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Research Article

Integral Property of Laplace Transform Function Figure with Parameter

Liu Zhiwei

Institute of Applied Technology, Hezhou University, Hezhou, 542899, China

Abstract This paper gives a definition to Laplace transform function figure with parameter and then deduces one of the integral properties of function figure. With this property, infinite integral problems with parameter of certain linear can be solved simply and concisely.

Keywords Laplace transform, function figure, integral order
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Introduction

Laplace transform function figure with parameter is defined as follows:

Supposing the function $f(x,t)$in the $x \in R, t \geq 0$ is consecutive, and the integral $\int_0^{+\infty} f(x,t)e^{-pt}\,dt$ exists, thus for t, the function $f(x,t)$ is referred to as Laplace transform existence, and the function $F(p,x) = \int_0^{+\infty} f(x,t)e^{-pt}\,dt$ is referred to asLaplace transform function of function $f(x,t)$about t, which is marked as:$F(p,x) = L[f(x,t)]$.

Therefore, the function $f(x,t)$ is referred to as inversion of Laplace transform (or primitive function) of $F(p,x)$ about t, which is marked as:

$f(x,t) = L^{-1}[F(p,x)]$.

To prove the integral property of the function$F(p,x)$, this paper studies the interchangeability of integral order of double infinite integral first.

Lemma Supposing the function $f(x,t)$ in$x \geq a, t \geq a$ is consecutive, the integral $\int_a^{+\infty} f(x,t)dt$ converges for any bounded interval of$[a,+\infty]$. Then supposing there is at least one existence between $\int_a^{+\infty}[\int_a^{+\infty}|f(x,t)|dx]dt$ and

$\int_a^{+\infty}[\int_a^{+\infty}|f(x,t)|dt]dx$, thus$\int_a^{+\infty}[\int_a^{+\infty}|f(x,t)|dx]dt= \int_a^{+\infty}[\int_a^{+\infty}|f(x,t)|dt]dx$.

Demonstration As for $y > a$, thus,

$\int_a^{+\infty}[\int_a^{y} f(x,t)dx]dt = \int_a^{y}[\int_a^{+\infty} f(x,t)dt]dx$.

Obviously, as for any of$\beta > \alpha$, the function $\Psi(x) = \int_a^{+\infty} f(x,t)dt$ for $\forall x \in [a,y]$ is consecutive, according to the condition:

$\lim_{\beta\to+\infty} \int_a^{\beta} f(x,t)dt = \int_a^{+\infty} f(x,t)dt$is proved to be true consistently for $\forall x \in [a,y]$, therefore

$$\lim_{\beta\to+\infty} \int_a^{y}[\int_a^{\beta} f(x,t)dt]dx = \int_a^{\beta}[\int_a^{+\infty} f(x,t)dx]dt \quad (1)$$

On the other hand $\int_a^{y}\left[\int_a^{\beta} f(x,t)dt\right]dx = \int_a^{\beta}\left[\int_a^{+\infty} f(x,t)dx\right]dt.$

$$\text{thus} \lim_{\beta\to+\infty} \int_a^{y}[\int_a^{\beta} f(x,t)dt]dx = \int_a^{+\infty}\left[\int_a^{y} f(x,t)dx\right]dt. \quad (2)$$

Comparing (1) and (2), therefore:

$\int_a^{+\infty}[\int_a^y|f(x,t)|dx]dt = \int_a^y[\int_a^{+\infty}|f(x,t)|dt]dx.$ (3)

Considering the function $F(y,t) = \int_a^y f(x,t)dx$ of y and , which satisfies the following three conditions:

1. The function is consecutive in $y \geq a, t \geq a$.

2. As for any of$y \geq a$, there is $|F(y,t)| \leq \int_a^y |f(x,t)|dx \leq \int_a^{+\infty}|f(x,t)|dx$, in the hypothetical case there is $\int_a^{+\infty}[\int_a^{+\infty}|f(x,t)|dx]dt$, $\int_a^{+\infty} F(y,t)dt$converges consistently for $y \geq a$ according to criterion of M.

3. When$y \to +\infty$, $F(y,t)$converges consistently to $\int_a^{+\infty} f(x,t)dx$ for any finite interval of t, therefore, taking the limit under the integral, thus

$\lim_{y\to+\infty} \int_a^{+\infty}[\int_a^y f(x,t)dt]dx = \int_a^{+\infty}[\int_a^{+\infty} f(x,t)dx]dt.$ (4)

On the other hand, according to (3):

$\lim_{y\to+\infty} \int_a^{+\infty}[\int_a^y f(x,t)dx]dt = \lim_{y\to+\infty} \int_a^y[\int_a^{+\infty} f(x,t)dt]dx = \int_a^{+\infty}[\int_a^{+\infty} f(x,t)dt]dx$ (5)

Comparing (4) and (5), it comes to a conclusion

$\int_a^{+\infty}[\int_a^{+\infty} f(x,t)dx]dt = \int_a^{+\infty}[\int_a^{+\infty} f(x,t)dt]dx.$

Integral property of Laplace transform function figure with parameter:

Theorem.

Supposing $F(p,x)$ is the Laplace transform function about$t$, the function $\Phi(t) = \int_0^{+\infty}|f(x,t)|dx$converges when $t \geq 0$ and is bounded, thus $F(p,x)$ is $[0,+\infty]$ integrable, and $\int_0^{+\infty} F(p,x)dx = L[\int_0^{+\infty} f(x,t)dx]$.

Demonstration Because$|f(x)e^{-pt}| \leq |f(x,t)|$ $(p > 0, t > 0)$, and

$\int_0^{+\infty}|f(x,t)|dx$ converges when $t \geq 0$, therefore $\int_0^{+\infty} f(x,t)e^{-pt}dx$

converges consistently in $[0,+\infty]$. According to the research of CaiWei (1984), $\int_0^{+\infty} f(x,t)e^{-pt}dt$ also converges consistently when $x \geq 0$.

Moreover, because $\Phi(t) = \int_0^{+\infty}|f(x,t)|dx$ is bounded in $t \geq 0$, thus $M > 0$, make$\forall t \in (0,+\infty)$, $\Phi(t) \leq M$ and because$0 < \Phi(t)e^{-pt} < Me^{-pt}$, obviously

$\int_0^{+\infty} Me^{-pt}dt$ $(p > 0)$ converges, therefore $\int_0^{+\infty} \Phi(t)e^{-pt}dt$ converges, that is $\int_0^{+\infty}[\int_0^{+\infty}|f(x,t)|dx]dt$ exists.

According to the lemma:

$$\int_a^{+\infty} F(p,x)dx = \int_0^{+\infty}[\int_0^{+\infty} f(x,t)e^{-pt}dt]dx$$

$$= \int_0^{+\infty}[\int_0^{+\infty} f(x,t)e^{-pt}dx]dt$$

$$= \int_0^{+\infty}[\int_0^{+\infty} f(x,t)dxe^{-pt}]dt$$

that is $\int_0^{+\infty} F(p,x)dx = L[\int_0^{+\infty} f(x,t)dx]$.

For the convenience of application, the result of the theorem is usually written as

$\int_0^{+\infty} f(x,t)dx = L^{-1}[\int_0^{+\infty} F(p,x)dx]$ (6)

Example: Integral Calculation $\int_0^{+\infty} \frac{costx}{a^2+x^2}dx$ $(a > 0, t > 0)$.

Solution: Because $F(p,x) = L\left[\frac{costx}{a^2+x^2}\right] = \frac{1}{a^2+x^2}L[costx] = \frac{p}{(a^2+x^2)((p^2+x^2)}$,

Therefore $\int_0^{+\infty} F(p,x)\,dx = \int_0^{+\infty} \frac{p}{(a^2+x^2)((p^2+x^2)}dx = \frac{\pi}{2a(a+p)}$

According to (6), $\int_0^{+\infty} \frac{costx}{a^2+x^2}dx = L^{-1}\left[\frac{\pi}{2a(a+p)}\right] = \frac{\pi}{2a}e^{-at}$.

Therefore, the results of this research demonstrate that it is concise to understand and solve the problems of infinite integral with parameter in science and engineering.

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