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Research Article

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Approximation Method for Solving Heat model with Derivative Boundary Conditions

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Abstract In this paper, a numerical solution of two-dimensional two-sided heat model has been presented. The algorithm for the approximate solution for this equation is based on modified decomposition method. The approximate method has been applied to solve a practical numerical example and the results have been compared with the exact solution. The results were presented in tables using the MathCAD 12 software package when it is needed.

Keywords derivative boundary condition problem two-dimensional, modified decomposition method two-sided, heat equation

Introduction

The Adomian Decomposition Method (ADM) has been applied to a wide class of problems in physics, biology and chemical reactions. The method provides the solution in a rapid convergent series with computable terms. This method was successfully applied to nonlinear differential delay equations [1], a nonlinear dynamic system [2], the heat equation [3,4], the wave equation [5], coupled nonlinear partial differential equations [6,7], linear and nonlinear integro-differential equations [8] and Airy's equation [9]. Different modifications of this method and their applications are given in [10-13].

In this paper, we present modified decomposition method for solving the heat equation with derivative boundary conditions:

$$\frac{d}{dt}\Omega(x,y,t)=\frac{d^2}{d_+x^2}\Omega(x,y,t)+\frac{d^2}{d_-x^2}\Omega(x,y,t)+\frac{d^2}{d_+y^2}\Omega(x,y,t)+\frac{d^2}{d_-y^2}\Omega(x,y,t)-\Omega(x,y,t)+k(x,y,t)$$

with the initial condition

$$\Psi(x,y,0)=f(x,y), 0\le x\le T, 0\le y\le T$$

and the derivative boundary conditions

$$\Omega_x(0,x,t)=\mathrm{N}_1(y,t), 0<t\le T$$
$$\Omega_y(y,0,t)=\mathrm{N}_2(x,t), 0<t\le T$$
$$\Omega_x(1,x,t)=N_3(y,t), 0<t\le T$$
$$\Omega_y(y,1,t)=\mathrm{N}_4(x,t), 0<t\le T$$

Where $f, N_1, N_2, N_3, N_4, \Omega$ and K are known functions. T is given constant. In the present work, we apply the modified Adomian's decomposition method for solving eq. (1). The paper is organized as follows: In section 2 the approximate method with derivative boundary conditions is presented. In section 3 numerical example is solved numerically using the modified decomposition method. Finally, we present conclusion about solution of the two-dimensional two-sided heat equation.

Approximate Method

In this section, we present modified decomposition method for solving two-dimensional two-sided heat equations with derivative boundary condition given in eq.(1). In this method we assume that:

$$\Omega(x,y,t)=\sum_{n=0}^{\infty}\Omega_n(x,y,t)$$

eq.(1) can be rewritten:

$L_t\Omega(x,y,t)=L_{+xx}\Omega(x,y,t)+L_{-xx}\Omega(x,y,t)+L_{+yy}\Omega(x,y,t)+L_{-yy}\Omega(x,y,t)-\Omega(x,y,t)+k(x,y,t)$(2)

Where

$L_t(\cdot)=\frac{\partial}{\partial t}(\cdot)$,$L_{xx}=\frac{\partial^2}{\partial x^2}$ and $L_{yy}=\frac{\partial^2}{\partial y^2}$

The inverse $L^{-1}=\int_0^t(\cdot)dt$(3)

$$L^{-1}(L_t\Omega((x,y,t)))=L^{-1}(L_{+xx}(\Omega(x,y,t))+L_{-xx}(\Omega(x,y,t)))+L^{-1}(L_{+yy}(\Omega(x,y,t))+L_{-yy}(\Omega(x,y,t)))-L^{-1}(\Omega(x,y,t))+L^{-1}(k(x,y,t))$$

Then, we can write,

$\Omega(\mathrm{x},\mathrm{y},\mathrm{t})=$

$\Omega(\mathrm{x},\mathrm{y},0)+L_t^{-1}\left(L_{+xx}\left(\sum_{n=0}^{\bowtie}\Omega_n\right)+L_{-xx}\left(\sum_{n=0}^{\bowtie}\Omega_n\right)\right)+L_t^{-1}\left(L_{+yy}\left(\sum_{n=0}^{\bowtie}\Omega_n\right)+L_{-yy}\left(\sum_{n=0}^{\bowtie}\Omega_n\right)\right)-L_t^{-1}(\Omega(x,y,t))+L_t^{-1}(\mathrm{k}(x,y,t))$(4)

The modified decomposition method was introduced by Wazwaz [14]. This method is based on the assumption that the function $\gamma(x,y)$ can be divided into two parts, namely $\gamma_1(x,y)$ and $\gamma_2(x,y)$. Under this assumption we set

$$\gamma(x,y)=\gamma_1(x,y)+\gamma_2(x,y)$$

Then the modification

$u_0=\gamma_1$

$$\Omega_1=\gamma_2+L_t^{-1}(L_{+xx}\Omega_0)+L_t^{-1}(L_{-xx}\Omega_0)+L_t^{-1}(L_{+yy}\Omega_0)+L_t^{-1}(L_{-yy}\Omega_0)-L_t^{-1}(\Omega_0)$$

$\Omega_{n+1}=L_t^{-1}\left(L_{+xx}\left(\sum_{n=0}^{\infty}\Omega_n\right)+L_{-xx}\left(\sum_{n=0}^{\infty}\Omega_n\right)\right)+L_t^{-1}\left(L_{+yy}\left(\sum_{n=0}^{\infty}\Omega_n\right)+L_{-yy}\left(\sum_{n=0}^{\infty}\Omega_n\right)\right)-L_t^{-1}\left(\sum_{n=0}^{\infty}\Omega_n\right)$, n>1

Numerical Example:

Consider two-dimensional two-sided heat equation withderivativeboundary condition for the equation (1):

$$\frac{d}{dt}\Omega(x,y,t)=\frac{d^2}{d_+x^2}\Omega(x,y,t)+\frac{d^2}{d_-x^2}\Omega(x,y,t)+\frac{d^2}{d_+y^2}\Omega(x,y,t)+\frac{d^2}{d_-y^2}\Omega(x,y,t)-\Omega(x,y,t)+2t+t^2+x+y$$

subject to the initial condition

$$\Omega(\mathrm{x},\mathrm{y},0)=x^2+y^2\ x,y\in(0,1),\ 0\le t\le T$$

and the derivative boundary conditions

$$\Omega_x(0,y,t)=y^2+t^2, 0<t\le T$$
$$\Omega_y(x,0,t)=x^2+t^2, 0<t\le T$$
$$\Omega_x(1,y,t)=1+y^2+t^2, 0<t\le T$$
$$\Omega_y(x,1,t)=1+x^2+t^2, 0<t\le T$$

We apply the above proposed method; we obtain:

$$\Omega_0(x,y,t)=x^2+y^2+t^2$$
$$\Omega_1(x,y,t)=0$$
$$\Omega_2(x,y,t)=0$$
$$\Omega_3(x,y,t)=0$$

Then the series form is given by:

$$\Omega(x,y,t) = \Omega_0(x,y,t) + \Omega_1(x,y,t) + \Omega_2(x,y,t) + \Omega_3(x,y,t)$$
$$= x^2 + y^2 + t^2$$

This is the exact solution:

$$\Omega(x,y,t) = x^2 + y^2 + t^2$$

Table 1 shows the approximate solutions for heat equation with derivative boundary condition obtained for different values and comparison between exact solution and approximate solution.

Table 1: Comparison between exact solution and approximation solution for example

x = y	t	Exact Solution	Modified Adomian Decomposition Method	$\lvert\Omega_{ex} - \Omega_{MADM}\rvert$
0	3	9.0	9.0	0.0000
0.1	3	9.02	9.02	0.0000
0.2	3	9.08	9.08	0.0000
0.3	3	9.18	9.18	0.0000
0.4	3	9.32	9.32	0.0000
0.5	3	9.50	9.50	0.0000
0.6	3	9.72	9.72	0.0000
0.7	3	9.98	9.98	0.0000
0.8	3	10.28	10.28	0.0000
0.9	3	10.62	10.62	0.0000
1	3	11.0	11.0	0.0000

Conclusion

In this paper, we have applied the modified decomposition method for the solution of the two-dimensional two-sided heat equation with derivative boundary condition. This algorithm is simple and easy to implement. The obtained results confirmed a good accuracy of the method.

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