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The Comparison between Analytical and Numerical Solutions of Blasius Equation on Boundary Layer Parallel Flow over a Flat Plate

Md. Mizanur Rahman

Lecturer, Dept. of CSE, Varendra University, Rajshahi-6204, Bangladesh

Abstract Some authors have solved the Blasius equation for particular case by analytically or numerically or both. None of them solved it for some special cases (like as laminar profile, linear profile, parabolic profile, cubic profile and sin-cos profile). In this paper, we have solved the Blasius equation of boundary layer flow over a flat plate for five different profiles by analytically and numerically. We have compared the results graphically and then obtained phenomenal result.

Keywords Blasius equation, Wang's transformation, the finite difference approximation to derivative formula and MATHLAB

Introduction

The following boundary value problem with boundary condition

$$f'''(\eta)+\beta f(\eta) f''(\eta)=0 \qquad \ldots(1.1)$$

$$\left.\begin{array}{ll} f=0,\ f'=0 & \text{when } \eta=0 \\ and\ \ f'=1 & \text{when } \eta=\infty . \end{array}\right\}$$

where $\beta>0$, plays an important role in the boundary layer theory of fluid dynamics and is known as the Blasius H. [1] equation when $\beta=1$. The main hurdle in the solution of the above problem is the absence of the second derivative $f''(0)$. Once this derivative has been correctly evaluated an analytical solution of the boundary value problem may be readily found. Blasius H. [2] found the following power series solution of the problem with $\beta=1/2$ is $f(\eta)=\sum_{k=0}^{\infty}\left(-\frac{1}{2}\right)^k \frac{B_k \sigma^{k+1}}{(3k+2)!}\eta^{3k+2}$ … (1.2)

where $B_0=B_1=1$, $B_k=\sum_{r=0}^{k-1}\binom{3k-1}{3r}B_r B_{k-r-1}$; $k\geq 2$ and σ represents the unknown $f''(0)$. Howarth [3] solved the Eq.(1.1) with $\beta=0.5$ numerically and found $\sigma=0.33206$. Asaithambi [4] solved the Blasius equation more accurately and obtained this number as $\sigma=0.332057336$. Several authors have devised numerical algorithms to find good approximations to $f''(0)$, as for example, Asaithambi [4] and references therein. Asaithambi [4] solved Eq. (1.1) by considering the condition $\beta=1$ and found $f''(0)$ where $f''(0)=0.469600$. Recently, Wang [5] has used an ingenious idea to find $f''(0)$ analytically for $\beta=1$. He used $x=f'(\eta)$ & $y=f''(\eta)$ to transform Eq.(1.1) to another equation for solved easily as follows,

$$\therefore y'' + \beta \frac{x}{y} = 0 \quad ; x \in [0,1) \qquad \ldots(1.3)$$

with boundary conditions

$$y(0) = f''(0)\,,\ y'(0) = 0 \ \&\ \lim_{x \to 1} y(x) = 0 \qquad \ldots(1.4)$$

Wang [5] used the Adomian decomposition method to solve Eq. (1.3) with $\beta = 1$ and found

$$y(x) = \sigma - \frac{x^3}{6\sigma} - \frac{x^6}{180\sigma^3} - \frac{x^9}{2160\sigma^5} - \frac{x^{12}}{19008\sigma^7} \cdots \cdots \cdots \cdots \cdots \qquad \ldots(1.5)$$

Wang [5] solved this equation retaining six terms of the series Eq. (1.5) and found $f''(0) = \sigma = 0.453539$. Recently, Hashim [6] improved this value to $f''(0) = 0.453539$ by finding terms of the series Eq.(1.5) up to x^{24} by the Adomain decomposition method (ADM) and then approximating this function by the {12/12} diagonal Pade' Approximant. Faiz and Wafaa [7] used "Pade' approximation" up to {23/23} pade' approximant and reached the successive result $f''(0) = 0.469009$ which was very difficult and lengthy. Ahmad [8] has shown that the exact value of $f''(0)$ lies between 0.4695975 and 0.4696064. They have found the value of $f''(0)$ only for $\beta = 1$ which had no special flow.

Now, we have found the second order boundary condition $f''(0)$ to solve the Eq. (1.1) with some special cases (like as laminar profile, linear profile, parabolic profile, cubic profile and sin-cos profile) by the series solution and used the finite difference approximation to derivative formula for numerical solution and MATLAB for both cases.

In this paper, the Blasius equation of boundary layer flow over a flat plate of the problem is presented followed by the series solution of Blasius equation and again review this series solution for different profiles. Using the finite difference method, the numerical solution of second order boundary condition $f''(0)$ of the Blasius equation has been solved for special cases and then drawn an acceleration versus velocity graph by series and numerical solution. Finally, we have compared between these two results graphically and then obtained phenomenal result which acceleration decreases when velocity increases and acceleration exists at zero velocity.

2. General Blasius equation of boundary layer flow over a flat plate

Consider a thin infinite flat plate submerged in steady incompressible plane parallel flow, whose undisturbed velocity is U. The fluid has low viscosity, and the plate is at rest in such a way that its plane coincides with the direction of U. Since the plate is of infinite length, the flow may be considered as two-dimensional. Let the origin of the coordinate system coincide with the front edge of the plate, the x-axis lying along the plate parallel to U and the y-axis normal to the plate. The velocity U of the potential flow is constant in this case so that $\frac{dU}{dx}$=0 and hence $\frac{dp}{dx} = 0$ (Since, for steady flow, $U\frac{dU}{dx} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$).

Thus the Prandtl boundary layer equations in the case under consideration are

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon \frac{\partial^2 u}{\partial y^2} \qquad \ldots(2.1)$$

and $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \qquad \ldots(2.2)$$

The boundary conditions to be satisfied by u and v are

$$\left.\begin{aligned} u = v = 0 \quad \text{when } y = 0 \\ u = U \quad \text{when } y = \infty. \end{aligned}\right\} \qquad ...(2.3)$$

The integration of Eq.(2.1) and Eq.(2.2) can be simplified by reducing the number of unknown with the help of the stream function ψ .

$$u = -\frac{\partial \psi}{\partial y}, \qquad v = \frac{\partial \psi}{\partial x}. \qquad \ldots(2.4)$$

Then, Eq.(2.2) is satisfied automatically by Eq.(2.4).

The order of the boundary layer thickness is $(\upsilon x / U)^{1/2}$ approximate, i.e. $\delta \approx (\upsilon x / U)^{1/2}$. Then $\delta =$ c $(\upsilon x / U)^{1/2}$ … (2.5)

where c is arbitrary constant which is not equal to zero.

The unknown numerical factor remaining in the Eq.(2.5) can be determined from the different profile flows.

Hence, we take the new dimensionless distance parameter $\eta = \frac{y}{\delta}$ so that

$$\eta = \frac{y}{c}\sqrt{\frac{U}{\upsilon x}} = \frac{y}{c} x^{-1/2} \sqrt{\frac{U}{\upsilon}}. \qquad \ldots(2.6)$$

In accordance with the procedure of the law of similarity, let the velocity profile be

$$\frac{u}{U} = F(\eta). \qquad \ldots(2.7)$$

Using Eq.(2.4), Eq.(2.6) and Eq.(2.7), the stream function is given by

$$\psi = -\int u\,dy = -\frac{Uc}{\sqrt{U/\upsilon x}} \int F(\eta)\,d\eta = -c\sqrt{U\upsilon x}\, f(\eta) \qquad \ldots(2.8)$$

where f(η) $= \int F(\eta)\,d\eta$. Then from Eq.(2.4), Eq.(2.6) and Eq.(2.8), we have

$$u = Uf'(\eta) \qquad \ldots(2.9)$$

$$v = \frac{c}{2}\sqrt{\frac{U\upsilon}{x}}[\eta f'(\eta) - f(\eta)] \qquad ...(2.10)$$

$$\frac{\partial u}{\partial x} = -\frac{U\eta}{2x} f''(\eta) \qquad \ldots(2.11)$$

$$\frac{\partial u}{\partial y} = Uf''(\eta) \times \frac{1}{c}\sqrt{\frac{U}{\upsilon x}} \qquad ...(2.12)$$

and $$\frac{\partial^2 u}{\partial y^2} = Uf'''(\eta) \times \frac{1}{c^2}\left(\sqrt{\frac{U}{\upsilon x}}\right)^2 = f'''(\eta)\frac{U^2}{\upsilon x c^2}. \qquad ...(2.13)$$

Substituting Eq.(2.9) to Eq.(2.13) in Eq.(2.1), we get after simplification, the following ordinary differential equation is obtained

$$\therefore \quad f''' + \beta f f'' = 0 \qquad \ldots(2.14)$$

where $\beta = \frac{c^2}{2}$, which is known as general Blasius equation.

Since c is not equal to zero and $\beta = \frac{c^2}{2}$, then $\beta > 0$

Using Eq.(2.6), we see that $y=0 \Rightarrow \eta=0$ *and* $y=\infty \Rightarrow \eta=\infty$. Then from Eq.(2.9) and Eq.(2.10), we find that $u=0$, $v=0$ at y=0 $\Rightarrow f=0, f'=0$ *at* $\eta=0$. Furthermore, Eq.(2.9) shows that $u=U$ $\Rightarrow f'=1$. Hence the boundary conditions Eq.(2.3) may be re-written as

$$\left.\begin{array}{l} f=0,\ f'=0 \text{ when } \eta=0 \\ and\ \ f'=1 \quad \text{ when } \eta=\infty\ . \end{array}\right\} \qquad ...(2.15)$$

Finally, we have

$$\therefore \quad f'''+\beta ff''=0 \qquad \ldots (2.16)$$

where $\beta>0$ and with boundary condition

$$\left.\begin{array}{l} f=0,\ f'=0 \text{ when } \eta=0 \\ and\ \ f'=1 \quad \text{ when } \eta=\infty\ . \end{array}\right\} \qquad \ldots (2.17)$$

This implies that the Eq.(2.16) with Eq.(2.17) is called general Blasius equation.

3. The series solution of the general Blasius equation:

The Blasius equation is of the following form

$$f'''+\beta ff''=0 \qquad ...(3.1)$$

With boundary conditions

$$\left.\begin{array}{l} f=0,\ f'=0 \text{ when } \eta=0 \\ and\ \ f'=1 \quad \text{ when } \eta=\infty\ . \end{array}\right\} \qquad ...(3.2)$$

Since Eq.(3.1) is a third-order non-linear equation, the three boundary conditions Eq.(3.2) are sufficient to determine the solution completely, but the general solution of Eq.(3.1) has not been possible in closed form. The power series solution like as Blasius H [2] as follows

$$f(\eta)=\frac{\sigma}{2\,!}\eta^2-\frac{\beta\sigma^2}{5\,!}\eta^5+\frac{11\beta^2\sigma^3}{8\,!}\eta^8-\frac{375\beta^3\sigma^4}{11\,!}\eta^{11}+\ \ldots\ldots\ldots \qquad \ldots(3.3)$$

$$\therefore\ f(\eta)=\sum_{k=0}^{\infty}(-\beta)^k\frac{B_k\sigma^{k+1}}{(3k+2)!}\eta^{3k+2} \qquad ...(\ 3.4),$$

where $B_0=B_1=1,\ B_k=\sum_{r=0}^{k-1}\binom{3k-1}{3r}B_rB_{k-r-1};k\geq 2$ and σ represents the unknown $f''(0)$ which is analytic(series) solution of general Blasius equation.

Where σ represents the unknown $f''(0)$. Howarth [3] solved the Eq.(3.1) with $\beta=0.5$ numerically and found $\sigma=0.33206$.

Recently, Wang [5] has used ingenious idea to find $f''(0)=\sigma$ analytically for $\beta=1$ and also used $x=f'(\eta)$ & $y=f''(\eta)$ to transform Eq.(3.1) to another equation for solved easily as follows,

$$\therefore\ y''+\beta\frac{x}{y}=0 \quad ;x\in[0,1) \qquad \ldots(3.5)$$

with boundary conditions

$$y(0)=f''(0)=\sigma \quad (say),\ y'(0)=0\ \&\ \lim_{x\to 1}y(x)=0 \qquad \ldots(3.6)$$

We see that the Eq.(3.5) is a non-linear equation, the three boundary conditions in Eq.(3.6) are sufficient to determine the solution completely, but the general solution of Eq.(3.5) has not been possible in closed form. Then, to solve the Eq.(3.5) with Eq.(3.6) for β . Let us consider the series solution of Eq.(3.5) be of the form

$$y(x)=\sum_{i=0}^{\infty}a_ix^i \quad ;a_0\neq 0 \qquad \ldots(3.7)$$

Using the Eq.(3.7) in Eq.(3.5), we get

$$\sum_{i=2}^{\infty} i(i-1)\,a_i x^{i-2} \times (\sum_{i=0}^{\infty} a_i x^i) + \beta x = 0$$

which is an identity, and hence all coefficients of the various powers of x must vanish identically. Thus, we obtain all coefficients by using "MATHLAB". Then, substituting these values in Eq.(3.7), we get

$$\begin{aligned}
y(x) = \sigma &- \frac{\beta x^3}{6\sigma} - \frac{\beta^2 x^6}{180\sigma^3} - \frac{\beta^3 x^9}{2160\sigma^5} - \frac{\beta^4 x^{12}}{19008\sigma^7} - \frac{2099\beta^5 x^{15}}{299376000\sigma^9} - \frac{3037\beta^6 x^{18}}{359251200\sigma^{11}} \\
&- \frac{7159\beta^7 x^{21}}{5588352000\sigma^{13}} - \frac{2923513\beta^8 x^{24}}{13881466368000\sigma^{15}} - \frac{99956627\beta^9 x^{27}}{2748530340864000\sigma^{17}} \\
&- \frac{5575396901\beta^{10} x^{30}}{747256686422400000\sigma^{19}} - \frac{148645596773\beta^{11} x^{33}}{95648855862067200000\sigma^{21}} \\
&- \frac{39353876027701\beta^{12} x^{36}}{120517558386204672000000\sigma^{23}} - \frac{45179608819126391\beta^{13} x^{39}}{654892412270636187648000000\sigma^{25}} \\
&- \frac{49502674070991554293\beta^{14} x^{42}}{3195547468863468158323517030400000\sigma^{27}} - \\
&\frac{172614515616791054195 3\beta^{15} x^{45}}{997691167670755909813862400000000\sigma^{29}} \\
&- \frac{1492881937570648400020141 9\beta^{16} x^{48}}{44932462845516905712559246540800000000\sigma^{31}} \\
&- \frac{457894257433357136987941892348 9\beta^{17} x^{51}}{7115280153901829604112319485968384000000000000\sigma^{33}} \\
&- \frac{258343646927217255288523862012 11\beta^{18} x^{54}}{2056962808127983467370652360489041920000000000000\sigma^{35}} \\
&- \frac{178229487992146942933027763816314 37\beta^{19} x^{57}}{72224078118989755506318345681491239895040000000000000\sigma^{37}} \\
&- \frac{879426703072239315750359220212745025 91\beta^{20} x^{60}}{180282410381632120475386947489568518045696000000000000000\sigma^{39}} \ldots\ldots\ldots \qquad \ldots(3.8)
\end{aligned}$$

where σ represents the unknown $f''(0)$, which is the series solution of general Blasius equation of boundary layer flow over a flat plate.

Let us truncate the series Eq.(3.8) after twenty one terms & substituting x=1 and using Eq.(3.6) in Eq.(3.8). We obtain an approximate value of σ after solving the equation by using MATLAB.

$$\sigma^{40}-\frac{\beta\sigma^{38}}{6}-\frac{\beta^{2}\sigma^{36}}{180}-\frac{\beta^{3}\sigma^{34}}{2160}-\frac{\beta^{4}\sigma^{32}}{19008}-\frac{2099\beta^{5}\sigma^{30}}{29937600}-\frac{3037\beta^{6}\sigma^{28}}{359251200}$$
$$-\frac{7159\beta^{7}\sigma^{26}}{5588352000}-\frac{2923513\beta^{8}\sigma^{24}}{13881466368000}-\frac{99956627\beta^{9}\sigma^{22}}{2748530340864000}$$
$$-\frac{5575396901\beta^{10}\sigma^{20}}{747256686422400000}-\frac{148645596773\beta^{11}\sigma^{18}}{9564885586206720000}$$
$$-\frac{39353876027701\beta^{12}\sigma^{16}}{120517558386204672000000}-\frac{45179608819126391\beta^{13}\sigma^{14}}{654892412270636187648000000}$$
$$-\frac{49502674070991554293\beta^{14}\sigma^{12}}{1955474688634681583235170304000000}-\frac{1726145156167910541953\beta^{15}\sigma^{10}}{99769116767075590981386240000000000}$$
$$-\frac{149288193757064840002014 19\beta^{16}\sigma^{8}}{449324628455169057125592465408000000000}$$
$$-\frac{45789425743335713698794189 23489\beta^{17}\sigma^{6}}{711528015390182960411231948596838400000000000}$$
$$-\frac{258343646927217255288523862 01211\beta^{18}\sigma^{4}}{2056962808127983467370652360489041920000000000000}$$
$$-\frac{1782294879921469429330277638 1631437\beta^{19}\sigma^{2}}{722240781189897555063183456814912398950400000000000}$$
$$-\frac{879426703072239315750359220212745025 91\beta^{20}}{1802824103816321204753869474895685180456960000000000000}=0 \quad \ldots\ldots(3.9)$$

If we use " MATHLAB" in Eq.(3.9), then the value of σ have forty values in the Eq.(3.9), one of them to be 0.330911671809445386596061432915 63 for $\beta=\frac{1}{2}$

i.e. σ = 0.3309117 and 0.46797977422047225176527652357888 for $\beta=1$ i.e. σ =0.4679798 which is more accuracy without loss time than by using "Pade' approximation" method.

4. The value of $f''(0)$ from the series solution of the Blasius equation for some special case on boundary layer flow over a flat late

We know that the boundary layer thickness for the laminar boundary layer flow at a plate at zero incidence which is

$$\delta_{99}(x)=5\sqrt{\frac{\upsilon x}{U}} \qquad \ldots (4.1)$$

Comparing Eq.(2.5) and Eq.(4.1), we get $c=5$ … (4.2)

Then we get from Eq.(4.2) and $\beta=\frac{c^{2}}{2}$ which is $\beta=12.5$ … (4.3)

Therefore, the general Blasius equation of boundary layer flow over a flat plate which is Eq.(4.1) with Eq.(4.3) becomes

$$f'''+12.5ff''=0 \qquad \ldots (4.4)$$

with boundary condition

$$\left.\begin{array}{ll} f=0,\ f'=0 & \text{when } \eta=0 \\ and\ \ f'=1 & \text{when } \eta=\infty . \end{array}\right\}$$

which is Blasius equation of laminar boundary layer flow over a flat plate.

If we use " MATHLAB" in Eq.(3.9) for $\beta = 12.5$, then the value of σ have forty values, one of them to be 1.6545583590472269329803071645781 i.e. $f''(0)$=1.6545584 for laminar boundary layer flow over a flat plate.

Similarly, we get the value of $f''(0)$ for some special cases as below

Table 4.1: for the values of β, we find out the values of $f''(0)$ by using MATLAB after the series solution.

Type of boundary layer flow	**Boundary layer thickness (δ)**	**The value of β**	**The value of $f''(0)$**
laminar profile	$5\sqrt{\frac{\upsilon x}{U}}$	12.500	1.6545584
linear profile	$3.46\sqrt{\frac{\upsilon x}{U}}$	5.986	1.1449735
parabolic profile	$5.48\sqrt{\frac{\upsilon x}{U}}$	15.015	1.8133839
cubic profile	$4.64\sqrt{\frac{\upsilon x}{U}}$	10.765	1.5354445
sin-cos profile	$4.795\sqrt{\frac{\upsilon x}{U}}$	11.496	1.5867206

5. The numerical solution of second order boundary condition of Blasius equation for some special case on boundary layer flow over a flat late

The Blasius equation after using Wang [5] transformation is of the following form

$$\therefore\ y'' + \beta\frac{x}{y} = 0 \quad ; x \in [0,1) \qquad \dots(5.1)$$

with boundary conditions

$$y(0) = f''(0) = \sigma \quad (say),\ y'(0) = 0\ \&\ \lim_{x\to 1} y(x) = 0 \qquad \dots(5.2)$$

We know that the term y'' can be expressed by numerically. The finite difference approximation to derivative formula as below

$$y' = \frac{y_{i+1} - y_{i-1}}{2h} \text{ and } y'' = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} \qquad \dots.(5.3)$$

where $h = x_i - x_{i-1} \quad ; i = 1,2,3,4 \dots\dots\dots n$

Using Eq.(5.3) in Eq.(5.1), we get

$$\frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} + \beta\frac{x_i}{y_i} = 0$$

$$\Rightarrow y_i\left(y_{i+1} - 2y_i + y_{i-1}\right) = -\beta x_i h^2 \qquad \dots(5.4).$$

We subdivide the interval [0,1) into six equal parts so that h=0.17 and $x_0 = 0,\ x_i = x_{i-1} + h;\ i = 1, 2, 3, 4, 5\ \&\ x_6 \to 1\ (i.e.\ x_6 = 1)$ and $y_0 = \sigma\ \&\ y_6 = 0$ … (5.5).

Now, we solve the Eq.(5.4) with Eq.(5.5) by using "MATLAB" in the computer, we get the 32 values of σ for $\beta = 12.5$ in which one value of σ is equal to 1.6378570758116367275156635865643 i.e. $f''(0)$ = 1.6378571 for $\beta = 12.5$.

Similarly, we get the value of $f''(0)$ for some special cases in numerical solution as below

Table 5.1: for the values of β, we find out the values of $f''(0)$ by using MATLAB after numerical method.

Type of boundary layer flow	Boundary layer thickness (δ)	The value of β	The value of $f''(0)$
laminar profile	$5\sqrt{\frac{\upsilon x}{U}}$	12.500	1.6378571
linear profile	$3.46\sqrt{\frac{\upsilon x}{U}}$	5.986	1.1334160
parabolic profile	$5.48\sqrt{\frac{\upsilon x}{U}}$	15.015	1.7950794
cubic profile	$4.64\sqrt{\frac{\upsilon x}{U}}$	10.765	1.5199455
sin-cos profile	$4.795\sqrt{\frac{\upsilon x}{U}}$	11.496	1.5707041

6. Compare these results with graphically

We see that the Table 4.1 and the Table 5.1 are probably same and for finding the error between the series solution and the numerical solution, we combine the above two table as below

Table 6.1: for the values of β, we compare the values of $f''(0)$ between the Series solution and Numerical solution.

Type of boundary layer flow	Boundary layer thickness (δ)	The value of β	The value of $f''(0)$ for series solution	The value of $f''(0)$ for numerical solution	Error between the series solution and the numerical solution
laminar profile	$5\sqrt{\frac{\upsilon x}{U}}$	12.500	1.6545584	1.6378571	0.0167013
linear profile	$3.46\sqrt{\frac{\upsilon x}{U}}$	5.986	1.1449735	1.1334160	0.0115575
parabolic profile	$5.48\sqrt{\frac{\upsilon x}{U}}$	15.015	1.8133839	1.7950794	0.0183045
cubic profile	$4.64\sqrt{\frac{\upsilon x}{U}}$	10.765	1.5354445	1.5199455	0.0154990
sin-cos profile	$4.795\sqrt{\frac{\upsilon x}{U}}$	11.496	1.5867206	1.5707041	0.0160165

Table 6.2: This table of several accelerations with velocities by using series solution

Velocity [$f'(\eta)$]	Acceleration of laminar profile [$f''(\eta)$] for β =12.500	Acceleration of linear profile [$f''(\eta)$] for β =5.986	Acceleration of parabolic profile [$f''(\eta)$] for β =15.015	Acceleration of cubic profile [$f''(\eta)$] for β =10.765	Acceleration of sin-cos profile [$f''(\eta)$] for β =11.496
0	1.654558	1.144974	1.813384	1.535444	1.586721
0.17	1.648368	1.140689	1.806599	1.529699	1.580784
0.34	1.604768	1.110518	1.758814	1.489239	1.538972
0.51	1.483976	1.026928	1.626427	1.377142	1.423132
0.68	1.236967	0.855996	1.355707	1.147916	1.186251
0.85	0.783198	0.541982	0.85838	0.726815	0.751087
0.999	8.39E-03	5.84E-04	9.24E-04	7.79E-03	8.05E-03

Table 6.3: This table of several accelerations with velocities by using numerical solution

Velocity [$f'(\eta)$]	Acceleration of laminar profile [$f''(\eta)$] for β =12.500	Acceleration of linear profile [$f''(\eta)$] for β =5.986	Acceleration of parabolic profile [$f''(\eta)$] for β =15.015	Acceleration of cubic profile [$f''(\eta)$] for β =10.765	Acceleration of sin-cos profile [$f''(\eta)$] for β =11.496
0	1.637857	1.133416	1.795079	1.519945	1.570704082
0.17	1.637857	1.133416	1.795079	1.519945	1.570704082
0.34	1.600361	1.107469	1.753984	1.485149	1.534745782
0.51	1.486118	1.02841	1.628774	1.37913	1.425185915
0.68	1.247901	0.863562	1.367691	1.158063	1.196736615
0.85	0.812834	0.562491	0.890861	0.754317	0.779507772
0.999	0	0	0	0	0

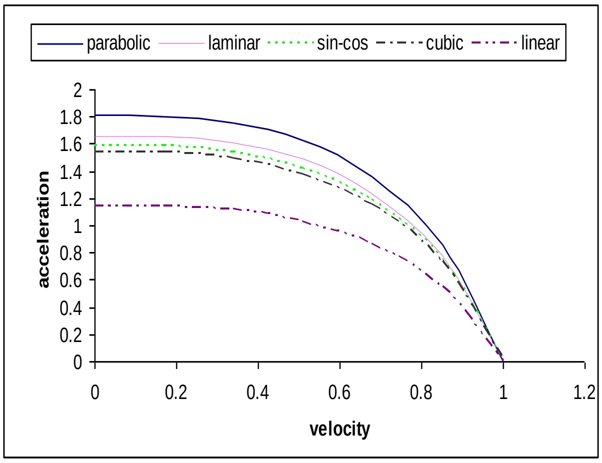

Figure 6.1: Effect of parabolic, laminar, sin-cos, cubic and linear profiles for series solution

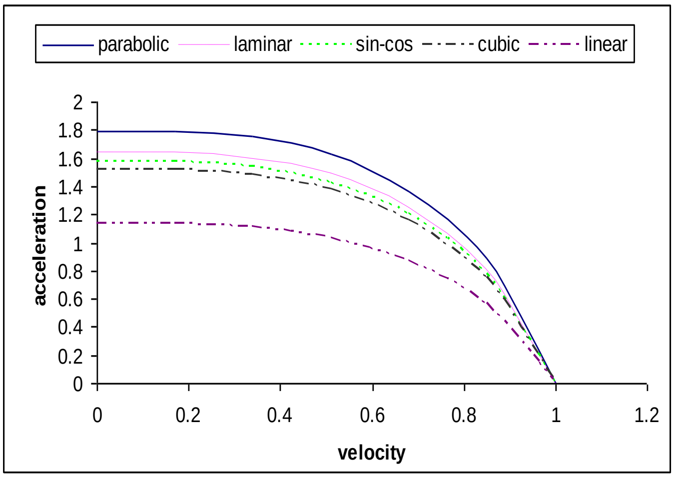

Figure 6.2: Effect of parabolic, laminar, sin-cos, cubic and linear profiles for numerical solution

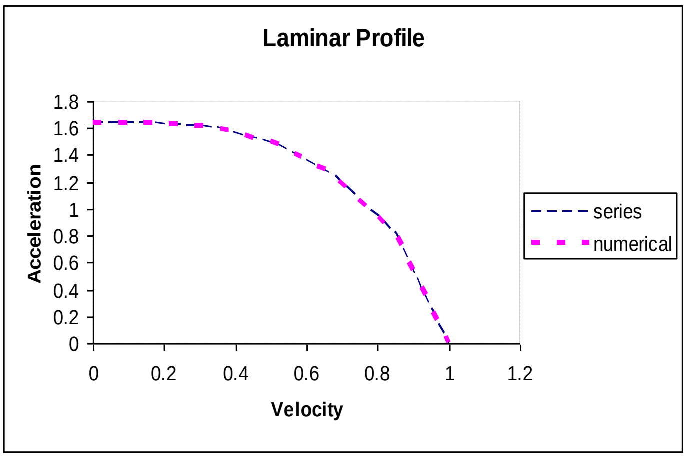

Figure 6.3: Effect of laminar profile ($\beta = 12.5$) for series and numerical Solution.

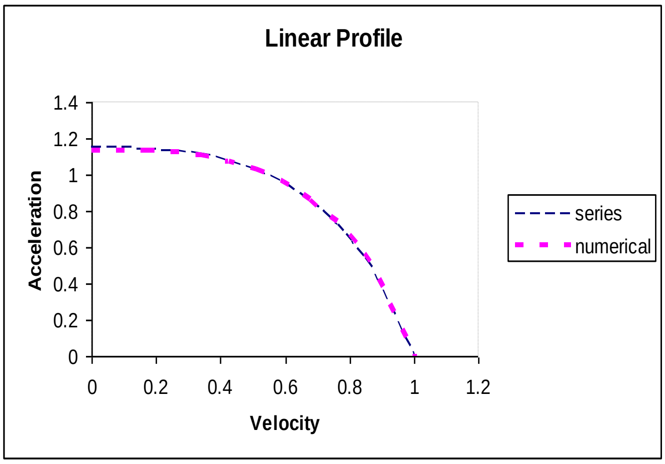

Figure 6.4: Effect of linear profile ($\beta = 5.986$) for series and numerical solution.

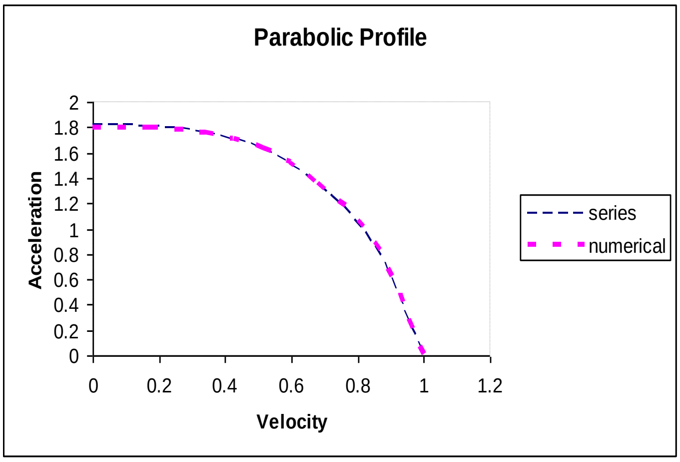

Figure 6.5: Effect of parabolic profile ($\beta = 15.015$) for series and numerical solution.

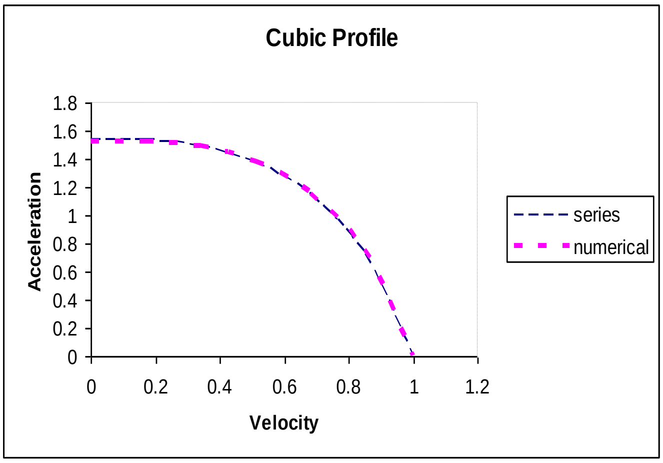

Figure 6.6: Effect of cubic profile ($\beta = 10.765$) for series and numerical solution.

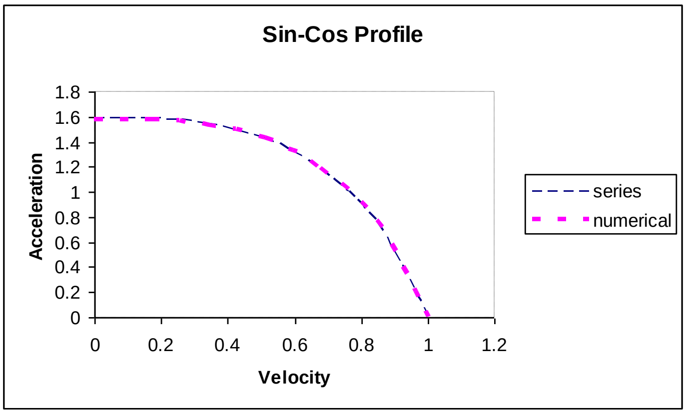

Figure 6.7: Effect of sin-cos profile ($\beta = 11.496$) for series and numerical solutio.

7: Conclusions

In this work, the finite difference approximation to derivative formula and “MATHLAB” are applied to solve the nonlinear Blasius equation. We have found the values of $f''(0)$ for some special cases (see Table 4.1 and 5.1) after solving the nonlinear Blasius equation by series solution and then by numerical solution with help of “MATHLAB” in both cases. In the Table 6.1, we may say that the error between the series solution and numerical solution is acceptable.

From the Table 6.2 or Table 6.3, we observed that acceleration decreased when velocity increased for different flows (like as Laminar Profile, Linear Profile, Parabolic Profile, Cubic Profile, Sin-Cos Profile).

Figures 6.1 to 6.7 show same result obtained by the analytical solution and the numerical solution. It has shown that acceleration would be high when velocity is zero for each case. A question may arise how does there exist an acceleration when velocity is zero? We can say for the question, in case of solid body, acceleration is zero when velocity is zero. But there exists some acceleration when velocity is zero for fluids which are lighter than air. In physically, when we keep fluid anywhere, it spreads for internal pressure at that instant. For this result, there exists acceleration at very small time to keep it anywhere instantly. For example, we choose $v_1 = 0.000004\ ms^{-1}$ at $t_1 = 0.000001\ s$ but $v_0 = 0\ ms^{-1}$ at $t_0 = 0\ s$. Therefore acceleration $= \dfrac{\Delta v}{\Delta t} = \dfrac{v_1 - v_0}{t_1 - t_0} = \dfrac{0.000004}{0.000001} = 4\ ms^{-2}$. This implies that there exists acceleration when velocity is zero only for fluid. Finally, in figures 6.1 to 6.7, we conclude that an acceleration must be existed and maximum for boundary layer flow (like as laminar profile, linear profile, parabolic profile, cubic profile and sin-cos profile) at zero velocity but an acceleration would be reduced to zero when velocity would be reached one unit. Therefore, we can say that an acceleration decreases when velocity increases in boundary layer flow.

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