



Study of the effects of shading under an energy recovery slab in contact with sawdust thermal insulation in a dynamic frequency regime

Mamadou Bamba Ndiaye*, Seydou Faye, Mor Ndiaye and Issa Diagne

*Laboratory of Semiconductors and Solar Energy, Physics Department, Faculty of Science and Technology, University Cheikh Anta Diop, Dakar, Senegal

Abstract In this article, we propose a study of the effects of shading, i.e. cloud cover under an energy-recovery slab in contact with thermal insulation. The expression of the temperature and the heat flux density are obtained from the heat equation and Fourier's law in frequency dynamics. The influences of parameters such as convective and radiative heat exchange coefficients are presented.

Keywords Shading effects - Energy recovery slab - thermal insulation - Sawdust - Frequency dynamic regime

Introduction

Faced with the multiple difficulties noted in the energy supply of our countries, it is necessary to optimize our consumption. In Senegal, energy consumption in housing for domestic use occupies an important part in the production of SENELEC. Thus, regulations on the thermal insulation of buildings will not only reduce the energy bill but also contribute to improving the thermal and acoustic comfort of our fellow citizens [1], [2].

It is in this perspective that we are interested in energy consumption, economy but also thermal insulation [3]. Mixing an insulating material with a building material is an additional process to improve its thermal insulation properties [4].

In the 1970s, an energy saving campaign was carried out in industrialized countries and led to a lack of development in the thermal insulation sector [5]. Thermal insulation is therefore very important in terms of environmental protection of the comfort of individuals and the economy of finances [6].

The purpose of this article is to study the effects of shading, i.e. cloud cover through the heat recovery slab and the thermal insulation from the temperature and the heat flux density.

Presentation of the model of study

The study model is a wall composed of two layers, and subjected to climatic stresses at the level of the external face. The other side is fixed to a thermal insulator, the sawdust attached to plywood.



The latter is in perfect contact with the external environment of a habitat.

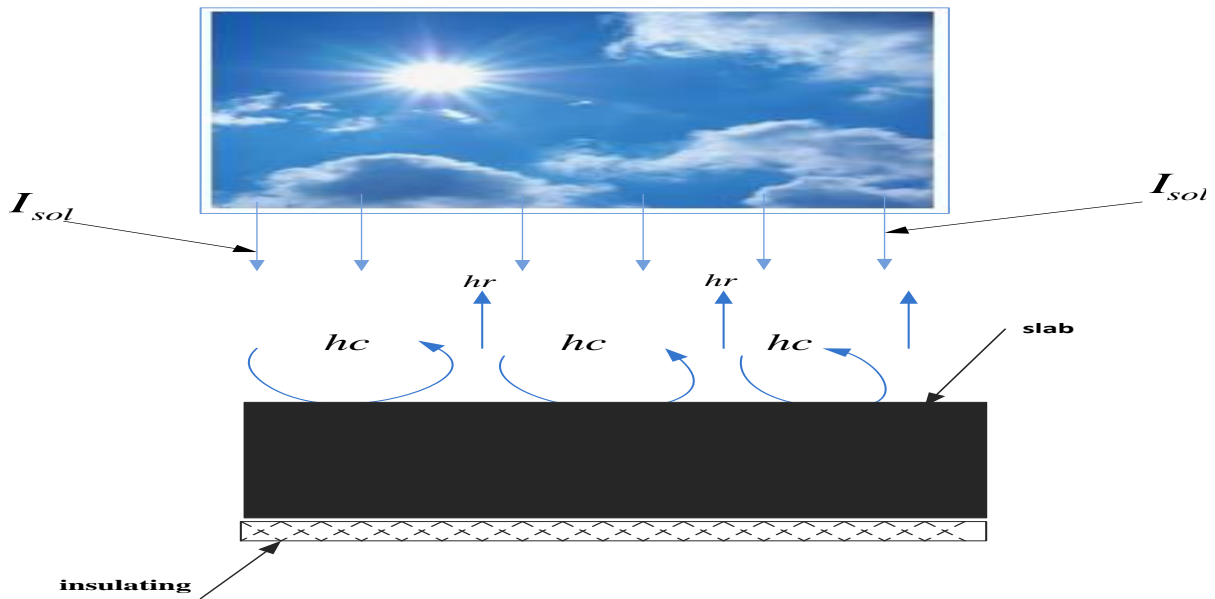


Fig. 1 Study model

Mathematical Model

Figure 2 presents the simplified diagram of the device composed of two layers: the energy-recovering concrete slab and the sawdust used as thermal insulation with respective thicknesses L_1 and L_2 as indicated in the figure already named.

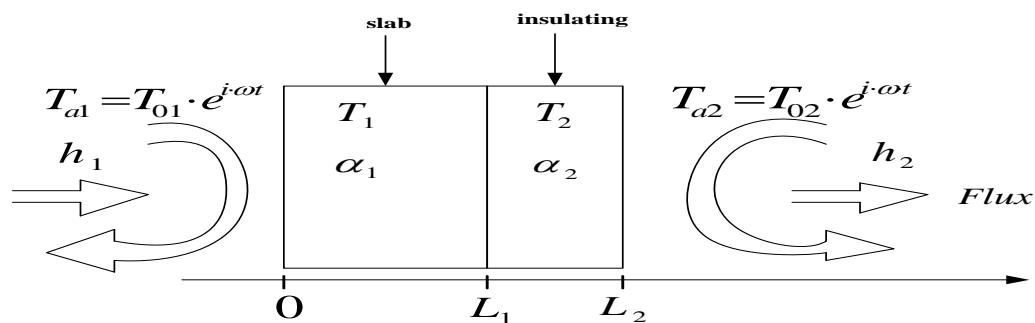


Fig. 2 Cross section of study model

T_{a1} : Ambient temperature of environment 1 (outside) in °C

T_{a2} : Ambient temperature of environment 2 (indoor) in °C

h_1 : Coefficient of thermal exchange at the front face

h_2 : Heat transfer coefficient on the rear face

The Expression of the temperature

When the two layers are subjected to external climatic stresses on both sides, there is a heat transfer governed by the heat equation given by:

$$\rho \cdot c \frac{\partial T}{\partial t} = \lambda \cdot \Delta T + P_p \tag{1}$$

ρ ($\text{Kg} \cdot \text{m}^{-3}$): the density of the material

C ($\text{J} \cdot \text{kg}^{-1} \cdot \text{K}^{-1}$): the specific heat of the material

λ ($\text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1}$): the thermal conductivity of the material

P_p ($W.m^{-1}$): the heat sink or internal heat source

T (K): the temperature at a considered point

Considering that there is no internal heat production [7,8] therefore $P_p = 0$. Thus, we establish two equations describing the heat transfer inside the energy recovery slab (2) and in the sawdust (3).

$$\frac{\partial^2 T_1(x, h_c, h_r, C_n, \omega, t)}{\partial x^2} - \frac{1}{\alpha_1} \frac{\partial T_1(x, h_c, h_r, C_n, \omega, t)}{\partial t} = 0 \quad (2)$$

$$\frac{\partial^2 T_2(x, h_c, h_r, C_n, \omega, t)}{\partial x^2} - \frac{1}{\alpha_2} \frac{\partial T_2(x, h_c, h_r, C_n, \omega, t)}{\partial t} = 0 \quad (3)$$

$$\text{With } \alpha_1 = \frac{\lambda_1}{\rho_1 \times C_1} \quad \alpha_2 = \frac{\lambda_2}{\rho_2 \times C_2}$$

α_1 : The thermal diffusivity coefficient relative to the slab ($m^2.s^{-1}$)

α_2 : The coefficient of thermal diffusivity relative to sawdust ($m^2.s^{-1}$)

$T_1(x, h_c, h_r, \omega, t, C_n)$: the temperature at a point considered in the heat recovery slab

$T_2(x, h_c, h_r, \omega, t, C_n)$: the temperature at a considered point in the insulation.

To solve these equations, it is first necessary to define the boundary conditions.

Interface between the external environment and the heat recovery slab

$$-\lambda_1 \left. \frac{\partial T_1}{\partial x} \right|_{x=0} = h_c (T_{a1} - T(0, t)) + h_r (T_s - T(0, t)) + I_{sol} \quad (4)$$

h_c : Coefficient of heat exchange by convection

h_r : Coefficient of heat exchange by radiation

T_{a1} : Room temperature of environment 1

T_s : Temperature provided by the sun in the ambient environment

I_{sol} : Solar flux arriving on the surface of the first layer

The expression of the solar flux is proposed by the following relation [9]:

$$I_{sol} = (0,828b + I) \cdot \left(1 - \frac{C_n}{100} \right) \quad (5)$$

It combines the expression of the solar flux I at the top of the atmosphere [10] with the coefficient of interception of the solar flux by the atmosphere $47,4 W.m^{-2}$ [11].

Interface between the heat recovery slab and the insulation

$$\lambda \left. \frac{\partial T_1}{\partial x} \right|_{x=L_1} = \lambda_2 \left. \frac{\partial T_2}{\partial x} \right|_{x=L_1} \quad (6)$$

$$T_1(L_1) = T_2(L_1) \quad (7)$$



Interface between the thermal insulation and the interior environment

$$-\lambda_2 \left. \frac{\partial T_2}{\partial x} \right|_{x=L_2} = h_2 (T_2 - Ta_2) \tag{8}$$

$$T_2(L_2) = T_p \tag{9}$$

T_p : temperature of the platform assumed constant in this work, the initial temperature and considered uniform inside the two layers $T_i = T_{0i}$.

To introduce the initial temperature T_i , one carries out the changes of variable of temperature

In the heat recovery slab

$$\bar{T}_1(x, h_c, h_r, \omega, t, C_n) = T_1(x, h_c, h_r, \omega, t, C_n) - T_{i1} \tag{10}$$

In the thermal insulation layer

$$\bar{T}_2(x, h_c, h_r, \omega, t, C_n) = T_2(x, h_c, h_r, \omega, t, C_n) - T_{i2} \tag{11}$$

The heat equations become:

In the layer of the heat recovery slab

$$\frac{\partial^2 \bar{T}_1(x, h_c, h_r, \omega, t, C_n)}{\partial x^2} - \frac{1}{\alpha_1} \frac{\partial \bar{T}_1(x, h_c, h_r, \omega, t, C_n)}{\partial t} = 0 \tag{12}$$

In the thermal insulation layer

$$\frac{\partial^2 \bar{T}_2(x, h_c, h_r, \omega, t, C_n)}{\partial x^2} - \frac{1}{\alpha_2} \frac{\partial \bar{T}_2(x, h_c, h_r, \omega, t, C_n)}{\partial t} = 0 \tag{13}$$

It can be written in the form

$$\frac{\partial^2 \bar{T}_i(x, h_c, h_r, \omega, t, C_n)}{\partial x^2} - \frac{1}{\alpha_i} \frac{\partial \bar{T}_i(x, h_c, h_r, \omega, t, C_n)}{\partial t} = 0 \tag{14}$$

With $i=1,2$ et $\alpha_i = \frac{\lambda_i}{\rho_i \times c_i}$

To solve the equations, we will use the method of separations of variables (depth x and time t). We will take into account the changes of the variables made with the temperature. Thus, we have:

$$\bar{T}_i(x, h_c, h_r, \omega, t, C_n) = X_i(x) \cdot Y_i(t) \tag{15}$$

After resolution (see appendix), we obtain the expression of the temperature:

In the energy recovery slab

$$T_1(x, h_c, h_r, \omega, t, C_n) = [A_1 \sinh(\beta_1(\omega) \cdot x) + B_1 \cosh(\beta_1(\omega) \cdot x)] \cdot e^{i\omega t} + T_{i1} \tag{16}$$

In the sawdust

$$T_2(x, h_c, h_r, \omega, t, C_n) = [A_2 \sinh(\beta_2(\omega) \cdot x) + B_2 \cosh(\beta_2(\omega) \cdot x)] \cdot e^{i\omega t} + T_{i2} \tag{17}$$

With $\beta_1(\omega) = \sqrt{\frac{\omega}{2\alpha_1}}(1 + i)$ $\beta_2(\omega) = \sqrt{\frac{\omega}{2\alpha_2}}(1 + i)$

The boundary conditions and the initial condition below, make it possible to obtain the coefficients

A_1, B_1, A_2 and B_2

$$\left\{ \begin{aligned} -\lambda_1 \frac{\partial \bar{T}_1(x)}{\partial x} \Big|_{x=0} &= h_c [T_{a1} - \bar{T}_1(0,t) - T_{i1}] + h_r [T_s - \bar{T}_1(0,t) - T_{i1}] + I_{Sol}; (18) \\ \lambda_1 \frac{\partial \bar{T}_1(x)}{\partial x} \Big|_{x=L1} &= \lambda_2 \frac{\partial \bar{T}_2(x)}{\partial x} \Big|_{x=L1}; (19) \\ \bar{T}_1(L_1) + T_{i1} &= \bar{T}_2(L_1) + T_{i2}; (20) \\ -\lambda_2 \frac{\partial \bar{T}_2(x)}{\partial x} \Big|_{y=L2} &= h_2 [T_2 - T_{a2}]; (21) \end{aligned} \right.$$

Thus, the coefficients A_1, B_1, A_2 and B_2 are obtained after calculation (see appendix).

Heat flux density is the heat flux per unit area. It is expressed in Watt per square meter w/m^{-2} given by Fourier's law:

$$\vec{\varphi} = -\lambda \overrightarrow{grad}T \tag{22}$$

Results and Discussion

Evolution of temperature and heat flux density as a function of decimal logarithm of the pulsation: under illumination

In the concrete slab

The figures below show the influence of the depth for different values of the thickness of the concrete slab on the temperature distribution as a function of the decimal logarithm of the pulsation in the illumination zone.

For different depth values of the energy recovery slab, the curves show the same profile.

For pulsations lower than $10^{-4} rad/s$, the temperature is constant and maximum turning around $45^\circ C$. These very low pulsations correspond to long periods of stress, resulting in significant heat diffusion. Then, for pulsations between 10^{-4} et $10^{-3} rad/s$, there is a drop in temperature. This variation describes a heat transfer phenomenon corresponding to a variable regime. And finally, for pulsations greater than $10^{-3} rad/s$, the temperature is almost observed and minimal. The exchange periods are short and the quantities of heat exchanged are less significant with the external environment.

The modulus of the heat flux density increases with the pulsation until it reaches a maximum value of about 800 and then a decrease is noted. This extremum corresponds to an important storage of energy and the decrease is due to the loss of heat.

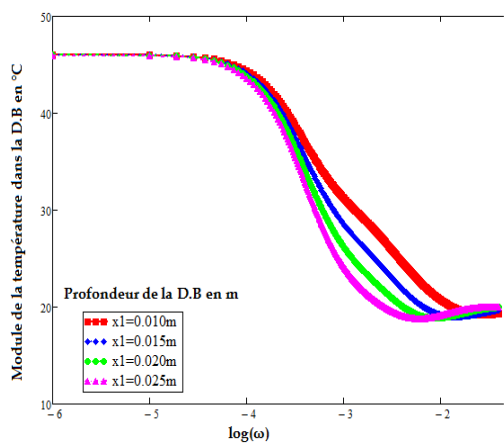


Figure 3-a

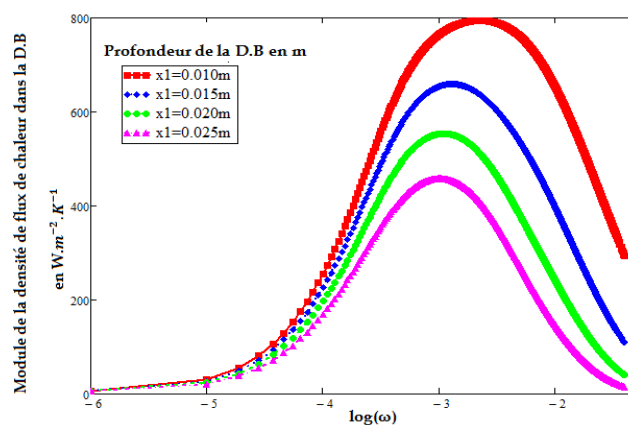


Figure 3-b

Fig. 3 Modulus of temperature (figure 2-a) and heat flux density (figure 2-b) as a function of the decimal logarithm of the pulsation $h_1=0,05 (W. m^{-2}. K^{-1})$; $h_r=100 (W. m^{-2}. K^{-1})$; $h_2=0,05$; $C_n=0\%$; $I=800 (W. m^{-2})$.

In the sawdust

The figures below show the influence of depth for different sawdust thickness values on the temperature distribution as a function of the decimal logarithm of the pulsation in the illumination zone.

The temperature curves at the level of the sawdust in the illumination zone, i.e. at a cloud cover of 0%, have the same appearance as that in the energy recovery slab.

It can be seen that for exciter pulses lower than $10^{-4}rad/s$ the transmitted temperature is important favoring a very low energy. It can be seen that the deeper you go, the lower the temperature. For pulsations between 10^{-4} et $10^{-3}rad/s$ there is a drop in temperature which corresponds to a diffusion of energy and finally for pulsations greater than $10^{-3}rad/s$ there is thermal equilibrium in the material.

The heat flux density modulus is quite low for pulses less than $10^{-4}rad/s$. it increases until it reaches a maximum. This extremum is greater for shallow depths and decreases considerably. Thus the quantity of heat is all the more important when one is on the surface, but in depth one notes a slight loss of heat.

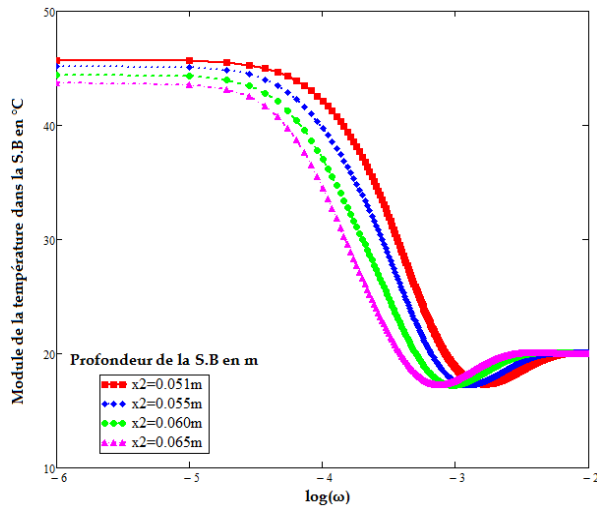


Figure 4-a

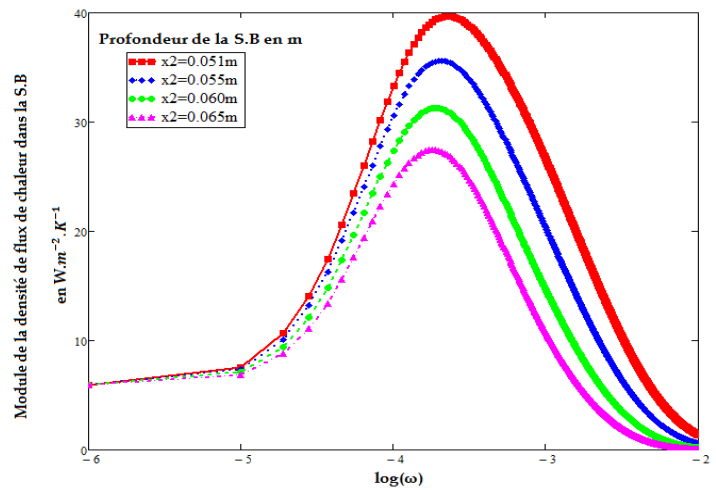


Figure 4-b

Fig. 4 Module of the temperature (figure 3-a) and the heat flux density (figure 3-b) according to the decimal logarithm of the pulsation. $h_1=0,05 (W. m^{-2}. K^{-1})$; $h_r=100 (W. m^{-2}. K^{-1})$; $h_2=0,05$; $C_n=0\%$; $I=800 (W. m^{-2})$.

Evolution of temperature and heat flux density as a function of decimal logarithm of the pulsation: under shade

In the concrete slab

The figures below show the influence of the depth for different values of the thickness of the concrete slab on the temperature distribution as a function of the decimal logarithm of the pulsation in the shadow zone, i.e. at a cloud cover of 100%.

For relatively low pulsation values correspond to a long period of thermal stress. The temperature at different depths and maximum and constant bordering that of the external environment about 36°C. These curves give practically the same profiles as the previous ones. But with the only difference that here we are in a shaded area corresponding to lower temperatures than in the lighting area.

The curves of this figure have the same profiles as those of figures (2a and 2b). We notice a drop in temperature for low frequencies corresponding to long periods of climatic stresses.

In the shadow zone, the curves of the heat flux density as a function of the decimal logarithm of the exciting pulsation under the influence of depth describe three phases:

The first where the heat flux density is low. Then, there is an increase reflecting a significant energy storage.

This amplitude is greater in the illumination zone, i.e. at 0% cloud cover and finally a return of it corresponds to a loss of thermal energy.



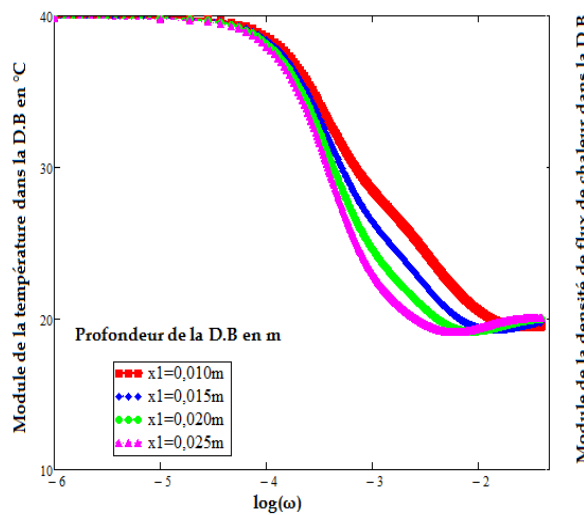


Figure 5-a

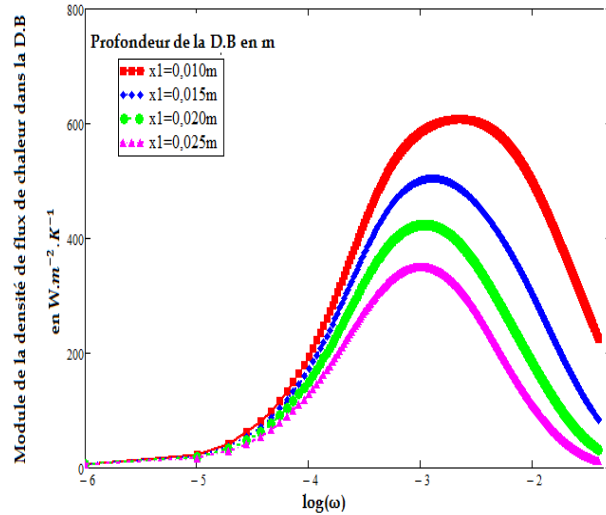


Figure 5-b

Fig. 5 Temperature modulus (figure 4-a) and heat flux density (figure 4-b) as a function of the decimal logarithm of the pulsation. $h_1=100$ ($W \cdot m^{-2} \cdot K^{-1}$); $h_r=0,05$ ($W \cdot m^{-2} \cdot K^{-1}$); $h_2=0,05$; $C_n=100\%$; $I=800$ ($W \cdot m^{-2}$).

In sawdust

The figures below show the influence of depth for different values of sawdust thickness on the temperature distribution as a function of the decimal logarithm of the pulsation in the shadow zone, i.e. at 100% cloud cover. For relatively low values of pulsation corresponding to a long period of climatic stress, the temperature at the levels of the different depths is maximum and decreases with depth.

These curves give practically the same profiles as the previous ones. But with the only difference that here we are in a shaded area, that is to say where the cloud cover is 100% corresponding to lower temperatures than in the illumination area.

For low frequencies there is a drop in the maximum temperature. The frequency band corresponding to the temperature variations remains practically constant for the different depths of the slab.

The modulus of the heat flux density as a function of the decimal logarithm of the exciting pulsation describes a thermal phenomenon for the same frequency bands previously defined.

The heat flux density is low for long periods of stress. It then increases until it reaches a maximum. We note that this extremum is greater in the light zone than in the shadow zone. However, the amount of heat is all the more important when you are on the surface.

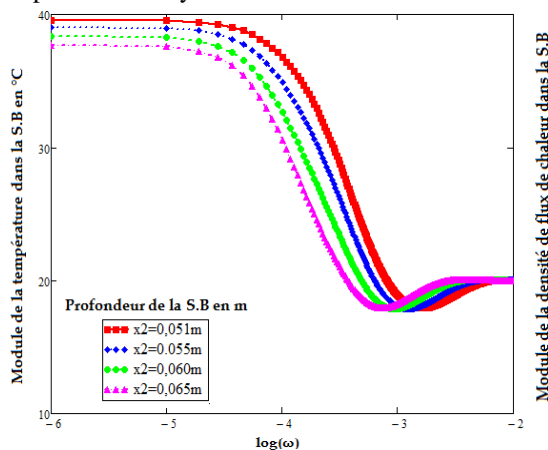


Figure 6-a

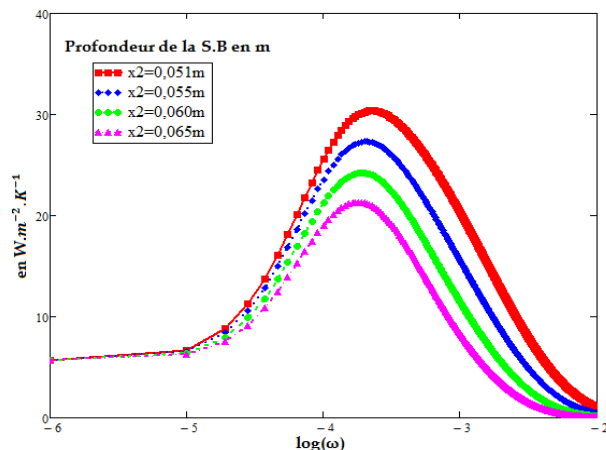


Figure 6-b

Fig. 6 Temperature module (figure 6-a) and heat flux density (figure 6-b) as a function of the decimal logarithm of the pulsation. $h_1=100$ ($W \cdot m^{-2} \cdot K^{-1}$); $h_r=0,05$ ($W \cdot m^{-2} \cdot K^{-1}$); $h_2=0,05$; $C_n=100\%$; $I=800$ ($W \cdot m^{-2}$).



Conclusion

The profiles of temperature modulus and heat flux density as a function of the excitatory pulsation under the influence of depth at the level of the concrete slab and sawdust in the illumination zone and in the zone of shading were presented. The study showed the important qualities of thermal insulating of material sawdust.

References

- [1]. P. Coulomb, G. Herrmann, S. Sarlin, "Thermal insulation in the design and upgrading of work premises"
- [2]. M. Abdesselam "A new approach to the climatic design of buildings. » Francophone Energies Liaison No. 42
- [3]. Diagne, M. Dieng, M. L. Sow, A. Wereme, F. Niang – G. Sissoko, "Estimation of the effective thermal insulation layer of a Kapok-plaster material in a dynamic frequency regime", CIFEM2010, Edition University of Rennes 1, pp 394-399, 2010
- [4]. Harouna Bal, Yves Jannot, Salif Gaye, Frank Demeurie, "Measurement and modeling of the thermal conductivity of a wet composite porous medium: Laterite based bricks with millet waste additive", *Construction and Building Materials* 41, pp 586–593, 2013.
- [5]. C. Langlais, S. Klarsfeld, *Thermal insulation at room temperature. Physical bases, Engineering techniques. Building*, 1999, Vol. CD2, No. C3370.
- [6]. C. Langlais, S. Klarsfeld, *Thermal insulation at room temperature. Properties, Engineering techniques. Functional Materials*, 2004, Vol. N1, No. BE9860.
- [7]. J. Fourier "Analytical theory of heat" 2004, Editons Jacques Gabay.
- [8]. A. Dieng, L. Ould Habiboulayh, A. S. Maïga, A. Diao and G. Sissoko, "Impedance spectroscopy method applied to electrical parameters: determination on bifacial silicom solar cell under magnetic field" 2007, *Journal des Sciences*, Vol. 7, No. 3, 48-52
- [9]. I. Diagne, M. Dieng, M.L. Sow, A. Wereme, F. Niang, G. Sissoko, (2014) Estimation of the effective thermal insulation layer of a kapok-plaster material in dynamic frequency regime
- [10].] N. B. Guttman, J. D. Matthews, "Computation of extraterrestrial solar radiation, solar elevation angle and true solar time of sunrise and sunset", *SOLMET Vol. 2- Final report*, National Climatic Center, U.S. Department of Commerce (1979), pp49-52.
- [11]. J. Schmetz, « Relationship between solar net radiative fluxes at the top of the atmosphere and at the surface », *Journal of the Atmospheric Sciences*, 50-8 (1993), pp1125

