



Modeling of Orthometric Heights from Multi-Networks of GNSS/Precise Levelling in Owerri and Environs, Imo State, Nigeria

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Abstract Orthometric heights based on GNSS require a geoid model to convert highly accurate ellipsoidal height (h) to the much-desired orthometric height (H) critical to many cadastral, surveying, mapping, engineering and environmental applications. The GNSS uses the default integrated global geoid models (EGM96/EGM2008) for ellipsoidal height conversion to orthometric but for local applications, global model is inadequate and hence development of local geoid models in the absence of a national geoid becomes very critical. The aim of this research is to model orthometric heights from multi-networks of GNSS/Precise levelling in Owerri and environs, Imo State by using ellipsoid heights (h) and the existing orthometric heights (H) from the relationship $N = (h - H)$ for geoid modelling. The objectives are to: investigate the physical status/stability of the existing orthometric heights in Owerri and environs; to carry out GNSS observations on existing controls for ellipsoidal height determination by relative technique, determine geoid undulation of existing controls using $N = (h - H)$; develop Microsoft excel program for interpolation of geoid undulation and hence model orthometric heights; to compare height obtained from model with existing orthometric height by statistical t test. This research adopted the dual-base reference stations approach, static 2 hours DGPS mode for data capture and connection to core stations. The polynomial models used to represent Owerri and environs surface are i) multi-quadratic model and ii) bi-cubic model. For each point, observation equation of the form $AX - L$ was derived. The least squares equation was solved using the online matrix solver (Huobi.pro) for (X) to determine the model coefficient parameters used to develop the geometric geoid model program using the Microsoft Excel 2010. The standard deviations of the geoid model determined orthometric height H are: $\sigma_{\text{multiquadratic}} = 11\text{cm}$ and $\sigma_{\text{bicubic}} = 14\text{cm}$. Computation of t from formula and compared with value of t from t table distribution revealed $[(H)_{\text{multiquadratic}} \text{ and } H_{\text{MSL}}]$ the possibility of coincidence/fit of the two surfaces though based on different vertical datum (geoid and MSL). From computed F1, test statistics and using the standard deviation, no other surface than multi-quadratic is needed to model orthometric heights. Coefficient of correlation (R) and coefficient of Determination (R^2) values of 0.995m and 99% respectively indicate the multi-quadratic model has a high predictive ability. Diagnostic tests confirmed that a multi-quadratic model at 95% confidence limits can be sufficiently adequate for geoid modelling. Also computing $N \sum_{i=1}^n [(ai)]^2$ and comparing with $(1.98/\sqrt{N})$ (N is no of controls=24) at 95% confirmed the validity of using the models for orthometric heights determination. The bias value of zero and skill parameter of one implies total agreement between technique, observations, processing and the model results. The developed model will serve as reliable alternative for orthometric height acquisition at centimeter level accuracy which is adequate for producing topographic maps (at 1m contour



interval), base maps for planning and production of large scale engineering plans. Kriging interpolation method was used to generate contour maps, geoidal maps and the digital elevation models. It is hereby recommended that the developed model (multi-quadratic) be adopted for elevation data acquisition for day to day geospatial data needs in cadastral, mapping, engineering/ Environmental applications.

Keywords Orthometric heights; Geoid modeling; GNSS; Ellipsoidal height; Geoidal maps

1. Introduction

The Nigerian geodetic network is a crucial framework for pinpointing locations on the Earth's surface, encompassing size, shape, orientation, and position parameters with respect to the real Earth [1]. This network's historical roots trace back to the colonial era when it was established to facilitate exploitation and resource evacuation in Nigeria [2]. Its development was initiated with a levelling network in 1891 and later expanded to include a triangulation network in 1912. These triangulations were computed based on the Clarke 1880 reference ellipsoid [2], furnishing provisional coordinates for mapping purposes [3]. Heights were determined through reciprocal vertical angle observations and, where feasible, from levelling data. The entire network was adjusted figurally, using the coordinates at L40 as the origin, and the mean datum was obtained at the north terminal of the Minna base, derived from values obtained through triangulation.

However, the Minna datum, established in 1928, has inherent limitations due to its arbitrary selection and the absence of triangulation to the south and west of Minna [2]. The network's shortcomings are further compounded by distortions inherent in the observations [3], rendering it inadequately equipped to meet the demands of modern positioning projects. Moreover, the geodetic reference system used in Nigeria is founded on the Clarke 1880 reference ellipsoid, with the origin not coinciding with the Earth's center of mass, but rather situated at one of the triangulation network's stations (Agajelu and Moka, 1989). These factors collectively contribute to significant distortions within the Nigerian geodetic network, underscoring the need for its improvement and transformation to a more robust geocentric datum.

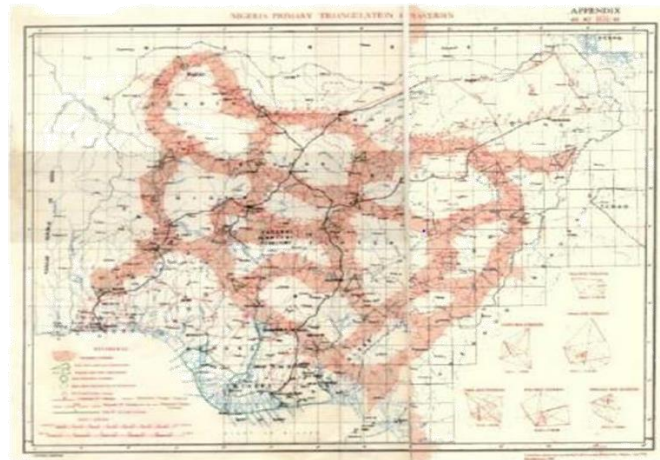


Figure 1: The Nigerian Primary Triangulation Network.

2. Study Area

The study area is Owerri and environs, Imo State, located in the South Eastern Nigeria. This area lies within latitudes 5.5096° N and longitude 7.0391° E respectively. It covers about 279,131.16 hectares of land with various land users such as academic, commercial, religious and residential. Its density is about 364sq.km, Area is 1,019.6 sq.km. It has an estimated population of about 1,401,873 (NPC 2006).



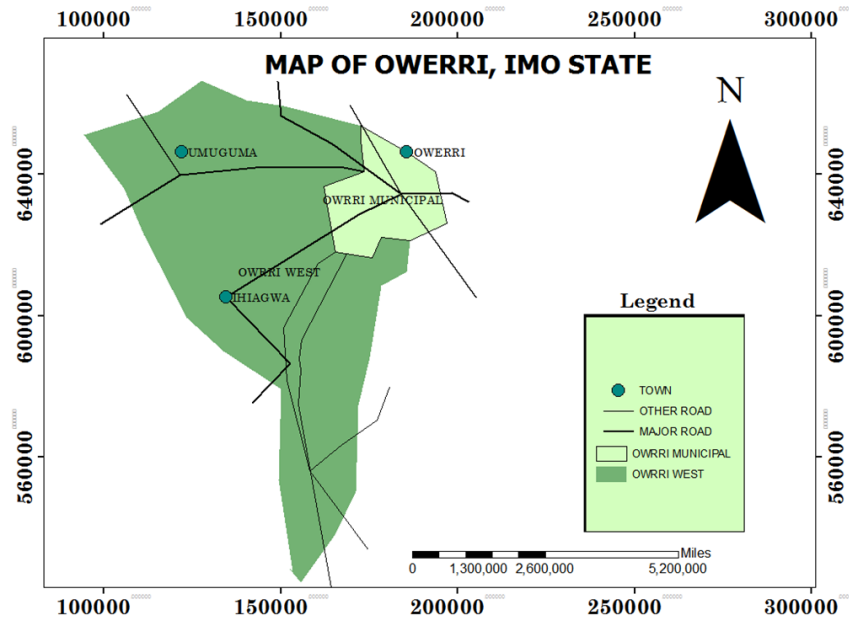


Figure 2: Map Study Area (Source: OSGOF)

3. Material and methods

3.1. Methodology

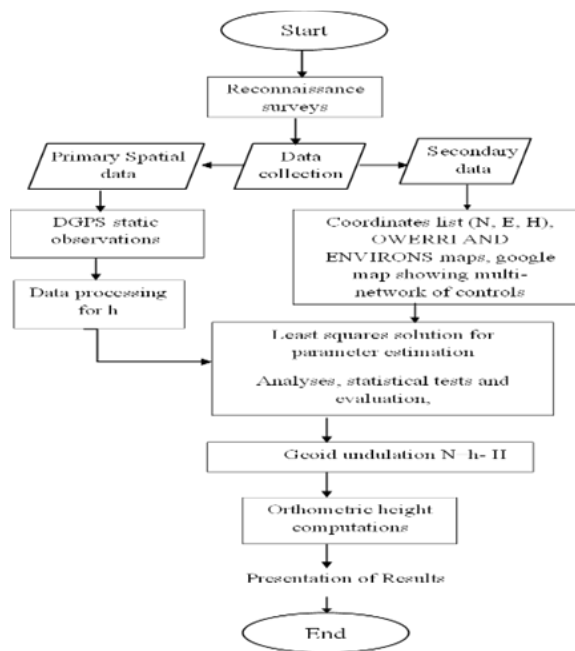


Figure 3: Flowchart of Methodology

4. Results and Discussion

Following the solution of the least squares problem, which determined the coefficients of the polynomial surface model equations, the practical interpolation equation for accurate geoidal undulation calculations within the defined geoid model limiting area (Owerri and environs) is derived. The polynomial constants obtained through this process are then substituted back into the original polynomial equations, resulting in the interpolation equation presented below:

• MULTI - QUADRATIC MODEL A CONSTANTS (9 coefficients)

$$X = \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \alpha_7 \\ \alpha_8 \end{pmatrix} = \begin{pmatrix} 24.224890121000000000 \\ -0.00002409340580587179 \\ -0.00008013620770038382 \\ 0.0000000009699046795 \\ 0.00000000370280953876 \\ 0.00000001166702184889 \\ -0.00000000000021600943 \\ -0.00000000000045716237 \\ 0.0000000000000000886 \end{pmatrix}$$

• BICUBIC MODEL B CONSTANTS (10 coefficients)

$$X = \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \alpha_5 \\ \alpha_6 \\ \alpha_7 \\ \alpha_8 \end{pmatrix} = \begin{pmatrix} 23.50081592604167925515 \\ 0.00004395285221636346 \\ 0.00009105227330487502 \\ -0.00000000156204910634 \\ -0.00000000352164634358 \\ -0.00000000178532065159 \\ 0.0000000000004116279 \\ 0.0000000000002433215 \\ 0.0000000000000729517 \end{pmatrix}$$

The geoidal map was generated by substituting the height (H) values of each point with the geoid undulations (N) and employing kriging interpolation software for surface visualization. It is widely acknowledged that kriging consistently produces surfaces closely aligned with the original surfaces. The outcomes of this process are visually represented in Figure 4.1 and Figure 4.2. Interestingly, the plotting of geoid heights derived from both the multiquadratic and bicubic geoid models yielded indistinguishable geoid surfaces, underscoring the robustness and accuracy of the geoid modelling approach employed.

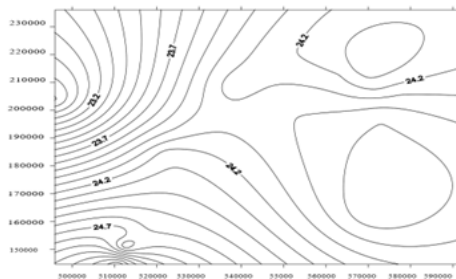


Figure 4.1: Multiquadratic Geoid Height Map

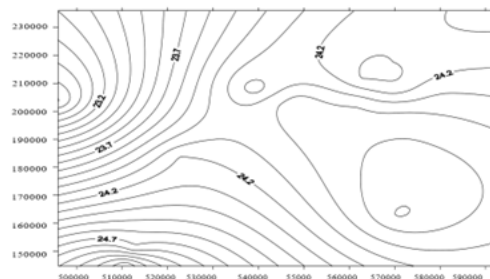


Figure 4.2: Bicubic Geoid Height Map

Orthometric heights, calculated using both models and the pre-existing mean sea level (MSL) orthometric heights, were utilized to create contour maps using kriging interpolation software. The outcomes, as depicted in

Figures 4.3 to 4.5, reveal that the contour maps generated from the two developed models closely mirror each other and align with the contour map of the existing orthometric heights.

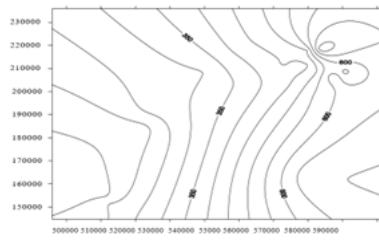


Figure 4.3: Multiquadratic Orthometric Height Map

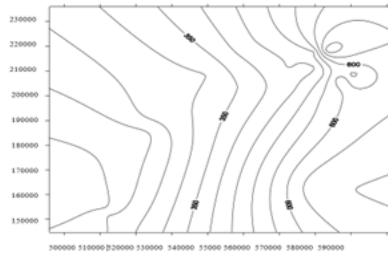


Figure 4.4: Bicubic Orthometric Height Map

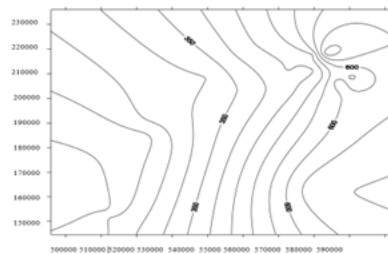


Figure 4.5: Existing Orthometric Height Map

To illustrate the disparities between the multiquadratic and bicubic models, a plot depicting geoid undulation values against control points is presented in Figure 4.6. A visual examination of this figure reveals a high degree of overlap between the two surfaces, affirming their interchangeability and suitability for orthometric height determination within Owerri and its surrounding areas.

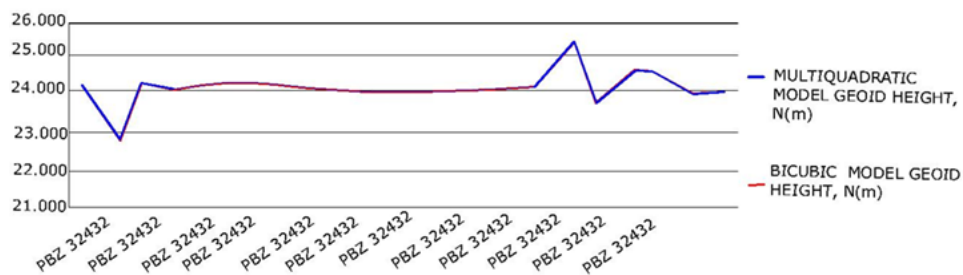


Figure 4.6: geoid undulation models from both multiquadratic and bicubic models

5. Conclusion

This research, focused on GNSS levelling in Owerri and its environs, offers a centimeter-level consistency in orthometric heights, making it valuable for geospatial applications. The developed geometric geoid models provide an alternative to conventional levelling, ensuring accuracy and global compatibility. Both multiquadratic and bicubic models can determine orthometric heights, with multiquadratic offering superior accuracy. This approach can replace traditional levelling over large areas, significantly expediting height data acquisition.

References

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