Available online www.jsaer.com

Journal of Scientific and Engineering Research, 2019, 6(4):167-174

Research Article

ISSN: 2394-2630
CODEN(USA): JSERBR

Separation Axiom (Hausdorff) on Fuzzy Bitopological Space in Quasi-coincidence Sense

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Abstract In this paper, we introduce some new definitions of Hausdorff separation (T_2 separation) on fuzzy bitopological space in quasi-coincidence sense and establish relations among them and their counterparts. We show that the notions are good extension, hereditary, productive and projective. In fact, we observe that these concepts are preserved under one-one, onto, fuzzy open and fuzzy continuous mappings. Finally, we discuss about initial and final fuzzy bitopological spaces on our notions using quasi-coincidence sense.

Keywords Fuzzy Bitopological Space; Quasi-coincidence; Fuzzy T_2 Bitopological Space; Good Extension; Mapping; Initial and Final Fuzzy Bitopologies.

1. Introduction

The fuzzy set was first explored in [35] and this concept extended to fuzzy topological spaces in [5]. More recently much research has been done extending the theory of fuzzy topological spaces in various directions; in particular, normality [13], uniformity [14] and regularity [2], topological representation [6], separations in fuzzy topological spaces [3, 11, 27, 29], fuzzy topological groups [7], fuzzy bitopological spaces [3, 4, 12, 16, 23], product of fuzzy topological spaces [15], strong-separation and strong countability [31], supra fuzzy topological spaces [9, 10, 20] and infra fuzzy topological spaces[8, 32]. One of the important parts in fuzzy mathematics is fuzzy bitopological space with separation axioms, which continuously attracted significant international attention.

The research for fuzzy bitopological spaces started in early nineties [16]. The topic of fuzzy bitopological spaces with separation axioms has become attractive area of research, as these spaces possess many desirable properties and can be found throughout various areas in fuzzy topologies. Recent progress has been made constructing separation axioms on fuzzy bitopological spaces [16, 24]. One most studied in separation axioms on fuzzy bitopological spaces is T_2separation [24].

The purpose of this paper is to further contribute to the development of fuzzy bitopological spaces, especially on fuzzy T_2bitopological spaces. In this paper, we define fuzzy T_2bitopological space in quasi-coincidence sense [24, 26, 28]. We show that the definitions of the T_2separation satisfy the good extension property. We also present the hereditary, order preserving, productive, and projective properties of these new concepts. In addition, we discuss the initial and final fuzzy topologies of the T_2 separation.

2. Basic notions and preliminary results

In this section, we review some concepts occurring in the papers [1, 35], which will be needed in the sequel. In this paper, X and Y always presented non-empty sets.

Definition 2.1 [35] A function u from X into the unit interval I is called a fuzzy set in X. For every $x \in X$, $u(x) \in I$ is called the grade of membership of x in u. Some authors say that u is a fuzzy subset of Xinstead of saying that u is a fuzzy set in X. The class of all fuzzy sets from Xinto the closed unit interval $I$is denoted by I^X.

Definition 2.2 [21] A fuzzy set u in X is called a fuzzy singleton if and only if $u(x) = r, 0 < r \leq 1$, for a certain $x \in X$ and $u(y) = 0$for all points y of X except x. The fuzzy singleton is denoted by x_r and x is its support. The class of all fuzzy singletons in X will be denoted by $S(X)$. If $u \in I^X$ and $x_r \in S(X)$, then we say that $x_r \in u$ if and only if $r \leq u(x)$.

Definition 2.3 [34] A fuzzy set u in X is called a fuzzy point if and only if $u(x) = r, 0 < r < 1$, for a certain $x \in X$ and $u(y) = 0$ for all points y of X except x. The fuzzy point is denoted by x_r and x is its support.

Definition 2.4 [16] A fuzzy singleton x_r is said to be quasi-coincidence with u, denoted by $x_r\ qu$if and only if $u(x) + r > 1$. If x_r is not quasi-coincidence with u, we write $x_r \bar{q} u$ and defined as $u(x) + r \leq 1$.

Definition 2.5 [5] Let f be a mapping from a set X into a set Y and v be a fuzzy subset of Y. Then the inverse of v written as $f^{-1}(v)$ is a fuzzy subset of X defined by $f^{-1}(v)(x) = v(f(x))$, for $x \in X$.

Definition 2.6 [22] The function $f : (X,t) \to (Y,s)$ is called fuzzy continuous if and only if for every $v \in s, f^{-1}(v) \in t$, the function f is called fuzzy homeomorphic if and only if f is bijective and both f and f^{-1} are fuzzy continuous.

Definition 2.7 [19] The function $f : (X,t) \to (Y,s)$ is called fuzzy open if and only if for every open fuzzy set u in $(X,t), f(u)$ is open fuzzy set in (Y, s).

Definition 2.8 [25] Let f be a real valued function on a topological space. If $\{x : f(x) > \alpha\}$ is open for every real α, then f is called lower semi continuous function.

Definition 2.9 [5] A fuzzy topology t on X is a collection of members of I^X which is closed under arbitrary suprema and finite infima and which contains constant fuzzy sets 1 and 0. The pair (X, t) is called a fuzzy topological space (fts, in short) and members of t are called t-open (or simply open) fuzzy sets. A fuzzy set μ is called a t- closed (or simply closed) fuzzy set if $1 - \mu \in t$.

Definition 2.10 [30] A bitopological space (X,S,T) is called pairwise- T_1 (PT_1in short) if for all $x, y \in X, x \neq y$, there exist $U \in S, V \in T$ such that $x \in U, y \notin U$and$x \notin V, y \in V$.

A fuzzy bitopological property P is called hereditary if each subspace of a fuzzy bitopological space with property P, also has property P.

Definition 2.11 [33] Let $\{(X_i, s_i, t_i) : i \in \Lambda\}$ is a family of fuzzy bitopological spaces. Then the space$(\Pi X_i, \Pi s_i, \Pi t_i)$is called the product fuzzy bitopological space of the family $\{(X_i, s_i, t_i) : i \in \Lambda\}$, where Πs_i and Πt_i denote the usual product fuzzy topologies of the families $\{\Pi s_i : i \in \Lambda\}$ and $\{\Pi t_i : i \in \Lambda\}$ of the fuzzy topologies respectively on X.

A fuzzy bitopological property P is called productive if the product of fuzzy bitopological spaces of a family of fuzzy bitopological space, each having property P, has property P.

A fuzzy bitopological property P is called projective if for a family of fuzzy bitopological space $\{(X_i, s_i, t_i) : i \in \Lambda\}$, the product fuzzy bitopological space $(\Pi X_i, \Pi s_i, \Pi t_i)$ has property P implies that each coordinate space has property P.

Definition 2.12 [17] Let (X,T) be an ordinary topological space. The set of all lower semi continuous functions from (X, T) into the closed unit interval I equipped with the usual topology constitutive a fuzzy topology associated with (X,T) and is denoted by $(X, \omega(T))$.

Definition 2.13 [18] The initial fuzzy topology on a set X for the family of fts$\{(X_i, t_i)\}_{i \in \Lambda}$ and the family of functions $\{f_i : X \longrightarrow (X_i, t_i)\}_{i \in \Lambda}$ is the smallest fuzzy topology on X making each f_i fuzzy continuous. It is easily seen that it is generated by the family $\{f_i^{-1}(u_i) : u_i \in t_i\}_{i \in \Lambda}$.

Definition 2.14 [18] The final fuzzy topology on a set X for the family of fts$\{(X_i, t_i)\}_{i \in \Lambda}$ and the family of functions $\{f_i : (X_i, t_i) \longrightarrow X\}_{i \in \Lambda}$ is the finest fuzzy topology on X making each f_i fuzzy continuous.

Definition 2.15 [23] A function f from a fuzzy bitopological space (X, s, t) into a fuzzy bitopological space (Y, s_1, t_1) is called fuzzy FP −continuous if and only if $f: (X, s) \to (Y, s_1)$ and $f: (X, t) \to (Y, t_1)$ are both fuzzy continuous.

Theorem 2.1 [4] A bijective mapping from an fts (X, t) to an fts (Y, s) preserves the value of a fuzzy singleton (fuzzy point). Note: pre-image of any fuzzy singleton (fuzzy point) under bijective mapping preserves its value.

3. Fuzzy T_2 bitopological space

In this section, we present some new notions on fuzzy T_2 bitopological spaces and their relevant results. We also discuss existing some well-known properties using these new concepts and establish relationships among them with other existing notions.

Definition 3.1 A fuzzy bitopological space (X, s, t) is called

(a) $FPT_2(i)$ if and only if for any pair $x_m, y_n \in S(X)$ with $x \neq y$, there exist $u, v \in s \cup t$ such that $x_m qu$, $y_n qv$and $u \cap v = 0$.

(b) $FPT_2(ii)$ if and only if for any pair $x_m, y_n \in S(X)$ with $x \neq y$, there exist $u, v \in s \cup t$ such that $x_m qu$, $y_n qv$ and $u\bar{q}v$.

(c) [16] $FPT_2(iii)$ if and only if for any pair of fuzzy singletons x_m, y_n in X with $x \neq y$, there exist $u, v \in s \cup t$ such that $x_m \in u$, $y_n \in v$ and $u\bar{q}v$.

(d) [1] $FPT_2(iv)$ if and only if for any pair of distinct fuzzy singletons x_m, y_n in X, there exist$u, v \in s \cup t$ such that $x_m \in u$, $y_n \in v$and$u \cap v = 0$.

Example 3.1 Let $X = \{x, y\}$ and $u, v \in I^X$ with $u(x) = 1, u(y) = 0,\ v(y) = 1, v(x) = 0$ and t be the fuzzy topology on X generated by $\{0, u, v, 1\}$ and s be the fuzzy topology on X generated by $\{constants\}$. Also, let $x_m, y_n \in S(X)$ with $x \neq y$, then $u(x) + m > 1$ and $v(y) + n > 1$ for $m, n \in (0,1]$. Thus $x_m qu$ and $y_n qv$. Again, $\min\{u(x), v(x)\} = 0$ and $\min\{u(y), v(y)\} = 0$ which imply that $u \cap v = 0$.Hence (X, s, t) is $FPT_2(i)$ as $u, v \in s \cup t$. Also, $u(x) + v(x) \leq 1$ and $u(y) + v(y) \leq 1$ which imply that $u\bar{q}v$. Therefore, (X, s, t) is $FPT_2(ii)$.

Theorem 3.1 Let (X, s, t) be a fuzzy bitopological space and $(X, s \cup t)$ be a fuzzy topological space. If(X, s, t) is FPT_2 then $(X, s \cup t)$is fuzzy T_2 topological space.

Proof: Let (X, s, t) is FPT_2. Since $s \subseteq s \cup t$ and $t \subseteq s \cup t$, it follows that $(X, s \cup t)$ is FT_2.

Theorem 3.2 If the fuzzy topological space (X, s) and (X, t) are both fuzzy $T_2(j)$ topological spaces, then their corresponding fuzzy bitopological space (X, s, t) is $FPT_2(j)$, for $j = i, ii$. But the converse is not true in general.

Proof: Let (X, s) and (X, t) are both $FT_2(j)$. Then their corresponding fuzzy bitopological space (X, s, t) is $FPT_2(j)$, for $j = i, ii$ as $s \subseteq s \cup t$ and $t \subseteq s \cup t$. To prove (X, s, t) is $FPT_2(j)$ does not imply (X, s) and (X, t) are both $FT_2(j)$, for $j = i, ii$, the following is it's a counter example.

Counter example: Let $X = \{x, y\}$ and $u, v \in I^X$and t be the fuzzy topology on X generated by $\{u, v\} \cup \{constants\}$, with $u(x) = 1, u(y) = 0$ and $v(y) = 1, v(x) = 0$. Also, let s be the fuzzy topology on X generated by $\{constants\}$. Then, for any $0 < m \leq 1$ and $0 < n \leq 1, u(x) + m > 1$ and $v(y) + n > 1$, which imply that $x_m qu, y_n qv$.Again, $\min\{u(x), v(x)\} = 0$ and $\min\{u(y), v(y)\} = 0$ which imply that $u \cap v = 0$.Hence (X, s, t) is $FPT_2(i)$as $u, v \in s \cup t$. Also, $u(x) + v(x) \leq 1$ and $u(y) + v(y) \leq 1$ which imply that $u\bar{q}v$. Therefore, $(X, s, t)$is $FPT_2(ii)$. But (X, s) is not $FT_2(j)$, for $j = i, ii$.

Theorem 3.3 For a fuzzy bitopological space (X, s, t) the following implications are true:

$$FPT_2(i) \Rightarrow FPT_2(ii) \nRightarrow FPT_2(i),$$
$$FPT_2(iv) \Rightarrow FPT_2(i) \Rightarrow FPT_2(iv),$$
$$FPT_2(i) \nRightarrow FPT_2(iii) \nRightarrow FPT_2(i).$$

Proof of $FPT_2(i) \Rightarrow FPT_2(ii)$: Let (X, s, t) be a $FPT_2(i)$. We have to prove that (X, s, t) is $FPT_2(ii)$. Let x_m, y_n be fuzzy singletons in X with $x \neq y$. Since (X, s, t) is FPT_2 (i) there exists $u, v \in s \cup t$ such that

$x_m q u, y_n q v$, and $u \cap v = 0$. To prove (X, s, t) is $FPT_2(ii)$, it is only needed to prove that $u\bar{q}v$. Now, $u \cap v = 0 \Rightarrow (u \cap v)(x) = 0 \Rightarrow min\,(u(x),\ v(x)) = 0 \Rightarrow u(x) + v(x) \leq 1 \Rightarrow u\bar{q}v$.

Hence, (X, s, t) is $FPT_2(ii)$.

To show $FPT_2(ii) \nRightarrow FPT_2(i)$, there is a counter example.

Counter Example: Let $X = \{x, y\}$ and $u, v \in I^X$ be given by $u(x) = 0.8, u(y) = 0.1$, $v(y) = 0.8$, $v(x) = 0.1$. Let us consider the fuzzy topology s on X generated by $\{0, u, v, 1\}$. Also, let t be the fuzzy topology on X generated by $\{constants\}$. For $0.2 < m \leq 1, 0.2 < n < 0.9, u(x) + m > 1 \Rightarrow x_m q u$and, $v(y) + n \leq 1 \Rightarrow y_n q v$. Now,$u(x) + v(x) \leq 1$ and $u(y) + v(y) \leq 1 \Rightarrow u\bar{q}v$. But $\min(u(x), v(x)) = 0.1 \neq 0 \Rightarrow (u \cap v)x \neq 0 \Rightarrow u \cap v \neq 0$.Hence,$(X,s,t)$is $FPT2(ii)$ butnot $FPT2(i)$. In the similar way one can prove the other implications.

Theorem 3.4 Let (X, S, T) be a bitopological space. Then (X, S, T) is PT_2 if and only if $(X, \omega(S), \omega(T))$is $FPT_2(j)$, where $j = i, ii, iii, iv$.

Proof: Let (X, S, T) be a PT_2 topological space. We shall prove that $(X, \omega(S), \omega(T))$ is $FPT_2(i)$. Let x, y in X with $x \neq y$. Since (X, S, T) be a PT_2 topological space hence there exists $U, V \in S \cup T$ such that $x \in U, y \in V$and $U \cap V = 0$.From the definition of lower semi continuous function, $1_U, 1_V \in (\omega(S) \cup \omega(T))$. Then $1_U(x) = 1 \Rightarrow 1_U(x) + m > 1 \Rightarrow x_m q 1_U$.

Similarly, we can prove that $y_n q 1_V$. Also, $1_U \cap 1_V = 0$. If $1_U \cap 1_V \neq 0$, then it is to be find ξ in X which implies

$(1_U \cap 1_V)(\xi) \neq 0 \Rightarrow 1_U(\xi) \neq 0$ and $1_V(\xi) \neq 0$

imply that $U(\xi) = 1$ and $V(\xi) = 1$

imply that $\xi \in U$, $\xi \in V \Rightarrow \xi \in U \cap V \Rightarrow U \cap V \neq \phi$, a contradiction. So, $1_U \cap 1_V = 0$. Hence $(X, \omega(S), \omega(T))$ is $FPT_2(i)$.

Conversely, let $(X, \omega(S), \omega(T))$ is $FPT_2(i)$. It is to be proved that (X, S, T) be a PT_2 topological space. Let x, y in X with $x \neq y$. Since $(X, \omega(S), \omega(T))$is $FPT_2(i)$, then for any fuzzy singletons x_m, y_n in X, there exists $u, v \in (\omega(S) \cup \omega(T))$ such that $x_m q u$,$y_n q v$ and $u \cap v = 0$.

Now, $x_m q u \Rightarrow u(x) + m > 1 \Rightarrow u(x) > 1 - m = \alpha \Rightarrow x \in u^{-1}(\alpha, 1]$

Similarly, we can prove that $y \in v^{-1}(\alpha, 1]$.

Again, $u \cap v = 0 \Rightarrow (u \cap v)(\xi) = 0 \Rightarrow \min\{u(\xi), v(\xi)\} = 0$.

We claim that $u^{-1}(\alpha, 1] \cap v^{-1}(\alpha, 1] = \phi$. For, if $\xi \in u^{-1}(\alpha, 1] \cap v^{-1}(\alpha, 1]$, then

$\xi \in u^{-1}(\alpha, 1]$ and $\xi \in v^{-1}(\alpha, 1] \Rightarrow u(\xi) > \alpha$ and $v(\xi) > \alpha \Rightarrow \min\{u(\xi), v(\xi)\} > \alpha$, a contradiction.

Thus, $u^{-1}(\alpha, 1] \cap v^{-1}(\alpha, 1] = \phi$. Also, $u^{-1}(\alpha, 1], v^{-1}(\alpha, 1] \in S \cup T$. Hence (X, S, T) be a PT_2 topological space.Proof for $j = ii, iii, iv$ is similar to above.

4. Hereditary, productive and projective properties

In this section, we describe the hereditary, productive and projective properties on our given concepts. The first theorem is on hereditary property and the second one is on productive and projective properties.

Theorem 4.1 If (X, s, t) be a fuzzy bitopological space and $A \subseteq X$, $s_A = \{u/A : u \in s\}$, $t_A = \{v/A : v \in t\}$ and (X, s, t) is $FPT_2(j)$ then (A, s_A, t_A) is $FPT_2(j)$, where $j = i, ii, iii, iv$.

Proof: We first prove this theorem for $j = ii$ and similar to others.

Let (X, s, t) is $FPT_2(ii)$ and x_m, y_n are fuzzy singletons in A with $x \neq y$. Since $A \subseteq X$, x_m, y_n are also fuzzy singletons in X. Also since (X, s, t) is $FPT_2(ii)$, there exists $u, v \in s \cup t$ such that $x_m q u$,$y_n q v$ and $u\bar{q}v$. For $A \subseteq X$, we have u/A, $v/A \in s_A \cup t_A$.

Now, $x_m q u \Rightarrow u(x) + m > 1, x \in X \Rightarrow u/A(x) + m > 1, x \in A \subseteq X \Rightarrow x_m q u/A$

And similarly $y_n q v/A$.

Again, $u\bar{q}v \Rightarrow u(x) + v(x) \leq 1 \Rightarrow u/A(x) + v/A(x) \leq 1 \Rightarrow u/A\bar{q}v/A$. Therefore, (A, s_A, t_A) is $FPT_2(ii)$.

Theorem 4.2 If $\{(X_i, s_i, t_i): i \in \Lambda\}$ is a family of fuzzy bitopological spaces then the product fuzzy bitopological space $(\Pi X_i, \Pi s_i, \Pi t_i) = (X, s, t)$ is $FPT_2(j)$ if and only if each coordinate space (X_i, s_i, t_i) is $FPT_2(j)$, where $j = i, ii, iii, iv$.

Proof: Let for all $i \in \Lambda, (X_i, s_i, t_i)$ is $FPT_2(ii)$ space. We have to prove that (X, s, t) is $FPT_2(ii)$. Let x_m, y_n be fuzzy singletons in X with $x \neq y$. Then $(x_i)_m, (y_i)_n$ are fuzzy singletons with $x_i \neq y_i$ for some $i \in \Lambda$.
Since(X_i, s_i, t_i) is $FPT_2(ii)$, there exists$u_i, v_i \in s_i \cup t_i$such that $(x_i)_m q u_i$,$(y_i)_n q v_i$ and $u_i \bar{q} v_i$. But we have $\pi_i(x) = x_i$ and $\pi_i(y) = y_i$.
Now, $(x_i)_m q u_i \Rightarrow u_i(x_i) + m > 1 \Rightarrow u_i(\pi_i(x)) + m > 1 \Rightarrow (u_i \circ \pi_i)(x) + m > 1 \Rightarrow x_m q (u_i \circ \pi_i)$.
Similarly, we can show that $y_n q (v_i \circ \pi_i)$.
Again, $u_i \bar{q} v_i \Rightarrow u_i(x_i) + v_i(x_i) \leq 1 \Rightarrow u_i\big(\pi_i(x)\big) + v_i\big(\pi_i(x)\big) \leq 1$
$\Rightarrow (u_i \circ \pi_i)(x) + (v_i \circ \pi_i)(x) \leq 1 \Rightarrow (u_i \circ \pi_i)\bar{q}(v_i \circ \pi_i)$. Hence$(X, s, t)$ is $FPT_2(ii)$.
Conversely, let the product fuzzy bitopological space (X, s, t) is $FPT_2(ii)$. It is to be proved that for all $i \in \Lambda, (X_i, s_i, t_i)$ is $FPT_2(ii)$ space. Let a_i be a fixed element in X_i. Let $A_i = \{x \in X = \Pi_{i\in\Lambda} \in X_i\colon x_j = a_j \ for\ some\ i \neq j\}$. Then $A_i$is a subset of X, and hence (A_i, s_{A_i}, t_{A_i}) is a subspace of (X, s, t). Since (X, s, t) is $FPT_2(ii)$, so(A_i, s_{A_i}, t_{A_i}) is $FPT_2(ii)$. Again, A_i is homeomorphic image of X_i. Therefore, for all $i \in \Lambda$, (X_i, s_i, t_i) is $FPT_2(ii)$. In the similar way we can prove the others.

5. Mappings in fuzzy T_2 bitopological space

We discuss in this section about order preserving property of the notions under one-one, onto, fuzzy open and fuzzy continuous mappings.

Theorem: 5.1 Suppose (X, s, t) and (Y, s_1, t_1) are two fuzzy bitopological spaces and $f: X \to Y$ is bijectiveand fuzzy open map. If (X, s, t) is $FPT_2(j)$ then(Y, s_1, t_1) is $FPT_2(j)$, where $j = i, ii, iii, iv$.

Proof: Let $(X, s, t)$is $FPT_2(i)$ and x'_m, y'_n be fuzzy singletons in Y with $x' \neq y'$. Since f is onto then there exist $x, y \in X$with $f(x) = x'$, $f(y) = y'$and $x_m, y_n$are fuzzy singletons in X with $x \neq y$as f is one-one.
Again, (X, s, t) is $FPT_2(i)$, there exists $u, v \in s \cup t$such that $x_m q u$, $y_n q v$ and $u \cap v = 0$.
Now, $x_m q u \Rightarrow u(x) + m > 1$. Now, $f(u)(x') = \{sup\, u(x) : f(x) = x'\} \Rightarrow f(u)(x') = u(x)$, for some x and $f(u)(y') = \{sup\, u(y) : f(y) = y'\} \Rightarrow f(u)(y') = u(y)$, for some y Also, since $f$is a fuzzy open hence$f(u) \in s_1 \cup t_1 \quad as \quad u \in s \cup t$.
Again, $u(x) + m > 1 \Rightarrow \big(f(u)\big)(x') + m > 1 \Rightarrow x'_m q\, f(u)$. Similarly, it is easy to show that $y'_n q\, f(v)$.
Again, $u \cap v = 0 \Rightarrow \min\{u(x), v(x)\} = 0$
$\Rightarrow \min\{\big(f(u)\big)(x'), \big(f(v)\big)(x')\} = 0 \Rightarrow f(u) \cap f(v) = 0$. Also since f is fuzzy open, $f(u), f(v) \in s_1 \cup t_1$.
Thus,$(Y, s_1, t_1)$is $FPT_2(i)$. Similarly, one can prove the others.

Theorem: 5.2 Suppose (X, s, t) and (Y, s_1, t_1) are two fuzzy bitopological spaces and $f: X \to Y$ is one-one and fuzzy FP −continuous map. If (Y, s_1, t_1) is $FPT_2(j)$, then (X, s, t) is $FPT_2(j)$, where $j = i, ii, iii, iv$.

Proof: Let (Y, s_1, t_1) is $FPT_2(ii)$ andx_m, y_n be fuzzy singletons in X with $x \neq y$. Then$\big(f(x)\big)_m$, $\big(f(y)\big)_n$are fuzzy singletons in Y with $f(x) \neq f(y)$ as $f$is one-one. Also, since (Y, s_1, t_1) is $FPT_2(ii)$, there exists $u, v \in s_1 \cup t_1$such that $\big(f(x)\big)_m\, q\, u$, $\big(f(y)\big)_n\, q\, v$ and $u \cap v = 0$.
Now,$\big(f(x)\big)_m\, q\, u \Rightarrow u\big(f(x)\big) + m > 1 \Rightarrow f^{-1}\big(u(x)\big) + m > 1 \Rightarrow \Big(f^{-1}(u)\Big)(x) + m > 1 \Rightarrow x_m\, q\big(f^{-1}(u)\big)$ and in the same way, it is easy to prove that $y_n\, q\big(f^{-1}(v)\big)$.
Again, $u\bar{q}v \Rightarrow u(f(x)) + v(f(x)) \leq 1 \Rightarrow f^{-1}(u(x)) + f^{-1}(v(x)) \leq 1 \Rightarrow f^{-1}(u)(x) + f^{-1}(v)(x) \leq 1 \Rightarrow f^{-1}(u)\bar{q}\, f^{-1}(v)$. As f is fuzzy FP −continuous mapping and $u, v \in s_1 \cup t_1$then $f^{-1}(u)$, $f^{-1}(v) \in s \cup t$. Therefore, $(X, s, t)$is $FPT_2(ii)$. The proof of others is similar to above.

6. Initial and finalfuzzy T_2 bitopological space

Initial and final fuzzy bitopologies are defined and discussed with some properties in this section.

Definition 6.1 The initial fuzzy bitopology on a set X for the family of fuzzy bitopological spaces $\{(X_i, s_i, t_i)\}_{i\epsilon\Lambda}$and the family of functions $\{f_i: X \longrightarrow (X_i, s_i \cup t_i)\}_{i\epsilon\Lambda}$ is the smallest fuzzy bitopology on X making each f_i fuzzy continuous. It is easily seen that it is generated by the family $\{f_i^{-1}(u_i): u_i \epsilon s_i \cup t_i\}_{i\epsilon\Lambda}$.

Definition 6.2 The final fuzzy bitopology on a set X for the family of fuzzy bitopological spaces $\{(X_i, s_i, t_i)\}_{i\epsilon\Lambda}$and the family of functions $\{f_i: (X_i, s_i \cup t_i) \longrightarrow X\}_{i\epsilon\Lambda}$ is the finest fuzzy bitopology on X making each f_i fuzzy continuous.

Theorem 6.1 If $\{(X_i, s_i, t_i)\}_{i\epsilon\Lambda}$ is a family of $FPT_2(j)$ fts and $\{f_i: X \longrightarrow (X_i, s_i \cup t_i)\}_{i\epsilon\Lambda}$, a family of one-one and fuzzy continuous functions, then the initial fuzzy bitopology on X for the family $\{f_i\}_{i\epsilon\Lambda}$ is $FPT_2(j)$, for $j = i, ii, iii, iv$.

Proof: We shall prove the above theorem for $j = ii$ and the remaining is similar. Let t, s be the initial fuzzy topologies on X for the family $\{f_i\}_{i\epsilon\Lambda}$. Let x_m, y_n be fuzzy singletons in X with $x \neq y$. Then f_i(x), f_i(y) ∈ X_i and f_i(x)≠ f_i(y) as f_i is one-one. Scince $(X_i, s_i, t_i)$is $FPT_2(ii)$, then for any two distinct fuzzy singletons $(f_i(x))_m$, $(f_i(y))_n$ in X_i, there exist fuzzy sets u_i, $v_i \in s_i \cup t_i$ such that $(f_i(x))_m$q u_i,$(f_i(y))_n$q v_i, and $u_i\bar{q}v_i$. Now,$\left(f_i(x)\right)_m$q $u_i \Rightarrow u_i\left(f_i(x)\right) + m > 1 \Rightarrow f_i^{-1}(u_i)(x) + m > 1$. This is true for every $i\epsilon\Lambda$. So, $inf\, f_i^{-1}(u_i)(x) + m > 1$. And $(f_i(y))_n qv_i \Rightarrow v_i\left(f_i(y)\right) + n > 1 \Rightarrow f_i^{-1}(v_i)(y) + n > 1$.This is true for every $i\epsilon\Lambda$.So, $inf\ f_i^{-1}(v_i)(y) + n > 1$.Again, $u_i\bar{q}v_i \Rightarrow u_i\left(f_i(x)\right) + v_i\left(f_i(x)\right) \leq 1 \Rightarrow f_i^{-1}(u_i)(x) + f_i^{-1}(v_i)(x) \leq 1$. So, $inf\ f_i^{-1}(u_i)(x) + inf\ f_i^{-1}(v_i)(x) \leq 1$. Let $u =\ inf\ f_i^{-1}(u_i)$ and $v =\ inf\ f_i^{-1}(v_i)$. Then u, $v \in s_i \cup t_i$ as f_i is fuzzy continuous. So $u(x) + m > 1$,$v(y) + n\ > 1$ and $u(x) + v(x) \leq 1$. Hence, $x_m qu$, $y_n qv$ and $u\bar{q}v$. Therefore, (X, s, t) is $FPT_2(ii)$.

Theorem 6.2 If $\{(X_i, s_i, t_i)\}_{i\epsilon\Lambda}$ is a family of $FPT_2(j)$ fts and $\{f_i: (X_i, s_i \cup t_i) \longrightarrow X\}_{i\epsilon\Lambda}$, a family of fuzzy open and bijective function, then the final fuzzy topology on X for the family $\{f_i\}_{i\epsilon\Lambda}$ is $FPT_2(j)$, for $j = i, ii, iii, iv$.

Proof: We shall prove the above theorem for $j = ii$ and the remaining is similar. Let s, t be the final fuzzy topologies on X for the family $\{f_i\}_{i\epsilon\Lambda}$. Let x_m, y_n be fuzzy singletons in X with$x \neq y$. Then f_i^{-1}(x), f_i^{-1}(y) ∈ X_i and f_i^{-1}(x)≠ f_i^{-1}(y) as f_i is bijective. Since (X_i, s_i, t_i) is $FPT_2(ii)$, then for any two distinct fuzzy singletons $(f_i^{-1}(x))_m$, $(f_i^{-1}(y))_n$ in X_i, there exist fuzzy sets u_i, $v_i \in s_i \cup t_i$ such that $(f_i^{-1}(x))_m$q u_i, $(f_i^{-1}(y))_n$q v_i and $u_i\bar{q}v_i$. Now, $(f_i^{-1}(x))_m$q $u_i \Rightarrow u_i\left(f_i^{-1}(x)\right) + m > 1 \Rightarrow f_i(u_i)(x) + m > 1$. This is true for every $i\epsilon\Lambda$. So, $inf\ f_i(u_i)(x) + m > 1$. And $(f_i^{-1}(y))_n qv_i \Rightarrow v_i\left(f_i^{-1}(y)\right) + n > 1 \Rightarrow f_i(v_i)(y) +> 1$. This is true for every $i\epsilon\Lambda$. So, $inf\ f_i(v_i)(y) + n > 1$. Again, $u_i\bar{q}v_i \Rightarrow u_i\left(f_i^{-1}(x)\right) + v_i\left(f_i^{-1}(x)\right) \leq 1 \Rightarrow f_i(u_i)(x) + f_i(v_i)(x) \leq 1$. This is true for every $i\epsilon\Lambda$. So,$inf\ f_i(u_i)(x) + inf\ f_i(v_i)(x) \leq 1$. Let $u = inf\ f_i(u_i)$ and $v =\ inf\ f_i^{-1}(v_i)$. Then $u, v\ \in s_i \cup t_i$ as f_i is fuzzy open. So, $u(x) + m > 1$,$v(y) + n\ > 1$ and $u(x) + v(x) \leq 1$. Hence, $x_m qu$, $y_n qv$ and $u\bar{q}v$. Therefore, (X, s, t) is $FPT_2(ii)$.

7. Conclusion

The main result of this paper is introducing some new definitions of fuzzyT_1 bitopological spaces in sense of quasi-coincidence. We present their good extension, hereditary, productive and projective properties. We compare our results with other existing notions and their counter parts' examples [1, 24, 30]. These concepts would be interesting to more expansion on fuzzy bitopological spaces than [30] and extending to general fuzzy topological space [5].

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