



Determination of the Characteristics of Insulation and Thermal Inertia of a Flat Wall from the Thermal-Electrical Analogy in Dynamic Frequency Regim

¹Papa Touty Traore, Imam Katim Toure, Papa Monzon Alassane Samake, Elhadji Abdoul Aziz Cisse, Issa Diagne

¹Cheikh Anta Diop University, Physics Departement, Dakar, Senegal
Email: papatoutytraore@gmail.com

*¹Physics department, Cheikh Anta Diop University, Senegal

Abstract In this paper we study, a thermal-electrical analogy for the determination of the characteristics of isolation and inertia of a plane of wall.

The thermal transmittance and the phase shift were determined respectively from the shunt and series resistors, and the Bode diagram of the phase of the thermal impedance under the influence of the heat exchange coefficient.

The resistive character of the material was shown from the Bode diagram of the thermal impedance and finally the determination of the specific heat was proposed from the Bode diagram of the heat capacity.

Keywords Thermal-electrical analogy – thermal impedance-thermal exchange coefficient-thermal capacity

Introduction

Faced with the ecological and environmental problems that arise today, the issue of energy management is more topical than ever. Most countries are now targeting the reduction of greenhouse gas emissions in the long term [1]. This drastic reduction in the building's energy needs can only be achieved through efforts on insulation, thermal inertia and energy saving.

We take a determination of the characteristics of insulation and thermal inertia of a wall plane based on kapok-plaster in frequency dynamic regime [2].

In this study that we conducted, a thermal-electrical analogy was made, allowing to determine the thermal transmission coefficient from the Nyquist diagram of the thermal impedance, and to explain the resistive [4] character of the material from the diagram. of Bode of the thermal impedance. The study of the Bode diagram of the phase of the thermal impedance and the thermal capacity made it possible to determine respectively [5] the phase shift and the specific heat explaining the good thermal inertia of the material

Heat transfer model through the wall

Diagram of the study device

The material is a flat wall whose thickness is of length L . It is subjected to thermal stresses at the level of the two faces (front and rear). We assume that the heat transfer along the direction (ox) is perpendicular to the external stresses and the initial temperature of the material is non-zero.



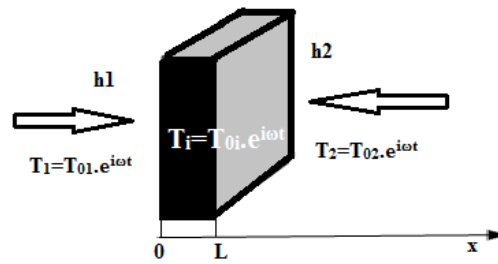


Figure 1: Flat plaster-kapok wall subjected to external stresses

Calculation of temperature and heat flux density

The heat equation determining the heat transfer in the wall (without heat source or heat sink) is given by equation (1):

$$\frac{\partial^2 T}{\partial x^2} - \frac{1}{\alpha} \cdot \frac{\partial T}{\partial t} = 0 \quad (1)$$

$$\text{with } \alpha = \frac{\lambda}{\rho \cdot C} \quad (2)$$

α is the thermal diffusivity.

λ is the thermal conductivity ($\text{W}\cdot\text{m}^{-1}$), ρ is the density ($\text{kg}\cdot\text{m}^{-3}$)

Boundary conditions

$$\lambda \frac{\partial T}{\partial x} = h_1 [T(0, t) - T_1] \quad (3)$$

$$\lambda \frac{\partial T}{\partial x} = h_2 [T_2 - T(L, t)] \quad (4)$$

$$T(x, t) = T_i \quad (5)$$

By making this change of variable $\bar{T} = T(x, t) - T_i$ equations (3), (4) and (5) become

$$\lambda \frac{\partial \bar{T}}{\partial x} = h_1 [\bar{T} + T_i - T_1] \quad (6)$$

$$\lambda \frac{\partial \bar{T}}{\partial x} = h_2 [T_2 - (\bar{T} + T_i)] \quad (7)$$

$$\bar{T}(x, 0) = 0 \quad (8)$$

The solution to equation (1) taking into account the change of variable is:

$$\bar{T}(x, h_1, h_2, \omega, t) = [A_1 \cdot \sinh(\beta(\omega, \alpha) \cdot x) + A_2 \cdot \cosh(\beta(\omega, \alpha) \cdot x)] \cdot e^{i\omega t} \quad (9)$$

$$T(x, h_1, h_2, \omega, t) = \bar{T}(x, h_1, h_2, \omega, t) + T_i \quad (10)$$

$$\beta(\omega, \alpha) = \sqrt{\frac{\omega}{2\alpha}} (1 + i) \quad (11)$$

The expressions of the coefficients A_1 and A_2 are determined from the boundary conditions. [6.7]

$$A_1 = f(x, \omega, h_1, h_2, \alpha) \quad (12)$$

$$A_2 = f(x, \omega, h_1, h_2, \alpha) \quad (13)$$

The heat flux density through the material is given by the expression

$$\varphi(x, h_1, h_2, \omega, t) = -\lambda \frac{\partial T}{\partial x} = -\lambda \times \beta [A_1 \cdot \cosh(\beta(\omega, \alpha) \cdot x) + A_2 \cdot \sinh(\beta(\omega, \alpha) \cdot x)] \cdot e^{i\omega t} \quad (14)$$



Thermal-electrical analogy

The table below summarizes some correspondences between electrical quantities [6] and Thermals.

Table 1: Correspondance between Electrical and Thermal parameters

Electrical Quantities		Thermal Quantities	
Expressions	Symbol	Expressions	Units
Intensity $I = \frac{dq}{dt}$	A	Flux density $\phi = -\lambda \frac{dT}{dx}$	$W.m^{-2}$
Change of potential ΔV	V	Change of temperature ΔT	$^{\circ}C$
Electrical impedance $Z = \frac{\Delta V}{I}$	Ω	Thermal impedance $Z = \frac{\Delta T}{\phi}$	$^{\circ}C / W.m^{-2}$
Electrical Capacity $C = \frac{Q}{V}$	F	Thermal capacity $C = \frac{\int \phi dt}{\Delta T}$	$W.m^{-2} . ^{\circ}C^{-1}$

Calculation of thermal impedance and heat capacity

a) Thermal impedance

The thermal resistance Re is obtained by:

$$\phi = \frac{T_{01} - T_{02}}{\frac{1}{S} \left[\frac{1}{h_1} + \frac{L}{\lambda} + \frac{1}{h_2} \right]} \tag{15}$$

We set ,

$$\Delta T = T_{01} - T_{02} = Re . \phi \tag{16}$$

Avec

$$Re = \frac{1}{S} \left[\frac{1}{h_1} + \frac{L}{\lambda} + \frac{1}{h_2} \right] \tag{17}$$

We generalize the notion of thermal resistance in thermal impedance Z in frequency dynamic regime where we have:

$$T(0, h_1, h_2, \omega, t) - T(x, h_1, h_2, \alpha, t) = Z . \phi \tag{18}$$

$$Z(x, \omega, h_1, h_2, t) = \frac{T(0, h_1, h_2, \omega, t) - T(x, h_1, h_2, \alpha, t)}{\phi(x, h_1, h_2, \alpha, t)} = \frac{\Delta T(x, h_1, h_2, \alpha, t)}{\phi(x, h_1, h_2, \alpha, t)} \tag{19}$$

b) Thermal capacity

The heat capacity is given by this relation from the electric thermal analogy (see table):

Thermal Impedance Results

Nyquist representation of thermal impedance

The Nyquist representation consists of plotting the imaginary part of the thermal impedance against its real part. Figure 2 gives the Nyquist representation under the influence of the exchange coefficient [7].

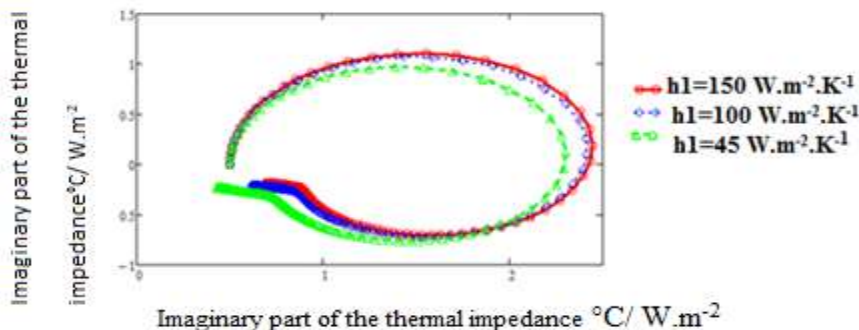


Figure 2: Evolution of the imaginary part of the thermal impedance as a function of its real part influence of the heat exchange coefficient

$$T_{01}=313 K ; T_i=293 K ; T_{02}=288 K ; x=0.05m, h_2= 5 W.m^{-2}.K^{-1}$$



The following table allows us to determine the values of the series and shunt resistances obtained from the Nyquist representation.

Table 1: Resistance values obtained from Nyquist representations.

Heat exchange coefficient at the front face ($W.m^{-2}.K^{-1}$)	45	100	150
Series resistance R_s ($K.m^2/W$)	0.5	0.5	0.5
Shunt resistance R_{sh} ($K.m^2/W$)	1.77	1.9	1.93
Thermal resistance $R_{th}= R_s+ R_{sh}$ ($K.m^2/W$)	2.27	2.4	2.43

The shunt resistance R_{sh} represents the behavior of the material to oppose heat loss. The series resistance R_s represents the behavior of the material to oppose the diffusion of heat.

The series resistance remains constant regardless of the value of the heat transfer coefficient. On the other hand, the shunt resistance and the thermal resistance increase when the heat exchange coefficient increases. The more the thermal resistance increases, the more the material is insulating. To explain this phenomenon, we will calculate the thermal transmission coefficient given by the following relationship:

$$U_p = \frac{1}{R_{th}} \tag{20}$$

The thermal transmission coefficient U_p indicates the level of insulation of the wall. The lower it is, the lower the losses. The following table gives us the values of the thermal transmittance for each value of the thermal resistance of the material.

Table 2: Thermal transmission coefficient dependent thermal resistance

$R_{th}(K.m^2/W)$	2.27	2.4	2.43
$U_p(W/ K.m^2)$	0.44	0.41	0.42

Bode diagram of thermal impedance

The thermal impedance modulus remains constant for $10^{-6} < \omega < 10^{-4} \text{ rad / s}$ is equal to $|Z| = 0.5K.m^2 / W$ and independent of the exciter pulse. The influence of the exchange coefficient is not obvious for low values of the pulsation so the material behaves like a pure resistance. When the exciter pulse is greater than 10^{-4} rad/s , the impedance modulus increases considerably with the exciter pulse until it reaches a peak and then decreases. The maximum of the thermal impedance modulus reflects a loss of heat flux density in the material at this level, the phenomenon of energy storage [8] is important, thus explaining the resistive nature of the material.

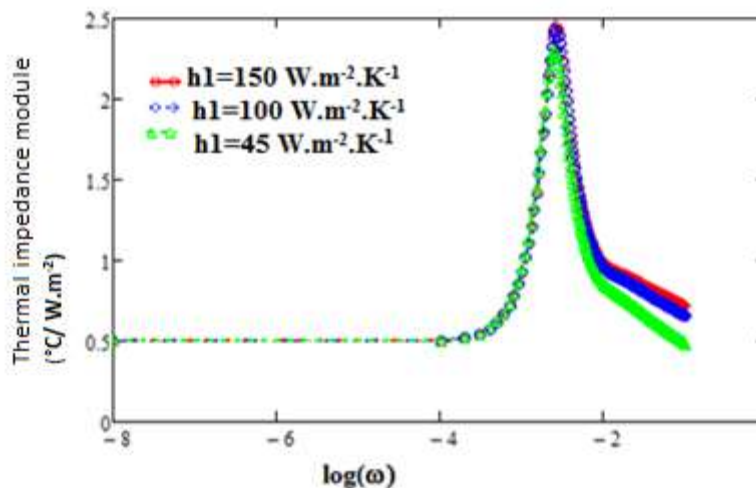


Figure 3: Evolution of the impedance modulus as a function of the decimal logarithm of the excitatory frequency $T_{01}=313 \text{ K}$; $T_i=293 \text{ K}$; $T_{02}=288 \text{ K}$; $x=0.05m$, $h_2= 5 \text{ W.m}^{-2}.K^{-1}$

Bode diagram of the thermal impedance phase

The Bode diagram of the phase of the thermal impedance gives the evolution of the phase according to the exciting frequency. The phase of the thermal impedance remains positive and increases slightly for $10^{-6} < \omega < 10^{-4} \text{ rad/s}$ at this level the influence of the exchange coefficient is not very obvious. When the exciting pulse is greater than 10^{-4} rad/s , the phase gradually increases and reaches a maximum corresponding to a phase value equal to 0.75 rad/s. Thus the material stores energy by inductive (if the phase is positive) and capacitive (if the phase is negative) effect.

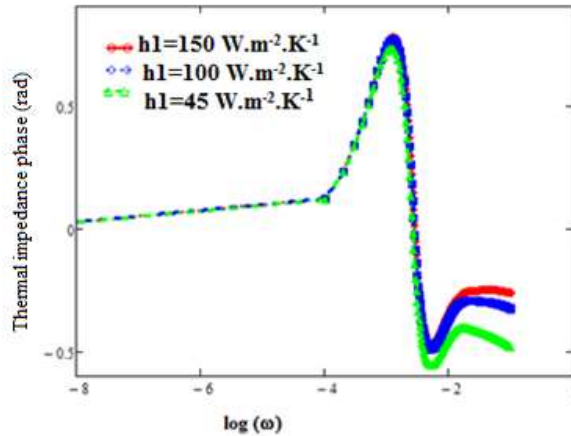


Figure 3: Variation of the phase of the thermal impedance of the material according to the decimal logarithm of the exciting frequency influence of the heat exchange coefficient $T_{01}=313 \text{ K}$; $T_i=293 \text{ K}$; $T_{02}=288 \text{ K}$; $x=0.05\text{m}$, $h_2= 5 \text{ W.m}^{-2}.\text{K}^{-1}$

The following expression gives the delay corresponding to the passage of the heat flux from the exterior to the interior called the phase shift Δt_f

$$\Delta t_f = \frac{P}{2\pi} \times \arg(Z) \tag{21}$$

Where P is the period (8h). The following table gives the different values of the phase shift under the influence of the heat exchange coefficient

Table 3: values of the phase shift under the different influence of the heat exchange coefficient

Heat exchange coefficient on the front face h (W.m ⁻² .K ⁻¹)	150			100			45		
Phase (rad)	0.11	0.75	0.21	0.11	0.74	0.18	0.11	0.7	0.2
Δt_f (h)	0.14	0.95	0.26	0.14	0.94	0.22	0.14	0.9	0.25

The phase shift Δt_f is more favorable to the high heat exchange coefficient because at this level the phase first reaches its maximum.

Heat capacity result

The thermal capacity modulus is maximum and constant for the values of the exciter frequency lower than 10^{-3} rad/s then decreases according to the exciter frequency. This maximum value of the capacitance modulus translates the material storage phenomena[9]. The higher the heat capacity, the greater the amount of heat the material can store. This therefore reflects the good quality of the thermal insulation[10] that is kapok-plaster. Because, an insulator is all the more efficient when it can store large quantities of heat on very thin thicknesses. This is what gives the material good thermal inertia [11].



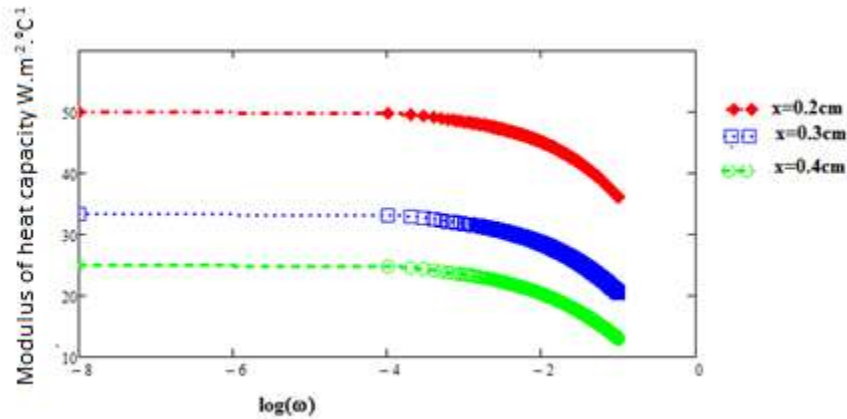


Figure 4: Variation of the modulus of the heat capacity of the material as a function of the decimal logarithm of the exciting frequency influence of the depth $T_{01}=313\text{ K}$; $T_i=293\text{ K}$; $T_{02}=288\text{ K}$; $h_1= 100\text{ W.m}^{-2}.\text{K}^{-1}$ $h_2= 5\text{ W.m}^{-2}.\text{K}^{-1}$

The specific heat c is given by the following expression:

$$c = \frac{C}{\rho \cdot A \cdot L}$$

A = The wall surface in m^2

ρ = Kg/m^3 [2]

L = wall thickness

Table 4: Different values of specific heat under the influence of thickness

Depth x (cm)	0.02		0.03		0.04	
C (W.s/K.m²)	48.8	43.87	32.23	25.55	18.1	11.66
c (W.s/K.kg)	11.77	10.54	7.75	6.13	4.35	2.8

Conclusion

The proposed study has made it possible to determine the insulation characteristics of a wall in a dynamic frequency regime.

A method for determining the thermal transmission coefficient has been proposed from the thermal resistance in order to show the good quality of the thermal insulation and to explain the resistive character of the material from the Bode diagram of the thermal impedance.

The study of the modulus of the phase of the thermal impedance made it possible to explain the inductive and capacitive phenomena of the material in order to determine the phase delay. A method of determining the specific heat was made from the expression of the heat capacity.

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