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## Constrained Predictive Control of PEM Fuel Cell System

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**Abstract** This paper is concerned with Constrained Model Predictive Control (MPC) of Proton Exchange Membrane Fuel Cells (PEMFC) system based on a benchmark model commonly studied in the dynamic PEMFC modeling/control literature. To achieve efficient performance under all operating conditions, a stack power system needs many components beyond the stack itself. This results in a complex system which requires a good control system to respond to stack load current change and to obtain a high operational performance. By adopting a predictive control strategy, the influence of input variable is exploited using the appropriate manipulated variables. The goals of this controller are to regulate parameters of interest in the fuel cell system, and thus, to increase the system operability. MATLAB-Simulink environment is used in this work to perform all the simulation. Results have been shown that only the predictive controller can manage the linear constraints imposed by the compressor without affecting the whole system performance. Because of the chosen research approach, the research results are obtained by simulation using MATLAB-Simulink environment. There no real system to test the proposed control methodology

**Keywords** Proton exchange membrane fuel cell; Predictive control

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### 1. Introduction

Reducing environmental pollution, have escalated the necessity of finding alternative green energy sources where fuel cells (FC) are viable choices [1]. FC provide a cleaner and more efficient form of power delivery compared to internal combustion engines [2]. Approximately, 70% of today's electrical power consumption is generated from fossil combustibles [3]. Currently fuel cells are employed in a variety of applications including stationary backup power, distributed power generation, portable electronic equipment, and in the transportation industry [1,2].

FC are electro chemical generators that convert chemical potential into electrical power. They are highly efficient and environmentally clean engines [1]. Compared to others power converters such as batteries and combustion engines, FC have high conversion efficiency and clean by-products. A variety of fuel cells for different applications are under development. The (PEMFC) operate at low temperatures, possess a compact design, and lack any corrosive fluid hazard [2]. FC, are composed of several peripheral components such as the humidifier, reformer, or heat exchanger. Figure 1 depicts a general schematic of an PEMFC system.

The heart of the PEMFC system is formed of a membrane placed between two electrodes. Hydrogen is used as a fuel, when oxidized, producing electrons and protons. The electrons are used in an external circuit, where they can perform work, while the protons travel through a membrane to the cathode. At the cathode water is formed by the reaction of oxygen with the electrons from the external circuit and protons from the anode.

In fuel cells system, maintaining optimal conditions requires good control actions performed by actuators like as pumps, valves, motors, vanes. As current is drawn, the control system must maintain optimal hydration of membrane, partial pressure of reactant gases and the temperature, to prevent performance degradation. These



important parameters must be regulated for many power and current levels. The resulting auxiliary actuator system, shown in Fig. 1, is needed to make fast and fine adjustments to satisfy performance, safety and reliability.

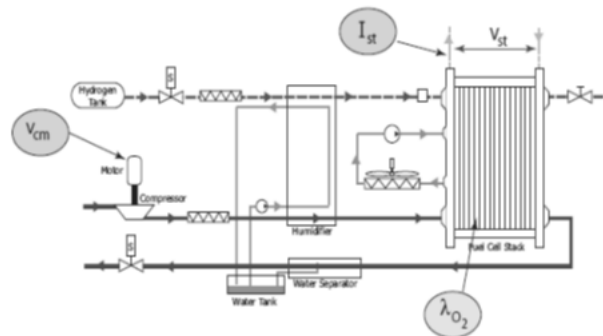


Figure 1: Schematic diagram of stack

There are many approaches to control PEMFC system. In [5], the oxygen excess ratio  $\lambda_{O_2}$  can be controlled to maximize the system power. In Refs. [2,5,6,7], the oxygen excess ratio is controlled by the compressor voltage. In Refs. [2,5], feedforward control was used, while in Refs. [6,7], (MPC) is used. These works indicate that there is a severe trade of between net power dynamic response and oxygen excess control.

In order to control  $\lambda_{O_2}$  in the cathode the voltage applied to the compressor is a suitable control variable, as it is showed in Refs. [2,5,6,7]. In this paper, we use the compressor voltage and a regulating valve for the cathode outlet flow as control variables to control the  $\lambda_{O_2}$  in the cathode.

The remainder of this paper is organized as follows: in Section 2, a nonlinear model of PEMFC together with its auxiliaries is reviewed. Constrained MPC principles are recalled in Section 3. In Section 4, simulation results are presented. Finally, the major conclusions are drawn in Section 6.

## 2. Model of the Fuel Cell System

In this section, the FC model used for the control study is presented. The schematics are shown in Figure 1. The focus of the work is not in the model itself, therefore, not all the details will be brought forth, but rather the model structure. For further information on the model equations, see [4].

In [4], fuel cell stack together with its auxiliaries is modelled based on electrochemical, thermodynamic and fluid flow principles. The model is developed for control design. All subsystem like the compressor, manifold dynamics, cooling system, the humidifier, membrane hydration, anode and cathode flow and stack voltage are modelled. Since the goal in this paper is control of air flow, we present the models, essential to capture the dynamics between the compressor and the air flow into the cathode.

Low partial  $O_2$  pressure decreases the fuel cell voltage and the generated power and can reduce the life of the stack. To prevent such a situation the  $O_2$  level in the cathode needs to be regulated. A single parameter can be defined to indicate the  $O_2$  level status in the cathode. Oxygen excess ratio (OER),  $\lambda_{O_2}$  is defined for this purpose as follows:

$$\lambda_{O_2} = \frac{W_{O_2,in}}{W_{O_2,rect}} \quad (1)$$

where  $W_{O_2,in}$  is the of flow of oxygen into the cathode and  $W_{O_2,rect}$  is the mass of oxygen reacted in the cathode. Therefore low values of  $\lambda_{O_2}$  are an indication of oxygen starvation. The rate of oxygen reacted,  $W_{O_2,rect}$ , depends on the load current,  $I_{fc}$ :

$$W_{O_2,rect} = M_{O_2} \frac{nI_{fc}}{4F} \quad (2)$$

where  $n$  is the number of cells in the stack and  $F$  is the Faraday number ( $F=96485$  Coulombs). Therefore, with increase in load current,  $\lambda_{O_2}$  decreases. To fix the level of OER, more air must be supplied to the fuel cell. The oxygen flow rate,  $W_{O_2,in}$  is a function of the air flow rate,  $W_{sm}$ :



$$W_{O_2,in} = 0.21 \frac{M_{O_2}}{M_a^{atm}} \frac{1}{1 + \Omega_{atm}} W_{sm} \quad (3)$$

where  $M_a^{atm}$  is the oxygen mass ratio in the dry atmospheric air and  $\Omega_{atm}$  is the humidity ratio of the atmospheric air. The mass out flow rate of the supply manifold,  $W_{sm}$  depends on the downstream (cathode) pressure and upstream (supply manifold) pressure,  $p_{sm}$  and temperature,  $T_{sm}$ . The cathode total pressure depends on the partial pressure of the (i) oxygen that is supplied,  $W_{O_2,in}$  reacted,  $W_{O_2,rc}$  and the oxygen removed, (ii) nitrogen that is supplied and removed and (iii) the water that is supplied, generated, transported through the membrane and removed. The additional cathode states of oxygen mass,  $m_{O_2}$ , nitrogen mass,  $m_{N_2}$ , water vapor mass,  $m_{w,ca}$  total return manifold pressure,  $p_{rm}$ , and anode states of hydrogen mass,  $m_{H_2}$  and water vapor,  $m_{w,an}$  are needed to capture the temporal dynamics of the total cathode pressure during a step change in current,  $I_{fc}$ . These detailed state equations are omitted here but can be found in [4]. However, to allow the reader understand how the control input affects the supply manifold flow,  $W_{sm}$  we add the following relations. Specifically, the supply manifold pressure,  $p_{sm}$  and mass,  $m_{sm}$  are related to the compressor's air flow,  $W_{cp}$  and temperature,  $T_{cp}$  with the following dynamics:

$$\frac{dp_{sm}}{dt} = K_{sm} (W_{cp} T_{cp} - W_{sm} T_{sm}) \quad (4)$$

$$\frac{dm_{sm}}{dt} = W_{cp} - W_{sm} \quad (5)$$

where  $K_{sm}$  is a coefficient determined by air specific heat coefficients and the manifold volume. The supply manifold temperature,  $T_{sm}$ , is defined by the ideal gas law. The compressor air flow,  $W_{cp}$ , and its temperature,  $T_{cp}$ , depend on the compressor rotational speed,  $\omega_{cp}$ :

$$J_{cp} \frac{d\omega_{cp}}{dt} = \frac{1}{\omega_{cp}} (P_{cm} - P_{cp}) \quad (6)$$

where  $J_{cp}$  is the compressor inertia and  $P_{cp}$  is the power absorbed by the compressor. The power supplied to the compressor,  $P_{cm}$ , is a function of compressor motor voltage,  $V_{cm}$ . In summary, the compressor voltage,  $V_{cm}$ , controls the speed of the compressor through the first-order nonlinear dynamics shown in (6). Speed of the compressor determines the compressor flow rate,  $W_{cp}$ , which then through equation (4) affects the supply manifold pressure,  $p_{sm}$ , which together with the cathode pressure, determines the supply manifold flow,  $W_{sm}$ , and finally flow rate of the oxygen into the cathode,  $W_{O_2,in}$ .

The fuel cell voltage  $v_{fc}$  is given in the form of polarization curves or a nonlinear map of current density,  $i$ , and other anode and cathode variables. Many fuel cells are connected in series to form a Fuel Cell Stack (FCS), hence, the total FCS voltage and power are  $V_{st}=nv_{fc}$  and  $P_{st}=nA_{fc}v_{fc}I_{fc}$ . The air compressor is a major contributor of parasitic loss in the fuel cell system. Therefore, the net power obtained from the fuel cell stack system is

$$P_{net} = P_{st}(x_{nl}, I_{st}) - P_{cm}(x_{nl}, V_{cm})$$

The set of equations described above, form a set of first-order nonlinear differential equations:

$$\begin{aligned} \dot{x}_{nl} &= h(x_{nl}, u, w) \\ u &= [V_{cm} \quad I_{fc}]^T \\ y &= [P_{net} \quad \lambda_{O_2} \quad V_{st}]^T \end{aligned} \quad (7)$$

where  $x_{nl}$  is the state vector of the nonlinear dynamic system. For the control design purpose, this augmented nonlinear system is linearized around a selected operating point. We define nominal stack current of  $I_{fc0}$ . The nominal value for oxygen excess ratio is selected at  $\lambda_{O_2}=2$ , which corresponds to maximum fuel cell net power for the nominal current. The compressor motor voltage needed, to supply the optimum air flow that corresponds to  $I_{fc0}$  and  $\lambda_{O_2}$ , is  $V_{cm0}$ . The linearized system has eight dynamic states and is described by:



$$\begin{aligned}\dot{x}_l &= Ax_l + Bu \\ y &= Cx_l + Du\end{aligned}\quad (8)$$

where the variables  $x_l$  and  $y$  show deviations from their nominal values. The linear state vector is:

$$x_l = \begin{bmatrix} m_{O_2} & m_{H_2} & m_{N_2} & w_{cm} & p_{sm} & m_{sm} & m_{w,an} & p_{rm} \end{bmatrix}^T \delta \quad (9)$$

where  $\delta$  stands for the deviation from the operating point. A discretized version of this linear model is used for control design in this paper. The nonlinear model (7) is used in nonlinear closed-loop simulations.

### 3. MPC Control Scheme With Constraints

MPC is a model-based control approach that utilizes a model of the system to project the future response as a function of control inputs and known disturbances; it then determines the optimal control inputs by minimizing a performance index over a finite prediction horizon. Pointwise-in-time constraints on the inputs, outputs, and states can be explicitly enforced in the optimization. The first control input from the calculated sequence of optimal inputs is applied to the system, and the optimization process is repeated at every time step in a receding horizon fashion. When the model and constraints are linear and the performance index is a quadratic function of the states and the inputs, the MPC problem can be cast as a quadratic programming problem for which efficient solutions exist. Model Predictive Control (MPC) has become the accepted standard for complex constrained multivariable control problems in the process industries. From a mathematical point of view, the problem is posed as to find the optimal control signal that minimizes the following finite horizon performance cost function:

$$J = \sum_{k=0}^{N_p-1} x_{t+k|t}^T Q x_{t+k|t} + \sum_{k=0}^{N_c-1} u_{t+k|t}^T R u_{t+k|t} \quad (10)$$

The variables  $x_{t+k|t}$  denote predicted states, given an input sequence  $u_{t+k|t}$ , a state estimate  $x_{t|t}$  and considering a discrete space state model of the system.  $N_p$ ,  $N_c$  are predictive and control horizon respectively,  $Q$ ,  $R$  positive defined matrices.

The cost function (10) subject to the model equation (8) and inequality constraints is minimized at each sample time to determine the sequence of the next NP control inputs  $U_i(k)=[u(k) \ u(k+1) \ \dots \ u(k+N-1)]$  over the future horizon P. When N the remaining control moves  $[u(k+N) \ u(k+N+1) \ \dots \ u(k+P-1)]$  are assumed to be zero. Here, the control input  $u$  (also called manipulated variable). According to the standard MPC design, only the first entry of the control sequence  $U_i(k)$  is applied to the system, the optimization horizon is moved one step forward, the model and constraints are updated if necessary, and the optimization process is repeated to obtain the next optimal control sequence  $U_i(k+1)$ . With a linear model of the process and linear constraints and the quadratic cost in (8), this dynamic optimization problem can be cast as quadratic program for which efficient real-time solutions exist.

### 4. Simulation Results

In this paper MPC of oxygen excess ratio in FC system is implemented using MATLAB MPC Toolbox [8]. The Fuel Cell System linear model used to implement the MPC is derived, through a linearization at operating point:  $P_{net0}=41\text{kW}$ ,  $\lambda_{O_2}=2$  and  $V_{st0}=235\text{V}$  in measured variables;  $I_{st0}=191\text{A}$  in measured input disturbances; and  $V_{cm0}=164\text{V}$  in manipulated variable. MPC weights  $Q$  and  $R$ , are tuned to desired control goals. Following the linear model in equation (8), there are three measured outputs (Stack Net Power, Oxygen Excess Ratio and Stack Voltage), but only the oxygen excess ratio measurement is controlled by the implemented MPC controller. Thus, after some "trial and error" experimentation, the weight associated to this variable has a value of 10 for a good performance control. The control horizon  $N_c$ , the prediction horizon  $N_p$ , and the matrices were adjusted, in the control algorithm, to achieve an adequate dynamic behaviour of the fuel cell system.

The air compressor voltage is modeled as a constraint input due to physical limits (maximum compressor voltage cannot exceed 230V, and voltage value is never negative). The oxygen excess ratio is modeled using output constraint (the operating range is between 1.5 and 3) in order to avoid starvation. However, this last



restriction can not be implemented because the electrochemical dynamics are much faster than fluid performances. It leads to the physical impossibility to prevent the drastic reduction in oxygen concentration when a step change in current occurs [9]. This constraint can only be satisfied when auxiliary components such as batteries or ultra capacitors are used. Nevertheless, as it is shown below, the oxygen concentration transient response afterwards the first reduction, is improved by the control techniques designed. Notice that this output constraint is implemented as a soft constraint in the MPC toolbox in order to prevent the infeasible solution. Fig. 3, shows the evolution of the oxygen excess ratio. A series of step changes in stack current are applied to the stack. This variable is considered as measured disturbance for MPC controller. The compressor voltage is the control action computed by MPC. Notice that the control goal is achieved, providing a maintained value "2.0" of oxygen excess ratio.

The simulation results of the controlled system with variations in the load current are presented in Figs. 2–6. In Fig. 2, the perturbation profile is showed. Figs. 3 and 4 show the controlled variables ( $\lambda_{O_2}$  and  $V_{st}$ ), while Figs. 5 and 6 show the manipulated variables ( $V_{cm}$  and  $A_t$ ). As can be observed in the figures, the implemented controller has a good disturbance rejection.

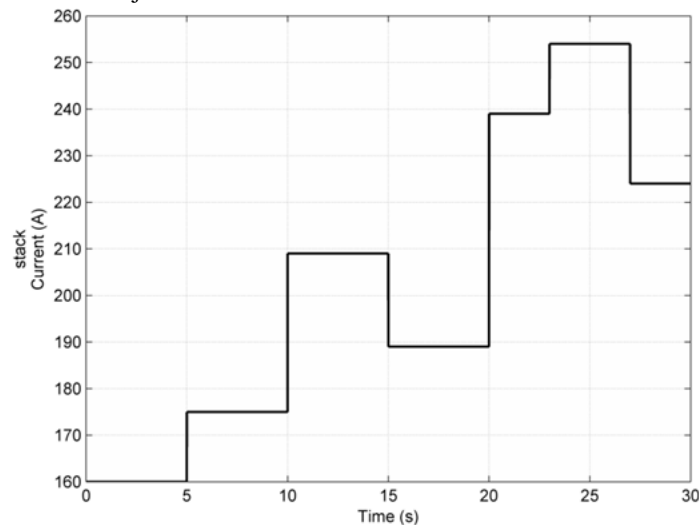


Figure 2: Fuel cell stack current.

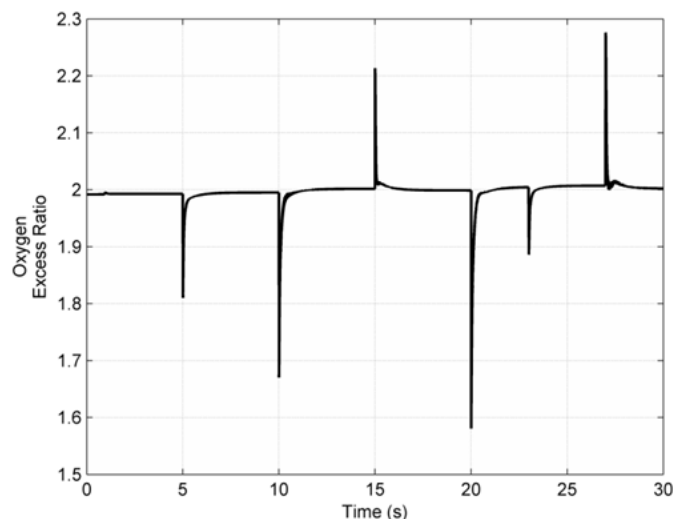


Figure 3: Oxygen excess ratio



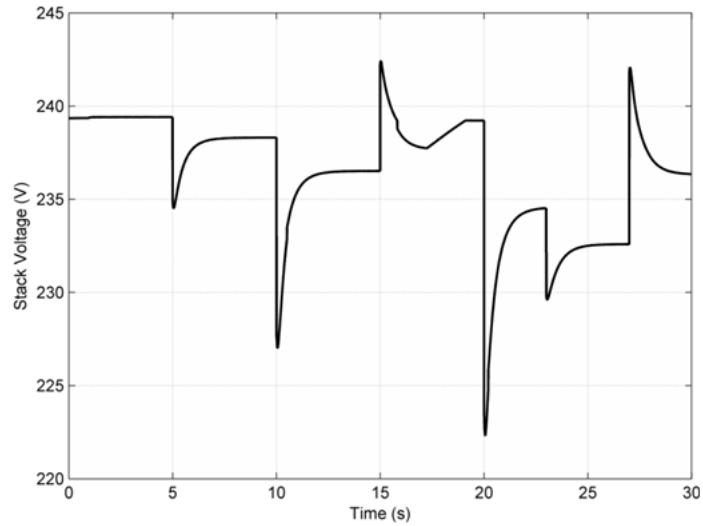


Figure 4: Stack Voltage.

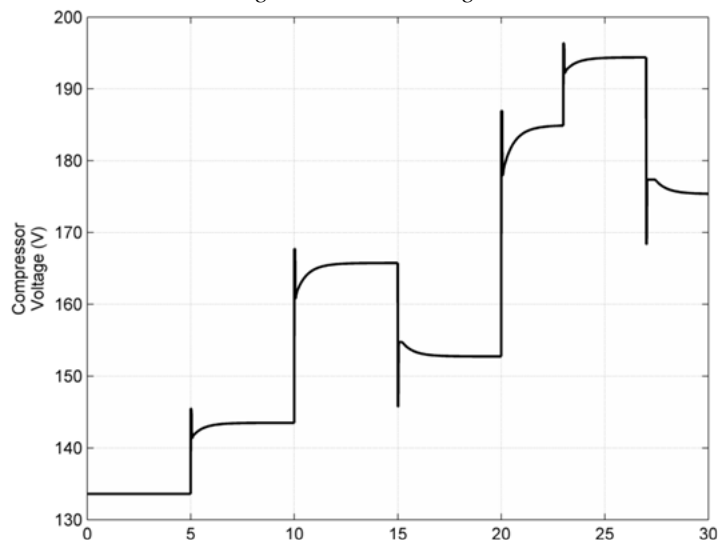


Figure 5: Compressor Voltage

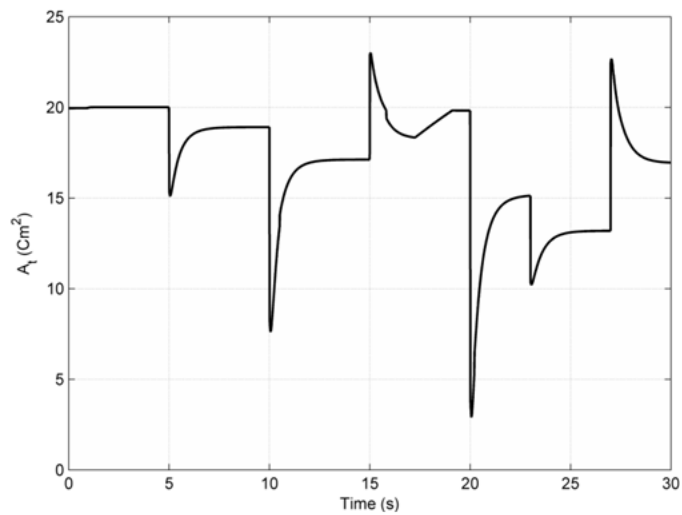


Figure 6: Cathode air flow opening valve area.

## 5. Conclusions

In this paper, a Model Predictive Control based technique for the Fuel Cell Systems has been presented. Results have been shown that only the predictive controller can manage the linear constraints imposed by the compressor without affecting the whole system performance.

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