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Research Article

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Influence of Gravity on Particles Propagators due to Changing Spacetime Dimensions

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Abstract We propose introducing gravitational effects into particle physics by using concept of spacetime dimensions. Passage of spacetime dimensions from four to five dimensions allows us to change force carrying particle propagators, namely photons and gravitons. These modifications lead to a finite and nonlocal theory of elementary particles, in particular, the nonlocal quantum electrodynamics due to G.V.Efimov with fundamental length $L_{Pl} = \sqrt{G\hbar/c^3}$.

Keywords Yukawa and particle propagators, Coulomb, Newtonian potentials. Spacetime dimensions, Nonlocal particle propagators, photon, Graviton, anomalous magnetic moment.

1. Four Dimensional Case and the Newtonian Potential

Construction of gravitational theory for elementary particles encounters very difficulties due to smallness of the gravitational coupling constant G and strong singularities in the interaction mechanism. Until now there is no idea how to unify gravity with other fundamental forces in nature.

Our approach is very modest. We introduce only modification of particle propagators by using five dimensional spacetime with fundamental length L_{Pl}. In five-dimensional spacetime the Newtonian law takes the form for any two bodies with masses M_1 and M_2:

$$F_5 = g_{true} \cdot \frac{M_1 \cdot M_2}{r^3}, \tag{1}$$

where

$$g_{true} = \frac{G}{c^2}\sqrt{G\hbar c}. \tag{2}$$

This force is equal to four-dimensional Newtonian one

$$F_N = G\frac{M_1 \cdot M_2}{r^2} \tag{3}$$

in a domain determined by the Planck length $r = L_{Pl}$:

$$g_{true} \cdot \frac{M_1 \cdot M_2}{L_{Pl}^3} = G\frac{M_1 \cdot M_2}{L_{Pl}^2}. \tag{4}$$

Here

$$\frac{g_{true}}{L_{Pl}} \equiv G \tag{5}$$

Equality (4) means that classical and quantum gravitational theories are unified at the Planck length. In our scheme, a role of five-dimensional spacetime is that it is responsible only for changing character of motion of elementary particles, as a result those causal or Green functions are modified in four-dimensional spacetime.

In turns out that interaction between elementary particles with modified propagators are finite and have nonlocal character.

2. Derivation of Modified Propagators for Force Carrying Particles in Four-Dimensional Spacetime

To obtain modified propagators for photons and gravitons due to gravitational effects, we act in the standard way. We know that the Coulomb, Yukawa and Newtonian potentials are related with force carrying particle's propagators by using the Fourier transformation in the static limit:

1. For a photon:

$$\frac{1}{\bar{p}^2} = c_1 \int d^3 r e^{i\bar{p}\bar{r}} \cdot U_C(r). \quad (6)$$

2. For a graviton:

$$\frac{1}{\bar{p}^2} = c_2 \int d^3 r e^{i\bar{p}\bar{r}} \cdot U_N(r). \quad (7)$$

3. For a scalar particle with mass m:

$$\frac{1}{m^2+\bar{p}^2} = c_3 \int d^3 r e^{i\bar{p}\bar{r}} \cdot U_Y \quad (8)$$

where c_1, c_2, and c_3 are some constants of normalization. Here we do not interested in tensor structures having in numerators in photon and graviton like $g_{\mu\nu}$ and $\Delta_{\mu\nu,\rho\delta} = g_{\mu\rho} g_{\nu\delta} + g_{\nu\rho} g_{\mu\delta} - \frac{2}{D-2} g_{\rho\delta} g_{\mu\nu}$.

In order to extend above formulas (6)-(8) in the five-dimensional spacetime case, we use two possibilies for construction of five-dimensional spacetime structure.

2.1. Direct product of four-and one-dimensional spaces $D_4 \otimes D_1$

In this case, formulas (6) and (7) take the forms:

$$\frac{1}{\bar{p}^2} \Rightarrow D_g^{\gamma}(\bar{p}^2) = c \int_0^{\infty} \frac{dr \cdot r^2}{r} \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \cdot \sin\theta \cdot e^{ipr\cos\theta} \times \int_{-1}^{1} d\lambda e^{ip\lambda l}, \quad (9)$$

where

$$p = \sqrt{p_1^2 + p_2^2 + p_3^2}, \quad l = L_{Pl} = \sqrt{\frac{G\hbar}{c^3}},$$

$$\frac{1}{l}\int_{-l}^{l} dx e^{ipx} = \int_{-1}^{1} d\lambda e^{ipl\lambda},$$

and we have used the Newtonian potential form

$$U_N(r) = const \cdot \frac{1}{r}. \quad (10)$$

Elementary calculations for the formula (9) lead to the following result:

$$D_g^{\gamma}(\bar{p}^2) = 8\pi c \frac{1}{\bar{p}^2} \frac{\sin pl}{pl}, \quad (11)$$

Here, for normalization we assume $8\pi c = 1$ and use the Mellin representation

$$V_1(p^2 l^2) = \frac{\sin pl}{pl} = \frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\xi \frac{(\bar{p}^2 l^2)^{\xi}}{\sin\pi\xi \cdot \Gamma(2+2\xi)} \quad (0 < \beta < 1) \quad (12)$$

which is useful for calculation purpose.

Notice that the Yukawa potential

$$U_Y(r) = \frac{g}{r} e^{mr} \quad (13)$$

leads to changing propagator for a scalar particle due to five-dimensional space:

$$D_m(\bar{p}^2) = \frac{V_1^m(\bar{p}^2 l^2)}{m^2+\bar{p}^2} \quad (14)$$

Thus, final results are:

$$\mathcal{D}_{\mu\nu}^{\gamma}(-p^2) = \frac{g_{\mu\nu}}{(2\pi)^4 i}\int d^4 p \frac{V_1(-p^2 l^2)}{-p^2-i\varepsilon} e^{ipx} \quad (15)$$

for nonlocal photons,

$$\mathcal{D}_{\mu\nu,\rho\delta}^{g}(-p^2) = \frac{\Delta_{\mu\nu,\rho\delta}}{(2\pi)^4 i}\int d^4 p \frac{V_1(-p^2 l^2)}{-p^2-i\varepsilon} e^{ipx} \quad (16)$$

for nonlocal graviton field,

$$\mathcal{D}_m(-p^2) = \frac{1}{(2\pi)^4 i}\int d^4 p e^{ipx} \frac{V_1^m(-p^2 l^2)}{m^2-p^2-i\varepsilon} \quad (17)$$

for a scalar particle with mass m and also for massive vector intermediate $W^{\pm}, Z^0$-bosons with the propagator

$$\mathcal{D}^m_{\mu\nu} = \frac{1}{(2\pi)^4 i}\int d^4 p e^{ipx}\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{m^2}\right)\frac{V_1^m(-p^2 l^2)}{m^2 - p^2 - i\varepsilon}.$$

These bosons carry electro-weak interactions. In all above formulas we have used notation:

$$-p^2 = -(p_0^2 - \vec{p}^2), \qquad px = p_0 x_0 - \vec{p}\vec{x}.$$

Here

$$V_1^m(-p^2 l^2) = \frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-\infty} d\xi \frac{(m^2-p^2)^\xi l^{2\xi}}{\sin\pi\xi\cdot\Gamma(2+2\xi)}.$$

2.2. Addendum (or addition) of four-and one-dimensional spaces $D_4 \cup D_1$

Instead of the formula (9) this case leads to the formula

$$\frac{1}{\vec{p}^2} \Rightarrow D_g^\gamma(\vec{p}^2) = c'\int_0^\infty \frac{dr\cdot r^3}{r^2}\int_0^{2\pi} d\varphi \int_0^\pi d\theta\cdot\sin\theta\cdot e^{ipr\cos\theta}\times\int_{-1}^1 d\theta_1 \sin^2\theta_1 e^{ipl\cos\theta_1} \tag{18}$$

Last integral takes the form

$$\mathcal{L} = \int_0^\pi d\theta_1\sin^2\theta_1 e^{ipl\cos\theta_1} = \int_{-1}^1 d\lambda\sqrt{1-\lambda^2}e^{ipl\lambda} = 2\int_0^1 d\lambda\sqrt{1-\lambda^2}\cos pl\lambda = \frac{\pi}{pl}J_1(pl), \tag{19}$$

where $J_1(x)$ is the Bessel function of the order 1 and we have used quantum potential form

$$U_5 = const\cdot\frac{1}{r^2}$$

leading to the quantum force (1) in five-dimensional spacetime case. Thus normalized modified propagator given by formulas (18) and (19) takes the form:

$$\mathcal{D}'^\gamma_g(\vec{p}^2) = \frac{V_2(\vec{p}^2 l^2)}{\vec{p}^2}, \tag{20}$$

where

$$V_2(p^2l^2) = \frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-\infty} d\xi\frac{(pl/2)^{2\xi}}{\sin\pi\xi\,\Gamma(1+\xi)\Gamma(2+\xi)}, \quad (0<\beta<1) \tag{21}$$

For this second case it should be change $V_1(-p^2l^2) \Rightarrow V_2(-p^2l^2)$ and therefore modified propagators for photons, gravitons and scalar particles are given by formulas (15)-(17) with $V_2(-p^2l^2)$ and $V_2^m(-p^2l^2)$ form factors.

Finally, notice that due to form factors (12) and (21) which are analytic functions on the left hand complex plane and there are no poles there, all Feynman diagrams [except vacuum polarization, like showing in Figure 1]

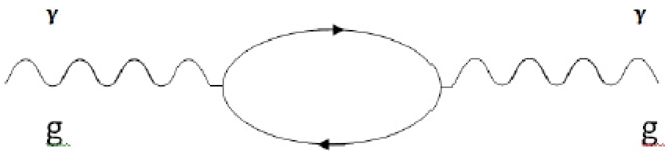

Figure 1

for electromagnetic, electro-weak and gravitational interactions between elementary particles, and for self interactions of scalar particles are finite.

3. Nonlocal Quantum Electrodynamics (NQED)

As an example, we study Feynman diagrams in nonlocal quantum electrodynamics in which the photon propagator is changed and spinor propagator does not modified because of conservation of electric charge which is broken for this case. In the language of Feynman diagrams if we change spinor propagator then the Ward-Takahashi identity does not valid.

3.1. Introduction

Lagrangian functions of the nonlocal quantum electrodynamics arisen from influence of gravity have similar structures as the local theory [1].

$$\begin{aligned}&L(x) = e{:}\overline{\psi}(x)\hat{A}(l,x)\psi(x){:} + e(Z_1-1)\times\overline{\psi}(x)\hat{A}(l,x)\psi(x){:}\\&-\delta m{:}\overline{\psi}(x)\psi(x){:} + (Z_2-1){:}\overline{\psi}(x)(i\hat{\partial}-m)\psi(x){:}\\&-(Z_3-1)\frac{1}{4}{:}F_{\mu\nu}(x)F^{\mu\nu}(x){:},\end{aligned} \tag{22}$$

where

$$\hat{A}(l,x) = A_\mu(l,x)\gamma^\mu,$$

and

$$\hat{\partial} = \gamma^{\mu}\frac{\partial}{\partial x_{\mu}}.$$

In our case of the nonlocal theory, renormalization constants $Z_1, Z_2, Z_3, \delta m$ are finite and moreover $Z_1 = Z_2$ due to the Ward-Takahashi identity. Here "Chronological" pairing (or T-product) of the fermonic field operators of electrons has the usual local form:

$$S(x-y) = \langle 0|T[\psi(x)\overline{\psi}(y)]|0\rangle = \frac{1}{(2\pi)^4}\frac{1}{i}\int d^4p \frac{e^{-ip(x-y)}}{m-\hat{p}-i\varepsilon} \qquad (23)$$

while "causal" function of the nonlocal electromagnetic field $A_\mu(l,x)$ in (22) takes the form (15). Notice that free Lagrangian of the electromagnetic field in (22) is constructed by using usual local tensor field:

$$F_{\mu\nu}(x) = \partial_\nu A_\mu(x) - \partial_\mu A_\nu(x).$$

Due to presence of the nonlocal photon propagator (15) or (20) in our theory all its matrix elements corresponding to any Feynman diagrams are finite except closed fermion loops which are as usually calculated by using the high dimensional regularization method of 't Hooft and Veltman [2].

Now we would like to calculate some primitive Feynman diagrams in NQED arisen from influence of gravity from four to five dimensions.

3.2. The Electron Self - Energy in NQED

The complete electron propagator in NQED is given by the sum

$$[-i(2\pi)^{-4}S_l'(p)] = [-i(2\pi)^{-4}S(p)] + [i(2\pi)^{-4}S(p)][i(2\pi)^4\Sigma_l(p)]$$
$$\times[-i(2\pi)^{-4}S(p)] + \cdots$$

where

$$S(p) = \frac{m+\hat{p}}{m^2-p^2-i\varepsilon}.$$

The sum is trivial and gives

$$S_l'(p) = [m-\hat{p}-\Sigma_l-i\varepsilon]^{-1}.$$

In lowest order there is a one - loop contribution to Σ_l, given by in Figure 2:

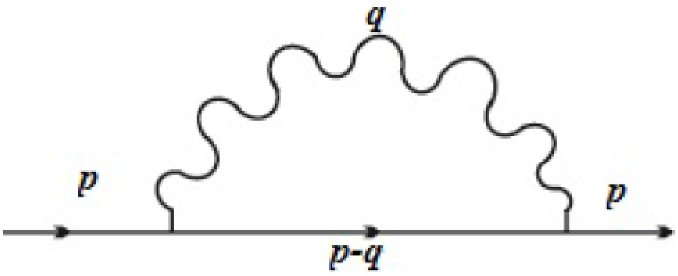

Figure 2: Diagram of Self - energy of a electron in NQED

$$-i{:}\,\overline{\psi}(x)\Sigma_l(x-y)\psi(y){:}$$

where

$$\Sigma_l(x-y) = -ie^2\gamma_\mu S(x-y)\gamma_\mu D^l(x-y) \qquad (24)$$

Passing to the momentum representation and making us of our regularization procedure δ that allows us to go to the Euclidean metric by using $k_0 \to \exp(i\pi/2)k_4$, one gets

$$\tilde{\Sigma}_l(p) = \frac{e^2}{(2\pi)^4}\int d^4k_E \frac{V_1(k_E^2 l^2)}{k_E^2}\gamma_\mu^{(E)}\frac{m-\hat{p}_E+\hat{k}_E}{m^2+(p_E-k_E)^2}\gamma\mu^{(E)}. \qquad (25)$$

Here $p_E = (-ip_0, p)$, $\gamma^{(E)} = (-i\gamma_0, \vec{\gamma})$ and $k_E = (k_4, k)$. Taking into account the Mellin representation (12) for the form - factor $V_1(k_E^2 l^2)$ and after some calculations, we have [3]:

$$\tilde{\Sigma}_l(p) = \frac{e^2}{8\pi}\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty} d\xi \frac{1}{\sin^2\pi\xi}\frac{v_1(\xi)(m^2e^2)^{\xi}}{\Gamma(1+\xi)}F(\xi,p) \qquad (26)$$

where

$$v_1(\xi) = \frac{1}{\Gamma(2+2\xi)},$$

$$F(\xi,p)=\frac{1}{\Gamma(1-\xi)}\int_0^1 du\left(\frac{1-u}{u}\right)^{\xi}\left(1-\frac{p^2}{m^2}u\right)^{\xi}\times(2m-\hat{p}u) \quad (27)$$

is a regular function in the half - plane $Re\,\xi>-1$. Assuming the value $m^2l^2=m^2L_{Pl}^2$ to be small, one can obtain:

$$\widetilde{\Sigma}_l(p)=\frac{e^2}{8\pi^2}\int_0^1 du(2m-\hat{p}u)\ln\left(1-\frac{p^2}{m^2}u\right)-$$
$$-\frac{e^2}{16\pi^2}\left[\left(3\ln\frac{1}{m^2l^2}+3v_{1'}(0)+3\psi(1)+1\right)+4m^2l^2v_1(1)\left(\ln\frac{1}{m^2l^2}-\frac{v_{1'}(1)}{v_1(1)}-\frac{5}{12}\frac{p^2}{m^2}\right)\right]-$$
$$-\frac{e^2}{16\pi^2}(m-\hat{p})\left[\left(\ln\frac{1}{m^2l^2}-v_{1'}(0)+1\right)-m^2l^2v_1(1)\frac{p^2}{3m^2}\right]. \quad (28)$$

3.3. Vertex Function and Anomalous Magnetic Moment of Leptons in NQED

In the momentum space and in the Euclidean metric, the vertex function takes the form [Figure 3]:

$$\widetilde{\Gamma}_\mu^l(p_1,p)=-\frac{e^2}{(2\pi)^4}\int d^4k_E\frac{V_1((p_E-k_E)^2l^2)}{(p_E-k_E)^2}\gamma_\nu\times$$
$$\times\frac{m-\hat{k}_E-\hat{q}_E}{m^2+(k_E+p_E)^2}\gamma_\mu\frac{m-\hat{k}_E}{m^2+k_E^2}\gamma_\nu \quad (29)$$

Again passing to the Minkowski metric and using the generalized Feynman parameterization

$$\frac{1}{a^{n_1}b^{n_2}c^{n_3}}=\frac{\Gamma(n_1+n_2+n_3)}{\Gamma(n_1)\Gamma(n_2)\Gamma(n_3)}\int_0^1\int_0^1\int_0^1 d\alpha d\beta d\gamma\delta(1-\alpha-\beta-\gamma)$$
$$\times\alpha^{n_1-1}\beta^{n_2-1}\gamma^{n_3-1}\frac{1}{[a\alpha+b\beta+c\gamma]^{n_1+n_2+n_3}} \quad (30)$$

one gets

$$\widetilde{\Gamma}_\mu^l(p_1;p)=-\frac{e^2}{8\pi}\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty}d\xi\frac{v_1(\xi)}{(\sin\pi\xi)^2}\frac{(m^2l^2)^\xi}{\Gamma(1+\xi)}F_\mu(\xi;p_1,p) \quad (31)$$

where

$$F_\mu(\xi;p_1,p)=\gamma_\mu F_1(\xi;p_1,p)+F_2(\xi;p_1,p).$$

Here

$$F_1(\xi;p_1,p)=\frac{1}{\Gamma(1-\xi)}\int_0^1\int_0^1\int_0^1 d\alpha d\beta d\gamma\delta(1-\alpha-\beta-\gamma)\alpha^{-\xi}Q^\xi,$$

$$F_2(\xi;p_1,p)=\frac{1}{\Gamma(-\xi)}\int_0^1\int_0^1\int_0^1 d\alpha d\beta d\gamma\delta(1-\alpha-\beta-\gamma)\alpha^{-\xi}Q^{\xi-1}\times$$
$$\times\frac{1}{m^2}\left[m^2\gamma_\mu-2mq_\mu+4m(\beta q_\mu-\alpha p_\mu)+\right.$$
$$\left.+(\alpha\hat{p}-\beta\hat{q})\gamma_\mu\hat{q}+(\alpha\hat{p}-\beta\hat{q})\gamma_\mu(\alpha\hat{p}-\beta\hat{q})\right], \quad (32)$$

$$Q=\beta+\gamma-\alpha\gamma\frac{p^2}{m^2}-\beta\gamma\frac{q^2}{m^2}-\alpha\beta\frac{(p+q)^2}{m^2}.$$

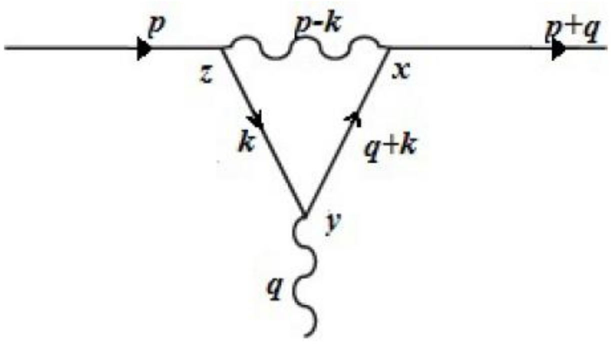

Figure 3: Vertex function in NQED

Let us calculate the vertex function (31) for two cases: first, when $q=0$ and p has an arbitrary value; second, when q is an arbitrary quantity and p,p_1 are situated on the m - mass shell. In the first case, assuming $q=0$ in the formula (32) and after some standard calculations [3], one gets

$$F_\mu(\xi;p_1,p)=\frac{1}{\Gamma(1-\xi)}\int_0^1 du\left(\frac{1-u}{u}\right)^\xi\left(1-u\frac{p^2}{m^2}\right)^\xi\times$$

$$\times\left[u\gamma_\mu+\frac{2\xi u p_\mu(2m-u\hat{p})}{m^2-up^2}\right]. \quad (33)$$

Comparing this formula with the expression (27) for the self-energy of the electron, it is easily seen that

$$F_\mu(\xi;p,p)=-\frac{\partial}{\partial p_\mu}F(\xi;p) \quad (34)$$

From this identity, we can obtain a very important conclusion. In the nonlocal theory of quanyum electrodynamics constructed using the concept of influence of gravity on the motion of a particle in four-dimensional spacetime, the Ward - Takahashi identity is valid

$$\tilde{\Gamma}^l_\mu(p,p)=-\frac{\partial}{\partial p_\mu}\tilde{\Sigma}_l(p) \quad (35)$$

In the second case, one can put

$$\bar{u}(\hat{p}_1)\tilde{\Gamma}^l_\mu(p_1,p)u(\hat{p})=\bar{u}(\hat{p}_1)\Lambda_\mu(q)u(\hat{p}) \quad (36)$$

where $\bar{u}(\hat{p}_1)$ and $u(\hat{p})$ are solutions of the Dirac equation

$$(\hat{p}-m)u(\hat{p})=0,\quad \bar{u}(\hat{p}_1)(\hat{p}_1-m)=0$$

Substituting the vertex function (31) into (36) and after some transformations, we have

$$\bar{u}(\hat{p}_1)F_\mu(\xi;p_1,p)u(\hat{p})=\bar{u}(\hat{p}_1)\Lambda_\mu(\xi;q)u(\hat{p}) \quad (37)$$

Here

$$\Lambda_\mu(\xi;q)=\gamma_\mu f_1(\xi;q^2)+\frac{i}{2m}\sigma_{\mu\nu}q_\nu f_2(\xi;q^2),$$

$$\sigma_{\mu\nu}=\frac{1}{2i}(\gamma_\mu\gamma_\nu-\gamma_\nu\gamma_\mu),$$

$$f_i(\xi;q^2)=\frac{1}{\Gamma(1-\xi)}\int_0^1\int_0^1\int_0^1 d\alpha d\beta d\gamma\delta(1-\alpha-\beta-\gamma)\times\alpha^{-\xi}L^{\xi-1}g_j(\alpha,\beta,\gamma,q^2),$$

$$L=\varepsilon\alpha+(1-\alpha)^2-\beta\gamma\frac{q^2}{m^2},$$

$$\begin{aligned}g_1(\alpha,\beta,\gamma,q^2)&=[(1-\alpha)^2(1-\xi)+2\alpha\xi]-\\&-[\beta\gamma+\xi(\alpha+\beta)(\alpha+\gamma)]\frac{q^2}{m^2},\\g_2(\alpha,\beta,\gamma,q^2)&=2\alpha(1-\alpha)(\xi).\end{aligned} \quad (38)$$

To avoid infrared divergences in the vertex function we have introduced here the parameter $\varepsilon=\mu^2_{ph}/m^2$, taking into account the "mass" of the photon. Finally, one gets

$$\Lambda_\mu(q)=\gamma_\mu F_1(q^2)+\frac{i}{2m}\sigma_{\mu\nu}q_\nu F_2(q^2), \quad (39)$$

where

$$F_j(q^2)=-\frac{e^2}{8\pi}\frac{1}{2i}\int_{-\beta+i\infty}^{-\beta-i\infty}\frac{d\xi}{(\sin\pi\xi)^2}\frac{v_1(\xi)}{\Gamma(1+\xi)}(m^2l^2)^\xi f_j(\xi,q^2) \quad (40)$$

It is easy to verify that the vertex function $\Lambda_\mu(q)$ satisfies the gauge invariant condition:

$$q_\mu\bar{u}(\hat{p}_1)\Lambda_\mu(q)u(\hat{p})=0 \quad (41)$$

Let us write the first terms of the decomposition for the functions $F_1(q^2)$ and $F_2(q^2)$ over small parameters m^2l^2 and q^2/m^2:

$$\begin{aligned}F_1(q^2)&=\frac{\alpha}{4\pi}\left[m-2\sigma-v_1{}'(0)+\frac{9}{2}-6C-3m^2l^2v_1(1)\right]+\\&+\frac{\alpha}{2\pi}\frac{q^2}{m^2}\left\{\frac{2}{3}\left(\frac{1}{2}\sigma-\frac{3}{8}\right)+\frac{m^2l^2}{3}\left[v_1(1)\left(-\chi+2C-\frac{13}{6}\right)+v'_1(1)\right]\right\}\end{aligned} \quad (42)$$

where $\sigma=\ln(m^2/\mu^2_{Ph})$, $C=0.577215\ldots$ is the Euler constant, $\alpha=e^2/4\pi$ and $\chi=\ln\left[\frac{1}{m^2l^2}\right]$, and

$$F_2(q^2)=-\frac{\alpha}{2\pi}\left(1-\frac{2}{3}v_1(1)m^2l^2\right) \quad (43)$$

From this last formula we can see that corrections to the anomalous magnetic moment (AMM) for leptons are given by

$$\Delta\mu=\frac{\alpha}{2\pi}\left[1-\frac{2}{3}v_1(1)m^2l^2\right] \quad (44)$$

The first term in (44) corresponds to the Schwinger [4] correction obtained in local QED. The second term is responsible from gravitational effects on the particle physics, where

$$l = L_{Pl} = \sqrt{\frac{G\hbar}{c^3}}.$$

In accordance with formfactors (12) and (21), functions $v_1(x)$ and $v_2(x)$ have the forms:

$$v_1(x) = \frac{1}{\Gamma(2+2x)}, \qquad v_2(x) = 2^{-2x}\frac{1}{\Gamma(1+x)\Gamma(2+x)}.$$

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