolve the optimization problem of seat allocation as formulated in Equations (7)-(10). Notably, the decision variables are $S_{i j}^{t}$, i.e. the total number of allocated seats between the $i$ th and $j$ th $(i<j)$ stops for the train no. $t$. Next, the penalty coefficient used to represent the reputation loss of the train's operator when the total number of seats provided by trains cannot fulfill passengers' demands, i.e. $R C$ shown in Equation (7), must also be determined in advance.

## 4. Case Study and Comparison

A case study is then conducted to verify the effectiveness and efficiency of the optimization procedure for seat allocation proposed in this study. In addition, the results of the implementation will be analyzed and compared.

## Case Study

With the completion of the railway around the Taiwan island, the electrification of the railway in the full range of trains' operations, as well as the double-tracking of the eastern train routes, the tourism industry in Yilan, Hualien, and Taitung in eastern Taiwan has developed rapidly in recent years. However, the capacity of passenger's transportation provided by the Taiwan Railway Administration (TRA) is insufficient seriously. Therefore, passengers have to scramble for tickets frequently during the consecutive holidays. The unbalance between the supply and demand of seats provided by the tilted trains, such as Taroko and Puyuma, is even more serious. The Taiwan Railway Administration has suffered huge losses both in terms of operation's profits and reputation of a firm. Therefore, this study aims to solve the seat allocation problem of tilted trains operating in the eastern area of Taiwan, thereby maximizing the operation revenue of the Taiwan Railway Administration and the convenience of passengers. Furthermore, the simulated optimal seat allocation obtained by the method proposed in this study is also compared with the results provided by the seat allocation strategy that is set up according to the proportions of passenger's demands between two stops.

## Data Collection

Among the trains provided by TRA between the east and west of Taiwan island, the so-called golden trains (that is, trains with fewer stops) have the most serious unbalance between the supply and demand of train's seats. Therefore, this study selects the trains no. 408 and 426 operated from Shulin to Taitung, as well as the trains no. 421 and 441 running in the range between Zhiben and Shulin as the research targets. First, the departure and destination, stops, total number of available seats, and fares between any paired stops for trains no.408, 426, 421 and 441 are first collected. The data regarding passengers' demands for seats in the online ticketing system are
also gathered. The passengers' demands between stops of any pair are estimated based on the reservation records in the online ticket reservation system by summing up the successful reservations and unsuccessful reservations.

## Data Clustering

The SOM neural network is then applied to cluster the data regarding passengers' demands between train stops for trains no. 408, 426, 421 and 441 on a daily basis within a week. The results are shown in Table 2.

Table 2: Clustering results for passengers' seat demands within a week

| Train No. 408 |  | Train No. 426 |  | Train No. 421 |  | Train No. 441 |  |
| :--- | :---: | :--- | :---: | :--- | :---: | :--- | :--- |
| Day | Cluster | Day | Cluster | Day | Cluster | Day | Cluster |
| Monday | 1 | Monday | 1 | Monday | 1 | Monday | 1 |
| Tuesday | 1 | Tuesday | 2 | Tuesday | 1 | Tuesday | 2 |
| Wednesday | 1 | Wednesday | 2 | Wednesday | 1 | Wednesday | 3 |
| Thursday | 2 | Thursday | 1 | Thursday | 2 | Thursday | 3 |
| Friday | 2 | Friday | 1 | Friday | 2 | Friday | 3 |
| Saturday | 3 | Saturday | 3 | Saturday | 2 | Saturday | 3 |
| Sunday | 3 | Sunday | 4 | Sunday | 2 | Sunday | 3 |

## Optimization of Allocating Seats

Next, the SOS algorithm is utilized to resolve the problem of seat allocation as formulated in Equations (7) to (10). The parameters $B F_{1}$ and $B F_{2}$ in SOS are set as 2. Notably, all of trains no. 408, 426, 421 and 441 are operated by the train type of Taroko, the total number of seats of these train is set as 376 . For each train number, the SOS algorithm is executed on a PC with an i7-4470 CPU and 32GB RAM for ten times. Table 3 summarizes the experimental results while ignoring the loss of a firm's reputation, i.e. the $R C$ in Equation (1) is 0. Similarly, the implementation results when considering the loss of a firm's reputation, i.e. the $R C$ in Equation (1) is set as 1, are shown in Table 4. The asterisk indicates the optimal implementation result of SOS among a specific cluster for a certain train number, thus the optimal allocation of a train's seats can be obtained as shown in Tables 5 and 6.

Table 3: Experimental results of $\operatorname{SOS}(R C=0)$
(A) Train No. 408

| Day | Monday, Tuesday, Wednesday |  | Thursday, Friday |  | Saturday, Sunday |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Execution <br> No. | Objective <br> function | CPU time <br> (seconds) | Objective <br> function | CPU time <br> (seconds) | Objective <br> function | CPU time <br> (seconds) |
| 1 | 281,254 | 21 | $305,312^{*}$ | 25 | $305,312^{*}$ | 23 |
| 2 | 293,315 | 33 | 305,312 | 19 | 305,312 | 22 |
| 3 | 293,315 | 30 | 305,312 | 20 | 300,618 | 22 |
| 4 | 293,315 | 24 | 305,312 | 18 | 305,312 | 25 |
| 5 | 282,174 | 23 | 305,312 | 29 | 305,312 | 22 |
| 6 | 281,937 | 32 | 305,312 | 19 | 305,312 | 23 |
| 7 | $293,596^{*}$ | 23 | 282,054 | 20 | 305,312 | 18 |
| 8 | 285,015 | 28 | 305,312 | 22 | 305,312 | 23 |
| 9 | 282,114 | 35 | 305,312 | 20 | 305,312 | 18 |
| 10 | 281,254 | 18 | 305,312 | 27 | 305,312 | 21 |

(B) Train No. 426

| Day | Monday, Thursday, <br> Friday |  | Tuesday, Wednesday |  | Saturday |  | Sunday |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Execution <br> No. | Objective <br> function | CPU time <br> (seconds) | Objective <br> function | CPU time <br> (seconds) | Objective <br> function | CPU time <br> (seconds) | Objective <br> function | CPU time <br> (seconds) |
| 1 | 286,829 | 21 | 290,992 | 22 | 306,012 | 27 | $305,312^{*}$ | 41 |
| 2 | $297,612^{*}$ | 22 | 297,408 | 21 | 305,312 | 21 | 305,312 | 25 |
| 3 | 292,831 | 32 | 297,612 | 26 | 305,982 | 17 | 305,312 | 27 |
| 4 | 297,612 | 27 | $297,877^{*}$ | 21 | 305,312 | 26 | 305,312 | 22 |
| 5 | 297,612 | 23 | 297,612 | 44 | 305,312 | 22 | 283,875 | 23 |
| 6 | 292,831 | 25 | 297,877 | 25 | 306,012 | 23 | 305,312 | 27 |
| 7 | 292,831 | 28 | 297,877 | 32 | $306,480^{*}$ | 28 | 305,312 | 33 |
| 8 | 286,829 | 33 | 297,612 | 27 | 306,012 | 23 | 295,279 | 39 |
| 9 | 297,612 | 29 | 297,408 | 23 | 305,374 | 30 | 302,562 | 28 |
| 10 | 297,612 | 26 | 297,408 | 35 | 305,312 | 30 | 305,312 | 22 |

(C) Train No. 421

| Day | Monday, Tuesday, Wednesday |  | Thursday, Friday, Saturday, Sunday |  |
| :---: | :---: | :---: | :---: | :---: |
| Execution <br> No. | Objective <br> function | CPU time <br> (seconds) | Objective <br> function | CPU time <br> (seconds) |
| 1 | 303,108 | 31 | 294,829 | 51 |
| 2 | 303,108 | 21 | $299,208^{*}$ | 49 |
| 3 | $305,312^{*}$ | 33 | 294,258 | 28, |
| 4 | 305,312 | 22 | 295,690 | 43 |
| 5 | 303,108 | 29 | 293,268 | 29 |
| 6 | 305,312 | 28 | 295,690 | 36 |
| 7 | 305,312 | 35 | 295,690 | 32 |
| 8 | 305,312 | 29 | 299,208 | 43 |
| 9 | 303,108 | 28 | 293,268 | 35 |
| 10 | 303,108 | 29 | 293,268 | 33 |

(D) Train No. 441

| Day | Monday |  | Tuesday |  | Wednesday, Thursday, Friday, <br> Saturday, Sunday |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Execution <br> No. | Objective <br> function | CPU time <br> (seconds) | Objective <br> function | CPU time <br> (seconds) | Objective <br> function | CPU time <br> (seconds) |
| 1 | $308,376^{*}$ | 45 | $304,184^{*}$ | 28 | $304,324^{*}$ | 33 |
| 2 | 304,649 | 29 | 304,184 | 33 | 395,490 | 45 |
| 3 | 304,184 | 28 | 304,184 | 23 | 300,655 | 45 |
| 4 | 305,084 | 28 | 304,184 | 34 | 301,718 | 67 |
| 5 | 305,084 | 21 | 304,184 | 22 | 296,956 | 28 |
| 6 | 305,084 | 35 | 304,184 | 36 | 296,956 | 56 |
| 7 | 308,376 | 29 | 304,184 | 34 | 300,655 | 58 |
| 8 | 308,376 | 32 | 304,184 | 37 | 304,324 | 47 |
| 9 | 308,376 | 39 | 304,184 | 25 | 304,324 | 62 |
| 10 | 304,184 | 30 | 304,184 | 28 | 300,655 | 52 |

Table 4: Experimental results of $\operatorname{SOS}(R C=1)$
(A) Train No. 408

| Day | Monday, Tuesday, <br> Wednesday |  | Thursday, Friday |  | Saturday, Sunday |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Execution <br> No. | Objective <br> function | CPU time <br> (seconds) | Objective <br> function | CPU time <br> (seconds) | Objective <br> function | CPU time <br> (seconds) |
| 1 | $-2,821,620$ | 19 | $-7,238,354^{*}$ | 20 | $-19,607,060$ | 18 |
| 2 | $-2,838,750$ | 29 | $-7,238,354$ | 19 | $-19,602,960^{*}$ | 17 |
| 3 | $-2,807,780^{*}$ | 30 | $-7,245,570$ | 33 | $-19,602,960$ | 16 |
| 4 | $-2,840,262$ | 21 | $-7,238,354$ | 19 | $-19,602,960$ | 17 |
| 5 | $-2,823,062$ | 20 | $-7,238,354$ | 19 | $-19,602,960$ | 18 |
| 6 | $-2,821,620$ | 25 | $-7,238,354$ | 20 | $-19,603,536$ | 21 |
| 7 | $-2,828,902$ | 22 | $-7,238,354$ | 20 | $-19,602,960$ | 18 |
| 8 | $-2,821,460$ | 19 | $-7,282,430$ | 20 | $-19,602,960$ | 19 |
| 9 | $-2,823,062$ | 25 | $-7,241,346$ | 27 | $-19,602,960$ | 21 |
| 10 | $-2,823,062$ | 35 | $-7,238,354$ | 20 | $-19,603,536$ | 22 |

(B) Train No. 426

| Day | Monday, Thursday, Friday |  | Tuesday, Wednesday |  | Saturday |  | Sunday |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Execution <br> No. | Objective <br> function | CPU time <br> (seconds) | Objective <br> function | CPU time <br> (seconds) | Objective <br> function | CPU time <br> (seconds) | Objective <br> function | CPU time <br> (seconds) |
| 1 | $-6,108,947^{*}$ | 20 | $-3,539,618^{*}$ | 23 | $-20,185,607^{*}$ | 19 | $-8,915,422^{*}$ | 21 |
| 2 | $-6,111,055$ | 21 | $-3,540,792$ | 19 | $-20,185,607$ | 25 | $-9,347,184$ | 22 |
| 3 | $-6,128,855$ | 22 | $-3,540,792$ | 20 | $-20,193,805$ | 22 | $-9,301,634$ | 22 |
| 4 | $-6,113,345$ | 22 | $-3,542,218$ | 23 | $-20,193,107$ | 25 | $-9,301,634$ | 19 |
| 5 | $-6,111,055$ | 18 | $-3,540,592$ | 22 | $-20,188,409$ | 24 | $-9,324,642$ | 22 |
| 6 | $-6,111,055$ | 23 | $-3,540,592$ | 20 | $-20,196,523$ | 26 | $-9,301,634$ | 20 |
| 237 | $-6,108,947$ | 20 | $-3,539,618$ | 20 | $-20,200,445$ | 25 | $-9,328,876$ | 23 |
| 8 | $-6,111,055$ | 22 | $-3,539,618$ | 25 | $-20,185,607$ | 23 | $-9,320,812$ | 25 |
| 9 | $-6,11,055$ | 21 | $-3,540,592$ | 21 | $-20,204,123$ | 25 | $-9,317,518$ | 22 |
| 10 | $-6,128,855$ | 25 | $-3,540,592$ | 26 | $-20,195,409$ | 20 | $-9,301,634$ | 20 |

(C) Train No. 421

| Day | Monday, Tuesday, Wednesday |  | Thursday, Friday, Saturday, |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |$|$| Execution <br> No. | Objective <br> function | CPU time <br> (seconds) | Objective <br> function | CPU time <br> (seconds) |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $-26,451,040$ | 18 | $-4,693,455$ | 23 |
| 2 | $-26,452,258$ | 29 | $-4,695,105$ | 25 |
| 3 | $-26,440,840^{*}$ | 19 | $-4,686,263^{*}$ | 22 |
| 4 | $-26,453,320$ | 23 | $-4,702,169$ | 24 |
| 5 | $-26,453,320$ | 24 | $-4,704,821$ | 39 |
| 6 | $-26,438,408$ | 30 | $-4,686,263$ | 35 |
| 7 | $-26,438,408$ | 25 | $-4,686,263$ | 29 |
| 8 | $-26,440,840$ | 19 | $-4,686,263$ | 33 |
| 9 | $-26,440,840$ | 23 | $-4,704,821$ | 29 |
| 10 | $-26,440,840$ | 26 | $-4,693,455$ | 31 |

(D) Train No. 441

| Day | Monday |  | Tuesday |  | Wednesday, Thursday, Friday, <br> Saturday, Sunday |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Execution <br> No. | Objective <br> function | CPU time <br> (seconds) | Objective <br> function | CPU time <br> (seconds) | Objective <br> function | CPU time <br> (seconds) |
| 1 | $-117,117,667^{*}$ | 22 | $-35,560,754$ | 19 | $-7,781,653^{*}$ | 25 |
| 2 | $-117,117,995$ | 24 | $-35,560,754$ | 18 | $-7,792,123$ | 34 |
| 3 | $-117,117,667$ | 21 | $-35,560,754$ | 26 | $-7,785,435$ | 32 |
| 4 | $-117,117,995$ | 20 | $-35,558,272^{*}$ | 26 | $-7,784,907$ | 23 |
| 5 | $-117,117,667$ | 23 | $-35,582,988$ | 31 | $-7,784,783$ | 28 |
| 6 | $-117,117,667$ | 22 | $-35,560,754$ | 30 | $-7,784,783$ | 33 |
| 7 | $-117,117,667$ | 23 | $-35,560,754$ | 25 | $-7,784,783$ | 26 |
| 8 | $-117,117,995$ | 25 | $-35,558,272$ | 32 | $-7,781,653$ | 28 |
| 9 | $-117,117,995$ | 19 | $-35,560,754$ | 26 | $-7,781,653$ | 25 |
| 10 | $-117,117,667$ | 21 | $-35,558,272$ | 28 | $-7,792,123$ | 32 |

Table 5: Optimal allocation of seats $(R C=0)$
(A) Train No. 408 (Monday, Tuesday, Wednesday)

| Destination | Banqian | Taipei | Songshan | Hualien | Yuli | Taitung |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Departure | Shulin | 55 | 194 | 53 | 7 | 0 |
| Banqian |  | 3 | 0 | 0 | 3 | 49 |
| Taipei |  |  | 97 | 0 | 0 | 100 |
| Songshan |  |  |  | 0 | 0 | 150 |
| Hualien |  |  |  |  | 7 | 0 |
| Yuli |  |  |  |  |  | 0 |

Profit $=$ NT\$293,596
(B) Train No. 408 (Thursday, Friday)

| Departure | Banqian | Taipei | Songshan | Hualien | Yuli | Taitung |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Shulin | 0 | 0 | 0 | 0 | 0 | 376 |
| Banqian |  | 0 | 0 | 0 | 0 | 0 |
| Taipei |  |  | 0 | 0 | 0 | 0 |
| Songshan |  |  |  | 0 | 0 | 0 |
| Hualien |  |  |  |  | 0 | 0 |
| Yuli |  |  |  |  |  | 0 |

Profit $=$ NT\$350,312
(C) Train No. 408 (Saturday, Sunday)

| Destination | Banqian | Taipei | Songshan | Hualien | Yuli | Taitung |  |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Departure |  | 0 | 0 | 0 | 0 | 376 |  |
| Shulin | 0 | 0 | 0 | 0 | 0 | 0 |  |
| Banqian |  | 0 | 0 | 0 | 0 | 0 |  |
| Taipei |  |  | 0 | 0 | 0 | 0 |  |
| Songshan |  |  |  | 0 | 0 | 0 | 0 |
| Hualien |  |  |  |  | 0 | 0 |  |
| Yuli |  |  |  |  |  | 0 |  |

Profit= NT\$350,312
(D) Train No. 426 (Monday, Thursday, Friday)

| Destination | Banqian | Taipei | Song- <br> Shan | Hua- <br> lien | Yuli | Tai- <br> tung | Zhiben |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Shulin | 0 | 0 | 0 | 0 | 50 | 326 | 0 |
| Banqian |  | 0 | 0 | 0 | 0 | 0 | 0 |
| Taipei |  |  | 0 | 0 | 0 | 0 | 0 |
| Songshan |  |  |  | 0 | 0 | 0 | 0 |
| Hualien |  |  |  |  | 0 | 0 | 0 |
| Yuli |  |  |  |  |  | 0 | 50 |
| Taitung |  |  |  |  |  |  | 50 |

Profit= NT\$297,612
(E) Train No. 426 (Tuesday, Wednesday)

| Departure Destination | Banqian | Taipei | Song- <br> shan | Hua- <br> lien | Yuli | Tai- <br> tung | Zhiben |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Shulin | 44 | 9 | 0 | 0 | 49 | 274 | 0 |
| Banqian |  | 0 | 0 | 0 | 0 | 0 | 44 |
| Taipei |  |  | 6 | 3 | 0 | 0 | 0 |
| Songshan |  |  |  | 0 | 1 | 5 | 0 |
| Hualien |  |  |  |  | 0 | 3 | 0 |
| Yuli |  |  |  |  |  | 0 | 50 |
| Taitung |  |  |  |  |  |  | 50 |

Profit $=$ NT\$297,877
(F) Train No. 426 (Saturday)

|  | Banqian | Taipei | Songshan | Hualien | Yuli | Taitung | Zhiben |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Shulin | 50 | 0 | 0 | 18 | 0 | 308 | 0 |
| Banqian | 0 | 0 | 0 | 0 | 0 | 0 | 50 |
| Taipei |  | 0 | 0 | 0 | 0 | 0 | 0 |
| Songshan |  |  | 0 | 0 | 0 | 0 |  |
| Hualien |  |  |  | 0 | 0 | 0 | 18 |
| Yuli |  |  |  |  | 0 | 0 | 0 |
| Taitung |  |  |  |  |  | 0 | 0 |

Profit= NT\$306,480
(G) Train No. 426 (Sunday)

| Departure Destination | Banqian | Taipei | Song- <br> shan | Hua- <br> lien | Yuli | Tai- <br> tung | Zhiben |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Shulin | 0 | 0 | 0 | 0 | 50 | 326 | 0 |
| Banqian |  | 0 | 0 | 0 | 0 | 0 | 0 |
| Taipei |  |  | 0 | 0 | 0 | 0 | 0 |
| Songshan |  |  |  | 0 | 0 | 0 | 0 |
| Hualien |  |  |  |  | 0 | 0 | 0 |
| Yuli |  |  |  |  |  | 0 | 50 |
| Taitung |  |  |  |  |  |  | 50 |

Profit= NT\$350,312
(H) Train No. 421 (Monday, Tuesday, Wednesday)

| $\text { Departure }{ }^{\text {Destination }}$ | Taitung |  | Hua- |  | Taipei | Banqian | Shulin |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Zhiben | 376 | 0 | 0 | 0 | 0 |  |  |
| Taitung |  | 0 | 300 | 0 | 0 |  | 76 |
| Yuli |  |  | 0 | 0 | 0 |  | 0 |
| Hualien |  |  |  | 0 | 0 |  | 300 |
| Songshan |  |  |  |  | 0 |  |  |
| Taipei |  |  |  |  |  |  | 0 |
| Bangian |  |  |  |  |  |  |  |

Profit= NT\$305,312
(I) Train No. 421 (Thursday, Friday, Saturday, Sunday)

| Departure Destination | Taitung | Yuli | Hua- <br> lien | Song- <br> dhan | Taipei | Banqian | Shulin |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Zhiben | 376 | 0 | 0 | 0 | 0 | 0 | 0 |
| Taitung |  | 0 | 0 | 0 | 76 | 300 | 0 |
| Yuli |  |  | 0 | 0 | 0 | 0 | 0 |
| Hualien |  |  |  | 0 | 0 | 0 | 0 |
| Songshan |  |  |  |  | 0 | 0 | 0 |
| Taipei |  |  |  |  |  | 0 | 76 |
| Banqian |  |  |  |  |  |  | 300 |

Profit= NT\$299,208
(J) Train No. 441 (Monday)

| Destination | Taitung | Yuli | Hua- <br> lien | Song- <br> shan | Taipei | Banqian | Shulin |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Zhiben | 14 | 0 | 0 | 0 | 108 | 254 | 0 |
| Taitung |  | 0 | 14 | 0 | 0 | 0 | 0 |
| Yuli |  |  | 0 | 0 | 0 | 0 | 0 |
| Hualien |  |  |  | 0 | 0 | 14 | 0 |
| Songshan |  |  |  |  | 0 | 0 | 0 |
| Taipei |  |  |  |  |  | 0 | 108 |
| Banqian |  |  |  |  |  |  | 268 |

Profit= NT\$308,376
(K) Train No. 441 (Tuesday)

| $\text { Departure } \text { Destination }$ | Taitung | Yuli | $\begin{array}{\|l\|l} \text { Hual } \\ \text { lien } \end{array}$ |  | Taipei | Banqian | Shulin |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Zhiben | 0 | 0 | 0 | , | 303 | 73 |  |
| Taitung |  | 0 | 0 | 0 | 0 | 0 |  |
| Yuli |  |  | 0 | 0 | 0 | 0 |  |
| Hualien |  |  |  | 0 | 0 | 0 |  |
| Songshan |  |  |  |  | 0 | 0 |  |
| Taipei |  |  |  |  |  | 301 |  |
| Banqian |  |  |  |  |  |  | 374 |

Profit= NT\$-35,558,272
(L) Train No. 441 (Wednesday, Thursday, Friday, Saturday, Sunday)

| Destination | Taitung | Yuli | Hua- <br> Departure <br> lien | Song- <br> shan | Taipei | Banqian | Shulin |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Zhiben | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Taitung |  | 0 | 0 | 0 | 0 | 76 | 300 |
| Yuli |  |  | 0 | 0 | 0 | 0 | 0 |
| Hualien |  |  |  | 0 | 0 | 0 | 0 |
| Songshan |  |  |  |  | 0 | 0 | 0 |
| Taipei |  |  |  |  |  | 0 | 0 |
| Banqian |  |  |  |  |  |  | 0 |

Profit= NT\$304,324

Table 6: Optimal allocation of seats $(R C=1)$
(A) Train No. 408 (Monday, Tuesday, Wednesday)

| Departination | Banqian | Taipei | Songshan | Hualien | Yuli | Taitung |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Dhulin | 114 | 195 | 0 | 0 | 0 | 67 |
| Banqian |  | 14 | 50 | 0 | 0 | 50 |
| Taipei |  |  | 100 | 0 | 39 | 70 |
| Songshan |  |  |  | 0 | 0 | 150 |
| Hualien |  |  |  |  | 0 | 0 |
| Yuli |  |  |  |  |  | 39 |

Profit $=$ NT\$-2,807,780
(B) Train No. 408 (Thursday, Friday)

| Destination | Banqian | Taipei | Songshan | Hualien | Yuli | Taitung |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Departure |  | 0 | 0 | 0 | 0 | 376 |
| Shulin | 0 | 0 | 0 | 0 | 0 | 0 |
| Banqian |  | 0 | 0 | 0 | 0 | 0 |
| Taipei |  |  | 0 | 0 | 0 | 0 |
| Songshan |  |  |  | 0 | 0 | 0 |
| Hualien |  |  |  |  | 0 | 0 |
| Yuli |  |  |  |  |  | 0 |

Profit $=$ NT\$-7,238,354
(C) Train No. 408 (Saturday, Sunday)

| Departure | Destination | Banqian | Taipei | Songshan | Hualien | Yuli |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: |
| D. Taitung |  |  |  |  |  |  |
| Shulin | 0 | 0 | 0 | 0 | 0 | 376 |
| Banqian |  | 0 | 0 | 0 | 0 | 0 |
| Taipei |  |  | 0 | 0 | 0 | 0 |
| Songshan |  |  |  | 0 | 0 | 0 |

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Profit= NT\$-19,602,960
(D) Train No. 426 (Monday, Thursday, Friday)

| Destination | Banqian | Taipei | Song- <br> shan | Hua- <br> lien | Yuli | Tai- <br> tung | Zhiben |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Departure | 0 | 0 | 0 | 0 | 50 | 326 | 0 |
| Shulin | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Banqian |  |  | 0 | 0 | 0 | 0 |  |
| Taipei |  |  | 0 | 0 | 0 | 0 | 0 |
| Songshan |  |  |  | 0 | 0 | 0 |  |
| Hualien |  |  |  |  | 0 | 0 | 0 |
| Yuli |  |  |  |  |  | 0 | 50 |
| Taitung |  |  |  |  |  |  | 50 |

Profit= NT\$-6,108,947
(E) Train No. 426 (Tuesday, Wednesday)

| Departure Destination | Banqian | Taipei | Song- <br> shan | Hua- <br> lien | Yuli | Tai- <br> tung | Zhiben |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Shulin | 46 | 0 | 0 | 0 | 49 | 281 | 0 |
| Banqian |  | 0 | 1 | 0 | 0 | 0 | 45 |
| Taipei |  |  | 0 | 0 | 0 | 0 | 0 |
| Songshan |  |  |  | 1 | 0 | 0 | 0 |
| Hualien |  |  |  |  | 1 | 0 | 0 |
| Yuli |  |  |  |  |  | 0 | 50 |
| Taitung |  |  |  |  |  |  | 50 |

Profit= NT\$-3,569,618
(F) Train No. 426 (Saturday)

| Departure Destination | Banqian | Taipei | Song- <br> shan | Hua- <br> lien | Yuli | Tai- <br> tung | Zhiben |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Shulin | 0 | 0 | 0 | 0 | 0 | 376 | 0 |
| Banqian |  | 0 | 0 | 0 | 0 | 0 | 0 |
| Taipei |  |  | 0 | 0 | 0 | 0 | 0 |
| Songshan |  |  |  | 0 | 0 | 0 | 0 |
| Hualien |  |  |  |  | 0 | 0 | 0 |
| Yuli |  |  |  |  |  | 0 | 0 |
| Taitung |  |  |  |  |  |  | 300 |

Profit= NT\$-20,185,607
(G) Train No. 426 (Sunday)

| Destination | Banqian | Taipei | Song- <br> Dhan | Hua- <br> lien | Yuli | Tai- <br> tung | Zhiben |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Shulin | 50 | 50 | 50 | 50 | 50 | 50 | 50 |
| Banqian |  | 50 | 50 | 50 | 50 | 50 | 50 |
| Taipei |  |  | 50 | 50 | 50 | 50 | 50 |
| Songshan |  |  |  | 50 | 50 | 50 | 50 |
| Hualien |  |  |  |  | 50 | 50 | 50 |
| Yuli |  |  |  |  |  | 50 | 50 |
| Taitung |  |  |  |  |  |  | 50 |

Profit= NT\$-8,915,422
(H) Train No. 421 (Monday, Tuesday, Wednesday)

| Destination | Taitung | Yuli | Hua- <br> Departure | Song- <br> lien <br> shan | Taipei | Banqian | Shulin |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Zhiben | 376 | 0 | 0 | 0 | 0 | 0 | 0 |
| Taitung |  | 0 | 300 | 0 | 76 | 0 | 0 |
| Yuli |  |  | 0 | 0 | 0 | 0 | 0 |
| Hualien |  |  |  | 0 | 0 | 300 | 0 |
| Songshan |  |  |  |  | 0 | 0 | 0 |
| Taipei |  |  |  |  |  | 0 | 0 |
| Banqian |  |  |  |  |  |  | 0 |

Profit= NT\$-26,440,840
(I) Train No. 421 (Thursday, Friday, Saturday,

Sunday)

| $\qquad$ | Taitung | Yuli | $\begin{array}{\|c\|} \hline \text { Hua- } \\ \text { lien } \\ \hline \end{array}$ | $\begin{array}{\|c} \hline \text { Song- } \\ \text { shan } \end{array}$ | Taipei | Banqian | Shulin |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Zhiben | 376 | 0 | 0 | 0 | 0 | 0 | 0 |
| Taitung |  | 4 | 20 | 0 | 52 | 300 | 0 |
| Yuli |  |  | 4 | 0 | 0 | 0 | 0 |
| Hualien |  |  |  | 24 | 0 | 0 | 0 |
| Songshan |  |  |  |  | 24 | 0 | 0 |
| Taipei |  |  |  |  |  | 76 | 0 |
| Banqian |  |  |  |  |  |  | 0 |

Profit $=$ NT $\$-4,686,263$
(J) Train No. 441 (Monday)

| Destination | Taitung | Yuli | Hua- <br> lien | Song- <br> Dhan | Taipei | Banqian | Shulin |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Departure |  | 0 | 0 | 0 | 0 | 376 |  |
| Zhiben | 0 | 0 | 0 | 0 | 0 |  |  |
| Taitung |  | 0 | 0 | 0 | 0 | 0 | 0 |
| Yuli |  |  | 0 | 0 | 0 | 0 | 0 |
| Hualien |  |  |  | 0 | 0 | 0 | 0 |
| Songshan |  |  |  |  | 0 | 0 | 0 |
| Taipei |  |  |  |  |  |  | 376 |
| Banqian |  |  |  |  |  |  | 0 |

Profit= NT\$-117,117,667
(K) Train No. 441 (Tuesday)

| Destination | Taitung | Yuli | Hua- <br> Donger <br> lien | Song- <br> shan | Taipei | Banqian | Shulin |
| :---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Zhiben | 0 | 0 | 0 | 0 | 303 | 73 | 0 |
| Taitung |  | 0 | 0 | 0 | 0 | 0 | 0 |
| Yuli |  |  | 0 | 0 | 0 | 0 | 0 |
| Hualien |  |  |  | 0 | 0 | 0 | 0 |
| Songshan |  |  |  |  | 0 | 0 | 0 |
| Taipei |  |  |  |  |  | 301 | 0 |
| Banqian |  |  |  |  |  |  | 374 |

Profit= NT\$-35,558,272
(L) Train No. 441 (Wednesday, Thursday, Friday, Saturday, Sunday)

| Destination <br> Departure | Taitung | Yuli | $\begin{array}{\|c\|} \hline \text { Hua- } \\ \text { lien } \end{array}$ | $\begin{gathered} \text { Song- } \\ \text { shan } \end{gathered}$ | Taipei | Banqian | Shulin |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Zhiben | 376 | 0 | 0 | 0 | 0 | 0 | 0 |
| Taitung |  | 0 | 0 | 76 | 0 | 300 | 0 |
| Yuli |  |  | 0 | 0 | 0 | 0 | 0 |
| Hualien |  |  |  | 0 | 0 | 0 | 0 |
| Songshan |  |  |  |  | 0 | 0 | 76 |
| Taipei |  |  |  |  |  | 0 | 0 |
| Banqian |  |  |  |  |  |  | 300 |

Profit $=$ NT\$-7,781,653

## Analysis and Comparison

Through analyzing and comparing Table 5 and 6 regarding the optimal allocations of seats for demands of different clusters for each train number, the findings in this study are summarized as follows:
(1) In the case of ignoring the loss of reputation of TRA and of only aiming to maximize the revenue, the algorithm tends to allocate seats for the passengers with travels of long distances in order to make more operating income.
(2) For trains no. 408, 426, 421 and 441, passengers of short distances can reserve seats relatively easily on normal working days without considering the loss of reputation. However, the best seat configuration of trains no. 408 and 426 on weekends especially will allocate trains' seats to passengers who are preparing for travels of long distances from the west to the east. In addition, the optimal policy of allocating seats for trains no. 421 and 441 will try to make more reservations of seats for passengers who have finished their trips and prepared for long-distance travels to return to their working places from the west to the east.
(3) In the case of considering the loss of reputation of TRA, the optimal seat configuration will be more dispersed than that ignoring the loss of reputation, so as not to be only income-oriented and ignore the demands of passengers with short distances.

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(4) For TRA, the capacity of passengers' transportation with/without taking the reputation into account is extremely not enough.

## Conclusions

The rational allocation of trains' seats is an important factor influencing the revenue form passengers' transportation, thus affecting the occupancy rate and profit of operating trains. However, the occupancy rate can be determined by many elements and make the problem of allocating seats a very difficult optimization problem. This study applies the Self-Organizing Map (SOM) neural network and Symbiotic Organisms Search (SOS) optimization algorithm to construct a systematic approach for resolving the problem of allocating trains' seats. First, the optimization problem is formulated by a mathematical model with considering the loss of reputation of a firm. The SOM is utilized to cluster passengers' demands in different days within a week for a train to form several groups. The SOS is then used to find the (near) optimal solution for the seats' allocation. The effectiveness and execution efficiency of the proposed method is demonstrated by taking the trains operated by the Taiwan Railway Administration (TRA) of the Ministry of Communications of the Republic of China as a case study. Based on the execution results, it tends to allocate seats for the passengers with travels of long distances for the purpose of making more operating income while not considering the loss of reputation of TRA and only trying to maximize the profit. For all of the golden trains, i.e. trains with fewer stops, considered in this study, passengers with shortdistance travels can get seats more easily on normal working days when not taking the loss of reputation into account. Next, the best configuration of seats for trains no. 408 and 426 especially on weekends tends to reserve seats for passengers that plan to have trips with long distances from the west to the east. Next, the policy for optimally allocating seats for trains no. 421 and 441 sailing from east to west can make more reservations of seats for passengers who have finished their journeys and returned to their working places for long distances from the west to the east. Furthermore, the optimal seat configuration will disperse the allocation of seats than the result provided by ignoring the loss of reputation, thus the passengers' demands of shorter distances can be met and the operating profit will not be the only orientation.

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