



Hedge Algebra: From Design to the First Steps Applying in Control Engineering

Trung Ngo Kien

Thai Nguyen University of Technology
Email: trungokien@tnut.edu.vn

Abstract Hedge Algebra (HA) is an algebraic structure for computing and simulating semantics of language so it can be considered as a basis of fuzzy logic. Using HA for designing controllers can create an algebraic structure in the form of a functional relationship, which allows to form a large set of linguistic values to describe input and output relationships. HA-based research is a new approach in the calculation of fuzzy controllers; therefore, researchers have had ideas applying it in the field of control. The remarkable success of the HA is based on functional reasoning methods that contain a wide variety of open elements. The user can optionally select different approaches in each step of the method. This article analyzes results obtained from the different approaches in control and automation.

Keywords Hedge-Algebra, linguistic domain, semantically quantifying, linear object, non-linear

1. Introduction

In fact, the value of linguistic variables has certain orders semantically. For example, it is obvious that “young” is smaller than old, “fast” is larger than “slow”.

A structure of Hedge Algebra consists of four parts including “X, G, H, ≤”. “X” is a domain of the linguistic variable; “≤” is a semantically ordering relation; “G” is the set of generators; “H” is the set of linguistic hedges (H can be negative or positive).

As a result, semantics of words represented by the structure HA is likely to be determined by their relative positions in linguistic domain, based on their natural meaning. Linguistic value is quantified by a real value in [0, 1].

Example: the linguistic variable “TRUE” can be considered as an algebraic structure, singed $AX=(X, G, H, \leq)$, where: “X” is the set of values (True, Very True, False, Little False...); “G” is the set of generators (True, False); “H” is the set of linguistic hedges (Very, More, Little, Less...); “≤” is an semantically ordering relation (Very False ≤ False ≤ True ≤ Very True).

2. Reasoning Method and Open Elements

A set of rules is shown in (1) where X_1, X_2, \dots, X_m and Y are linguistic variables; A_{ij}, B_i ($i = 1 \dots n; j = 1 \dots m$) are proportional values.

If $X_1=A_{11}$ and ... and $X_m=A_{1m}$ then $Y=B_1$

If $X_1=A_{21}$ and ... and $X_m=A_{2m}$ then $Y=B_2$

.....(1)

If $X_1=A_{n1}$ and ... and $X_m=A_{nm}$ then $Y=B_n$

The main idea of this method is that each clause “if ... then” will determine a point in Descartes space $Dom(X_1)*\dots*Dom(X_m)*Dom(Y)$. $Dom(X_i), Dom(Y)$ are linguistic domains of linguistic variables X_i and Y respectively and considered as HAs.



According to the HA approach, there are some steps that can be taken as follows:

Step 1: Determine AX_i , AY that are HAs for linguistic variables X_i , Y respectively quantify input values.

Step 2: build semantic quantifying mappings vAX_i and vAY , which means projections from HAs including AX_i , AY into $[0,1]$. With the use of v , each point of (1) can be expressed by another point in “super face” in real space. The set of these points shapes a model called Semantization Associate Memory (SAM).

Step 3: use a combination method to convert SAM model into a real curve in the plane namely semantic quantifying curve. Therefore, the original reasoning problem will turn into the linear interpolating problem. Determining and computing outputs according to linear interpolating method on this curve will bring reasoning results.

Recently outstanding successes of HA are based on functional reasoning method including open elements to help researchers develop and choose different accesses depending on study purpose. A number of studies using HA mainly focus on information technology and control sectors. For example, control problems for inverted pendulums, earthquake prediction or disease diagnosis... Hence, this paper analyzes and appreciates existing results of various studies, leading to the development of methods applied in the industry.

2.1. The research direction of the selection of HA parameters

The problem of determining the parameters of HA (including the fuzziness measurement of elements and hedges).

From the model (1), we see that this model includes $(m+1)$ linguistic variables. HA parameters consist of the fuzzy measure of elements: $fm_{AX_i}(c^-)$, $fm_{AX_i}(c^+)$ in the relation: $fm_{AX_i}(c^-) + fm_{AX_i}(c^+) = 1$. And the fuzzy measure of particles: $\mu_{AX_i}(h_j)$

Satisfying the condition:

$$\sum_{j=-q_i}^{-1} \mu_{AX_i}(h_j) = \alpha$$

$$\sum_{j=1}^{p_i} \mu_{AX_i}(h_j) = \beta$$

And $\alpha + \beta = 1$

The open factor here is the research using different algorithms to seek a set of HA parameters optimally to meet demands of the control system.

2.1.1. Using intuition to select parameters

Some researches [3],[8] have had an intuitive selection of HA parameters. For example, the ability to sense fuzzy measure of elements “true” or “false” is equal, or fuzzy measure of particles “very”, “more”, “little”, “possibly” is equal. Study results have applied for some control problems having simple mathematical models.

Remark: choosing parameters based on intuition in this study is simple but without mathematical basis.

2.1.2. Using optimal search tools

In [6], [10], the parameters of HA are determined by genetic algorithm, aiming at minimizing the error of the method. These study results have adopted to control problems having simple mathematical models and to monotonic or oscillating systems without overshoot. When applying to control problems of non-linear objects, it is difficult to gain good results because the objective function does not reflect accurately the quality of transition process.

Remark: The choice of these parameters bases on a strict mathematical basis. However, sometimes the value of parameters is determined more differently than the intuitive sense of people. (For example, it is difficult to accept if the fuzzy measure of “true” is 0.9; while “false” is 0.1).

2.2. Research direction offers reasoning method

For multi-input control problems, input-output relationship is a multi-dimensional grid. By constructing semantic quantifying mappings from HA (AX_i , AY) on $[0,1]$, each point of (1) can be represented by a set of hyper surface points in real space (SAM model).



The open factor here is to search for combination methods or directly deduced methods from the SAM model without combination methods.

2.2.1. Using combination methods:

For control systems with multiple input languages, all inputs can be combined into only one input in order to convert input-output relationship into a curve in 2-dimension space. Some recent researches have used the combination method to put the point of the SAM model into one point in the plane. Then points of the SAM model create a curve called the quantitative semantic curve, and the reasoning problem becomes a typically interpolate problem on the curve. In [3], [8] used the combination method:

AND = "PRODUCT" or AND = "MIN"

Study results in [10] are also adopted the combination method with coefficient [18] to convert a point $(x_1, x_2, \dots, x_m) \in \mathbb{R}^m$ with coefficients (w_1, w_2, \dots, w_m) satisfying a condition that $(w_1 + w_2 + \dots + w_m = 1)$ to vectors $(w_1x_1 + w_2x_2 + \dots + w_mx_m) \in \mathbb{R}^m$. As a result, the parameters of this method are combination coefficients.

Remark: in the combination method, the curve represented the input-output relationship is unlikely to describe totally the semantics of original rules, easily cause information loss and even cause conflicts between the rules, leading to inaccurate inferences. Selecting different combination methods results in different input-output relationships, that affects the output. Thus, choosing combination operators for each particular problem is also a complex one.

2.2.2. Using Radial Based Function (RBF)

In [6], [9], the neural network RBF is adopted to solve the directly reasoning problems from the quantitatively semantically curved surface and the SAM table. The points in the real "super face" given by the SAM table will be used as a sample for network training. Then neural network will approximate the "super face" and the output interpolation is corresponds to the input.

Remark: the advantage of this method is that information does not disappear during the reasoning process because the linear interpolation is not used. However, when optimizing the parameters of the reasoning problems, with each set of parameters, we get different "super - surfaces" and the network training process must be implemented before calculating objective function from the reasoning problem. Therefore, this method leads to more difficulties and more time consumption in training the network and optimizing parameters.

2.2.3. Inference in three dimensions

For two - input problems, the structure of input/output relationship is a three - dimension curve surface. When compared with the combination of inputs, in which the information can be lost and the network training is seen as difficult, the direct inference on the "super - face" is a good solution to overcome the above limitations. In [12], the inference in 3D - space is a direct linear interpolation method in three dimensions [16] to solve the approximate reasoning problems. In order to accurately approximate the curve surface, we can choose the nonlinear interpolation methods; however, there are more disadvantages due to the large number of calculations.

Remark: The method mentioned in [12] is regarded as more effective, simple and beneficial for a number of calculations. The inference results show that the information is not lost during the reasoning process, and the operation process gets quicker and easier due to without network training.

2.3. Experiment

Applying HA on specific physical models is being studied by many scientists.

The study results using HA have just applied for control problems with simple mathematical models by mathematical analysis and simulation [6], [8], [10].

The model [9] has studied the applications of fuzzy controllers and access to HA in a simple physical model such as the control problem for water level.

The model [12] uses HA for the exciting voltage controllers for self - exciting generators. However, the method [12] need to be studied, experimented on more complex control problems and to meet the real time requirements of industrial systems.



3. Experimental Results in Control

From the above analysis, the researches using HA are conducted successfully when solving the control problems with a simple mathematical model, but it seems difficult to apply for complex control problems. For nonlinear control problems, the HA method is still new and is an open research direction. Thus, the research and development of HA used in the control field become necessary. In the hope of using the HA as a study to design controllers in automated systems, the paper proposes some early results of the studies applying HA to a wide number of industrial objects [7].

3.1. Linear object with varied parameter

Controlling linear object with varying parameter J [7], [15] is shown in Fig. 1, simulated results as shown in Fig. 2, 3.

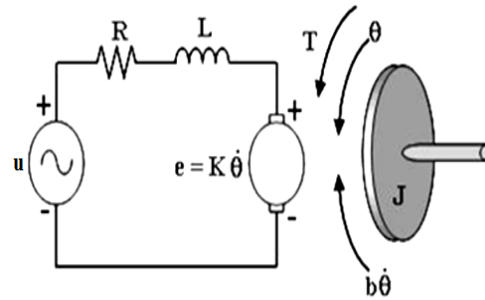


Figure 1: The replacement diagram of a DC with varying parameter J

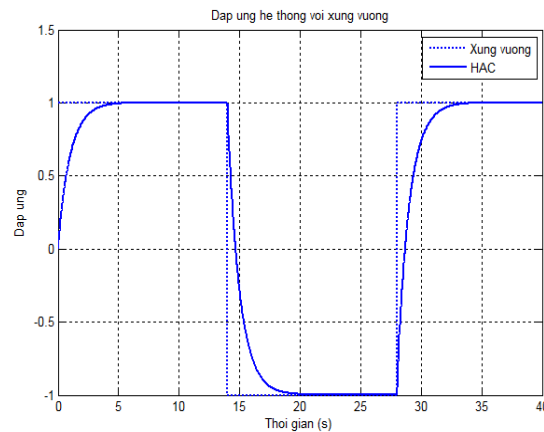


Figure 2: The response of the system with square pulses

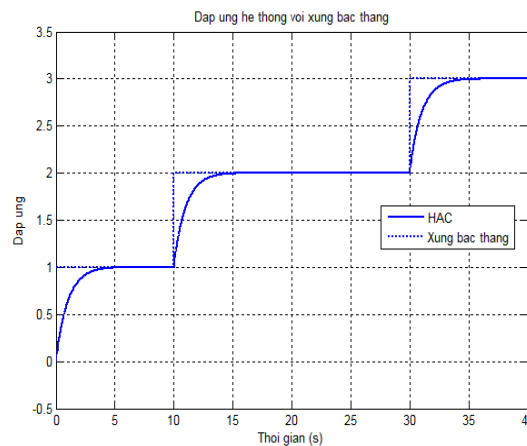


Figure 3: The response of the system with step pulses

3.2. Object with great delay

For controlled objects with great delay, simulation results are displayed in Fig. 4.

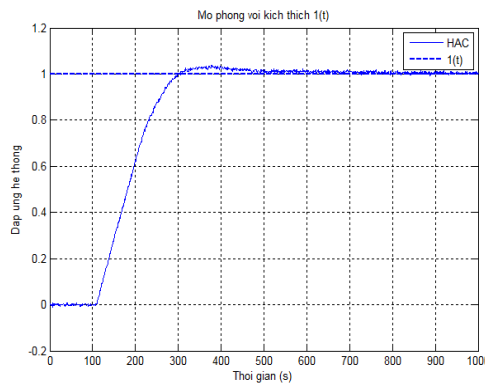


Figure 4: The response of the system with exciting function $1(t)$

3.3. Ball and Beam system

The simulation result of controlling the Ball and Beam system (Fig. 5) is represented in Fig. 6.

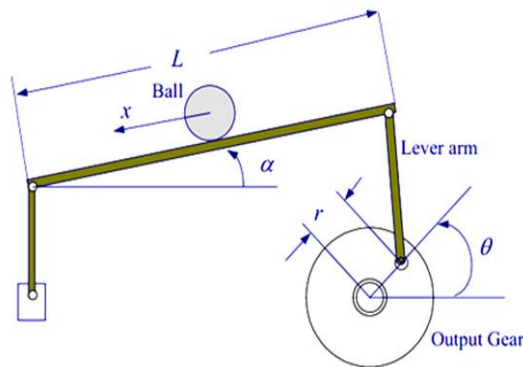


Figure 5: The description diagram of the Ball and Beam system

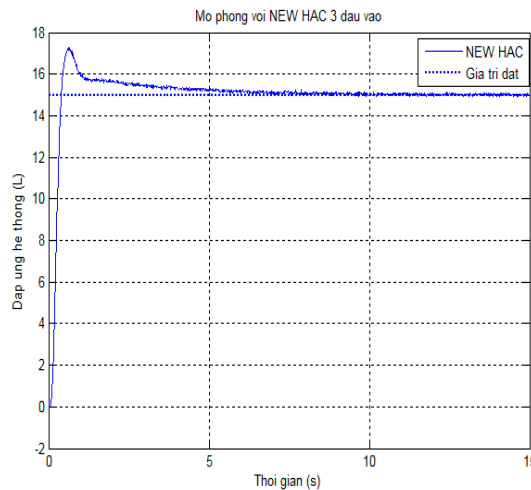


Figure 6: The response of the system

3.4. Nonlinear object

MEDE5 (Mechatronic Demonstrate Setup-2005) was designed by the Control Engineering group of the Department of Electrical Engineering in The Faculty of Electrical Engineering, Mathematics and Computer Science of The University of Twente as shown in Fig. 8. The mechanical part of the setup is designed mimicking printer technology. It consists of a slider which can move back and forth over a rail. In the model, a

DC motor, rail and slider are fixed on a flexible frame which causes vibration during the acceleration of the mass. The model can be applied to real applications such as 2-D, 3-D printers, CNCs and accurate control systems. If control theories are designed well then acceleration and deceleration processes of the slider will be smoother and the vibration of the frame will be reduced. The above system is verified by certain control methods that are integrated with PID, MRAS in order to drive the system to a set position or follow the reference orbit. The control system of nonlinear objects and simulation results are shown in Fig. 7, 8, 9, 10.

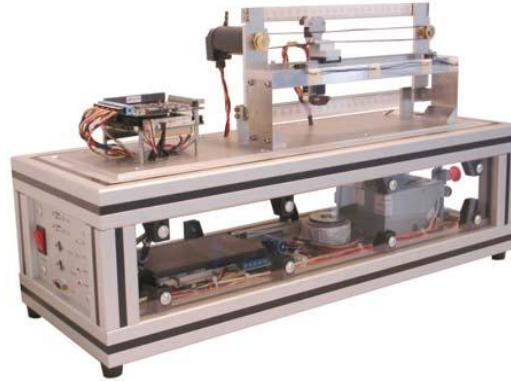


Figure 7: The model of system MEDE5

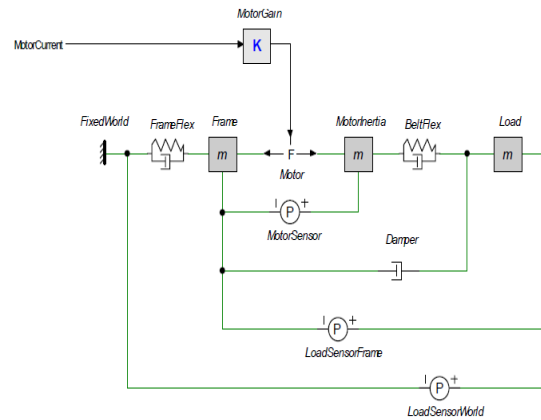


Figure 8: Modeling the mechanical structure of MEDE5

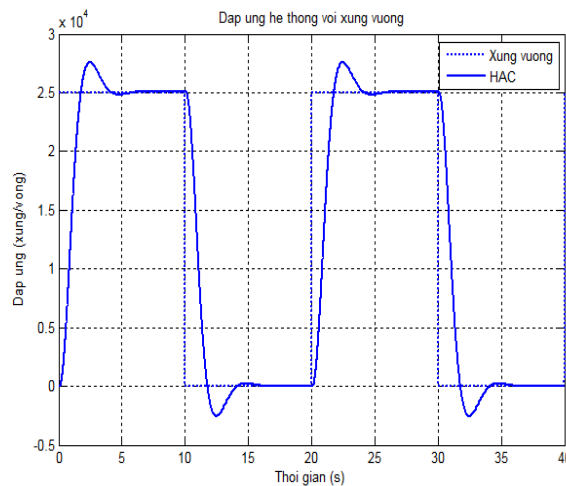


Figure 9: The response of the system with square pulses

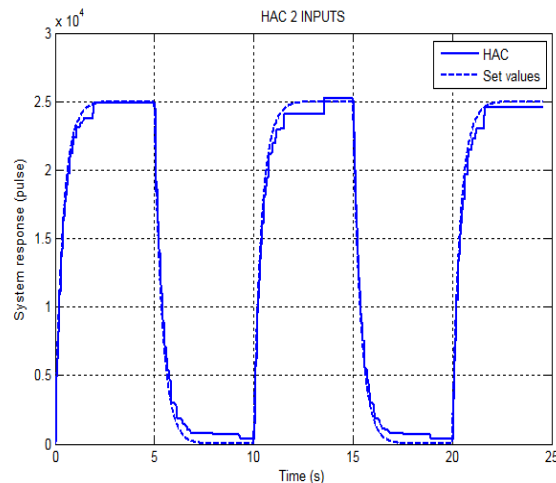


Figure 10: HAC performance

Remarks: simulation and experiment results for different object classes have verified the effectiveness of the method. The control system using HA has a good performance and meets the technical requirements.

4. Conclusions

This paper has proposed a new method to design a controller in which HA is used as a soft-computing tool applied in control. There are few factors affecting the reasoning process because it considers each language domain of language variables as an HA. The original reasoning problem turns to ordinary reasoning one by the quantitative semantic function.

During the design process, the advantages of HA are that using HA can create an algebraic structure in the form of a function relationship, allowing to build a great set of arbitrary language value for describing input-output relations.

The controllers using HA which does not solve identify problems or use the parameter estimators have a good performance and meet the technology requirements. Research results open the applicability of HA in the industry.

References

- [1]. C. C. Bui, H. P. Nguyen, D. P. Nguyen, X. M. Phan and V. H. Chu, *Fuzzy and Application*, Vietnam: Science and Technics Publishing House, 1999.
- [2]. D. C. Nguyen, "Advanced Controllers for Electromechanical motion Systems," The Doctor thesis, the University of Twente, the Netherlands, 2008.
- [3]. C. H. Nguyen and Wechler W, "An algebraic approach to structure of sets linguistic truth values," *Fuzzy sets and systems*, vol. 35, pp. 218 – 293, 1990.
- [4]. C. H. Nguyen, K. D. Tran and V. L. Xuan, "Fuzziness measure, quantified semantic mapping and interpolative method of approximate reasoning in medical expert systems," *The Journal of computer science and cybernetics*, vol. 18, No. 3, pp. 237 – 252, 2002.
- [5]. C. H. Nguyen, N. L. Vu and V. L. Xuan, "Optimal hedge-algebra-based controller: Design and application," *Fuzzy Sets and Systems*, vol. 159, No. 8, pp. 968 – 989, 2008.
- [6]. T. H. Pham, "Developing fuzzy reasoning methods using Hedge Algebra and applications," The Doctor thesis, Information Technology Institute, Hanoi, 2010.
- [7]. T. K. Ngo, "The research to improving hedge algebra based controller for nonlinear object", The Doctor thesis, ThaiNguyen University, ThaiNguyen, 2014.
- [8]. N. L. Vu, "The control using fuzzy logic, neural networks and Hedge Algebra," *The Journal of Science and Technology*, Science and Technics Publishing House, 2006.
- [9]. D. M. Nguyen, "The access to Hedge Algebra in the fuzzy control," The Doctor thesis, Information Technology Institute, Hanoi, 2013.



- [10]. X. V. Le, “Quantifying semantic values of linguistic variables based on Hedge Algebra and applications,” The mathematical doctor thesis, Information Technology Institute, Hanoi, 2009.
- [11]. Abdulrahman A. A. E, “Fuzzy control for nonlinear ball and beam system,” *International Journal of Fuzzy Logic Systems*, Vol. 3, No. 1, pp. 25 – 32, 2013.
- [12]. Dinko Vukadinović, Mateo Bašić, C. H. Nguyen, N. L. Vu, T. D. Nguyen, “Hedge-algebra-based voltage controller for a self-excited induction generator,” *Control Engineering Practice*, Vol. 30, pp. 78 – 90, 2014.
- [13]. D. A. Nguyen, H. L. Bui, N. L. Vu and D. T. Tran, “Application of hedge algebra-based fuzzy controller to active control of a structure against earthquake,” *Struct Control Health Monit*, vol. 20, pp. 483 - 495, ISSN 1545 – 2255, 2013.
- [14]. Farhan A. S, “Mechatronics design of Ball and Beam system: Education and Research,” *Control Theory and Informatics*, Vol. 3, No. 4, pp. 1 - 27, ISSN 2224 - 5774, 2013.
- [15]. Louchene, Benmakhlouf and Chaghi, “Solar tracking system with fuzzy reasoning applied to scisp set”, *Revue des Energies Renouvelables*, Vol. 10, No. 2, pp. 231 – 240, 2007.
- [16]. S.R. Buss, *A Mathematical Introduction with OpenGL*, New York: Cambridge University Press, 2003.
- [17]. Twillert H., “Demonstrator of Advanced Controllers,” The M.Sc. thesis, University of Twente, The Netherlands, 2005.
- [18]. Yager R. R., “Aggregation operators and fuzzy systems modeling,” *Fuzzy Sets and Systems*, vol. 67, pp. 129 - 145, 1994.

