Available online www.jsaer.com

***Journal of Scientific and Engineering Research*, 2019, 6(12):257-267**

Research Article

ISSN: 2394-2630
CODEN(USA): JSERBR

Heat transfer on peristaltic transport of a Visco-Elastic Rivlin-Ericksen Fluid in an Asymmetric Channel with Wall Properties

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Abstract The effect of heat transfer on the peristaltic transport of a Rivlin-Ericksen fluid with wall properties in a two dimensional flexible channel under long wave length approximation has been studied. A perturbation method of solution is obtained in terms of wall slope parameter and closed form of expressions has been derived for stream function, temperature and heat transfer coefficient. The effects of elastic parameters and pertinent parameters on the coefficient of heat transfer have been computed numerically. It is observed that heat transfer coefficient decreases in the region $0 \le y \le 0.75$ and increases in $0.75 \le y \le 1$ with increase in Prandtl and Eckert number.

Keywords Peristaltic transport, Heat transfer, Rivlin- Ericksen fluid, Temperature

1. Introduction

The study of the mechanism of peristalsis, in both physiological and mechanical situations, has become the object scientific research. From fluid mechanical point of view peristaltic motion is defined as the flow of generated by a wave traveling along the walls of an elastic tube. In physiology it may be described as a progressive wave of area contraction or expansion along a length of a distensible tube containing fluid provided with transverse and muscular fibers. It consists in narrowing and transverse shortening of a portion of the tube which then relaxes while the lower portion becomes shortened and narrowed. The mechanism of peristalsis occur for urine transport from kidney to bladder through the ureter, movement of chime in the gastro-intestinal tract, the movement of spermatozoa in the ducts efferent's of the mail reproductive tract, movement of ovum in the fallopian tube, vasomotion in small blood vessels, the food mixing and motility in the intestines, blood flow in cardiac chambers etc. Also bio-medical instruments such as heart-lung machine use peristalsis to pump blood while mechanical devices like roller pumps use this mechanism to pump and other corrosive fluids.

The problem of the mechanism of peristalsis transport has attracted the attention of many investigators. Fung and Yih [1], Shapiro and Jaffrin et al. [2] have studied peristaltic pumping with long wavelength at low Reynolds number. Haroun [3], Ebaid [4], Mishra & Rao [5] have studied peristalsis under different conditions. Mittra and Prasad [6] studied peristaltic transport in a two-dimensional channel considering the elasticity of the walls under the approximation of small amplitude ratio with dynamic boundary conditions, Raghunath Rao [7] studied peristaltic transport of a newtonian fluid with wall properties in an asymmetric channel and Ali et al. [8] studied peristaltic flow of a micropolar fluid in an asymmetric channel.

The interaction of peristalsis and heat transfer has become highly relevant and significant in several industrial processes also thermo dynamical aspects of blood become significant in process like haemodialysis and oxygenation when blood is drawn out of the body. Keeping these things in view, Srinivas and Kothandapani [9] investigated the peristaltic transport of a Newtonian fluid with heat transfer in an asymmetric channel.

Radhakrishnam acharya and Srinivasulu [10] investigated the influence of wall properties on peristaltic transport with heat transfer. Sobh *et al* [11] studied heat Transfer in Peristaltic flow of Viscoelastic Fluid in an Asymmetric Channel. Raghunath Rao *et al* [12] investigated the effect of heat transfer on peristaltic transport of Viscoelastic fluid in a channel with wall properties and S. Nadeem *et al* [13] investigated the influence of inclined magnetic field on peristaltic flow of a Williamson fluid model in an inclined symmetric or asymmetric channel.

The present research aimed is to investigate the interaction of peristalsis for the flow of a Visco-elastic Rivlin Ericksen fluid with wall properties in an asymmetric flexible channel under long wavelength approximation. A perturbation method of solution is obtained in terms of wall slope parameter and closed form of expressions has been derived for temperature distribution and heat transfer coefficient. The effects of elasticity parameters and pertinent parameters on temperature distribution and heat transfer coefficient have been computed numerically.

2. Formulation of the Problem

Consider a peristaltic flow of a Visco- Elastic Rivlin Ericksen fluid in an asymmetric channel of width d_1+d_2, the walls of the channel are assumed to be flexible and are taken as a stretched membrane on which traveling sinusoidal waves of moderate amplitude are imposed.

The geometry of flexible walls are represented by

$$h_1\,(x,t)=d_1+a_1\ \text{Cos}\,\frac{2\pi}{\lambda}(x-c\,t)\ ,\qquad \text{upper wall} \tag{1}$$

$$h_2\,(x,t)=-\,d_2-a_2\ \text{Cos}\left[\frac{2\pi}{\lambda}(x-c\,t)+\theta\right],\qquad \text{lower wall} \tag{2}$$

Where a_1, a_2 are the amplitudes of the peristaltic waves, '*c*' is the wave velocity, 'λ' is the wave length, *t* is the time and $\theta\ (0\le\theta\le\pi)$ is the phase difference. It should be noted that $\theta=0$ corresponds to symmetric channel with waves out of phase, $\theta=\pi$ with waves in phase, and further a_1, a_2, d_1, d_2 and θ satisfy the following inequality, Mishra and Rao [14].

$$a_1^{\,2}+a_2^{\,2}+2a_1a_2\cos\theta\le\left(d_1+d_2\right)^2$$

Making use of above relations the equations governing the two-dimensional flow of Rivlin -Ericksen fluid are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3}$$

$$\left(\frac{\partial u}{\partial t}+u\,\frac{\partial u}{\partial x}+v\,\frac{\partial u}{\partial y}\right)=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\beta\,\frac{\partial^2}{\partial y^2}\left(\frac{\partial u}{\partial t}+u\,\frac{\partial u}{\partial x}+v\,\frac{\partial u}{\partial y}\right) \tag{4}$$

$$\left(\frac{\partial v}{\partial t}+u\,\frac{\partial v}{\partial x}+v\,\frac{\partial v}{\partial y}\right)=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\upsilon\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\beta\,\frac{\partial^2}{\partial y^2}\left(\frac{\partial v}{\partial t}+u\,\frac{\partial v}{\partial x}+v\,\frac{\partial v}{\partial y}\right) \tag{5}$$

Equation of energy

$$\rho\,C_p\left(\frac{\partial T}{\partial t}+u\,\frac{\partial T}{\partial x}+v\,\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\upsilon\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)^2-\beta\left(\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial t\,\partial y}+u\,\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\,\partial y}+v\,\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\right) \tag{6}$$

Where *u, v* are the velocity components, '*p*' is the fluid pressure, 'ρ' is the density of the fluid, 'υ' is the coefficient of kinematic viscosity, *T* is the temperature, 'C_p' is the specific heat at constant pressure and '*k*' is the coefficient of thermal conductivity.

The governing equation of motion of the flexible wall may be expressed as

$$L\begin{Bmatrix}h_1\\ h_2\end{Bmatrix}=p-p_0 \tag{7}$$

Where 'L' is an operator, which is used to represent the motion of stretched membrane with damping forces such that

$$L \equiv -T^{*}\frac{\partial^2}{\partial x^2}+m\frac{\partial^2}{\partial t^2}+C\frac{\partial}{\partial t} \tag{8}$$

Here T^* is the elastic tension in the membrane, m is the mass per unit area and C is the coefficient of viscous damping forces, p_0 is the pressure on the outside surface of the wall due to tension in the muscles. For simplicity, we assume p_0 = 0. The horizontal displacement will be assumed zero. Hence the boundary conditions for the fluid are

$$u=0 \quad \text{at} \quad \begin{cases} y=h_1 \\ y=h_2 \end{cases} \tag{9}$$

Continuity of stresses requires that at the interfaces of the walls and the fluid p must be same as that which acts on the fluid at $y=h_1$ & $y=h_2$. The use of 'x' momentum equation the dynamic boundary conditions at flexible walls are

$$\frac{\partial}{\partial x}L\begin{Bmatrix} h_1 \\ h_2 \end{Bmatrix}=\frac{\partial p}{\partial x}=\rho\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right) \quad at \quad \begin{cases} y=h_1 \\ y=h_2 \end{cases} \tag{10}$$

The conditions on temperature are

$$\left.\begin{array}{l} T=T_0 \quad \text{on} \quad y=h_1 \\ T=T_1 \quad \text{on} \quad y=h_2 \end{array}\right\} \tag{11}$$

In view of the incompressibility of the fluid and two-dimensionality of the flow, we introduce the Stream function 'ψ' such that

$$u=\frac{\partial\psi}{\partial y} \quad and \quad v=-\frac{\partial\psi}{\partial x}$$

and introducing non-dimensional variables

$$x'=\frac{x}{\lambda},\ y'=\frac{y}{d},\ u'=\frac{u}{c},\ v'=\frac{v}{c\,\delta},\psi'=\frac{\psi}{cd},t'=\frac{ct}{\lambda},\ h_1'=\frac{h_1}{d_1},\ h_2'=\frac{h_2}{d_1},\ p'=\frac{p\,d^2}{\mu\,c\,\lambda},\theta=\frac{T-T_0}{T_1-T_0} \tag{12}$$

in equations of motion and the conditions (1) – (6) & (8) – (11) and eliminating p , we finally get (after dropping primes)

$$h_1(x,t)=1+a\ \text{Cos}\,2\pi(x-t) \tag{13}$$

$$h_2(x,t)=-d-b\,\text{Cos}\left[2\pi(x-t)+\theta\right] \tag{14}$$

$$R\delta\left(\frac{\partial}{\partial t}\left(\frac{\partial\psi}{\partial y}\right)+\frac{\partial\psi}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial\psi}{\partial y}\right)\right)-\frac{\partial\psi}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial\psi}{\partial y}\right)\right)\right)=-\frac{\partial p}{\partial x}+\left(\frac{\partial^2}{\partial y^2}\left(\frac{\partial\psi}{\partial y}\right)+\delta^2\frac{\partial^2}{\partial x^2}\left(\frac{\partial\psi}{\partial y}\right)\right)$$
$$+S\,R\,\delta\left(\frac{\partial^2}{\partial y^2}\left(\frac{\partial}{\partial t}\left(\frac{\partial\psi}{\partial y}\right)+\frac{\partial\psi}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial\psi}{\partial y}\right)\right)-\frac{\partial\psi}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial\psi}{\partial y}\right)\right)\right)\right) \tag{15}$$

$$R\delta^3\left(\frac{\partial}{\partial t}\left(\frac{\partial\psi}{\partial x}\right)+\frac{\partial\psi}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial\psi}{\partial x}\right)\right)-\frac{\partial\psi}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial\psi}{\partial x}\right)\right)\right)=-\frac{\partial p}{\partial y}+\delta^2\left(\frac{\partial^2}{\partial y^2}\left(\frac{\partial\psi}{\partial x}\right)+\delta^2\frac{\partial^2}{\partial x^2}\left(\frac{\partial\psi}{\partial x}\right)\right)$$
$$+S\,R\,\delta^3\left(\frac{\partial^2}{\partial y^2}\left(\frac{\partial}{\partial t}\left(\frac{\partial\psi}{\partial x}\right)+\frac{\partial\psi}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial\psi}{\partial x}\right)\right)-\frac{\partial\psi}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial\psi}{\partial x}\right)\right)\right)\right) \tag{16}$$

$$R\,p_r\delta\left(\frac{\partial}{\partial t}\left(\frac{\partial\theta}{\partial t}\right)+\frac{\partial\psi}{\partial y}\left(\frac{\partial\theta}{\partial x}\right)-\frac{\partial\psi}{\partial x}\left(\frac{\partial\theta}{\partial y}\right)\right)=\left(\frac{\partial^2\theta}{\partial y^2}+\delta^2\frac{\partial^2\theta}{\partial x^2}\right)+E\left(\frac{\partial^2\psi}{\partial y^2}+\delta^2\frac{\partial^2\psi}{\partial x^2}\right)$$
$$-S\,p_r R\,E\delta\left(\frac{\partial^2\psi}{\partial y^2}\frac{\partial^3\psi}{\partial t\,\partial y^2}+\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial y^2}\frac{\partial^3\psi}{\partial t\,\partial y^2}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}\frac{\partial^3\psi}{\partial y^3}\right) \quad (17)$$

$$\frac{\partial\psi}{\partial y}=0 \quad on \quad \begin{cases} y=1+a\ Cos\,2\pi\,(x-t) \\ y=-d-b\ Cos\left[\,2\pi\,(x-t)+\theta\right]\end{cases} \quad (18)$$

$$\left(\frac{\partial^3\psi}{\partial y^3}+\delta^2\frac{\partial^3\psi}{\partial x^2\,\partial y}\right)-R\,\delta\left(\frac{\partial^2\psi}{\partial y\,\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\,\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}\right)$$
$$=\left(E_1\frac{\partial^3}{\partial x^3}+E_2\frac{\partial^3}{\partial x\,\partial t^2}+E_3\frac{\partial^2}{\partial x\,\partial t}\right)\begin{Bmatrix}h_1\\ h_2\end{Bmatrix} \quad at\ \ y=\begin{cases}1+a\ Cos\ 2\pi(x-t)\\ -d-b\ Cos\left[2\pi\,(x-t)+\theta\right]\end{cases} \quad (19)$$

$$\left.\begin{matrix}\theta^*=0 \quad on \quad y=h_1\\ \theta^*=1 \quad on \quad y=h_2\end{matrix}\right\} \quad (20)$$

Eliminating p from the equations (15) – (16), we get

$$R\,\delta\left(\left(\frac{\partial}{\partial t}\left(\nabla^2\psi\right)\right)+\frac{\partial\psi}{\partial y}\left(\frac{\partial}{\partial x}\left(\nabla^2\psi\right)\right)-\frac{\partial\psi}{\partial x}\left(\frac{\partial}{\partial y}\left(\nabla^2\psi\right)\right)\right)=\left(\frac{\partial^2}{\partial y^2}\left(\nabla^2\psi\right)+\delta^2\left(\frac{\partial^2}{\partial x^2}\left(\nabla^2\psi\right)\right)\right)$$
$$+R\,S\,\delta\left(\left(\frac{\partial}{\partial t}\left(\frac{\partial^4\psi}{\partial y^4}\right)\right)+\frac{\partial\psi}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial^4\psi}{\partial y^4}\right)\right)+\left(\frac{\partial^4\psi}{\partial y^4}\right)\left(\frac{\partial^4\psi}{\partial x\partial y}\right)-\left(\frac{\partial^2\psi}{\partial y^2}\right)\left(\frac{\partial^4\psi}{\partial x\,\partial y^3}\right)-\frac{\partial\psi}{\partial x}\left(\frac{\partial^5\psi}{\partial y^5}\right)\right) \quad (21)$$
$$+2R\,S\,\delta^3\left(\left(\frac{\partial}{\partial t}\left(\frac{\partial^4\psi}{\partial x^2\partial y^2}\right)\right)+\left(\frac{\partial^4\psi}{\partial y^4}\right)\left(\frac{\partial^2\psi}{\partial x^2}\right)+\frac{\partial\psi}{\partial y}\left(\frac{\partial^5\psi}{\partial x^2\,\partial y^3}\right)-\left(\frac{\partial^2\psi}{\partial x^2}\right)\left(\frac{\partial^4\psi}{\partial x\,\partial y^3}\right)-\frac{\partial\psi}{\partial x}\left(\frac{\partial^5\psi}{\partial x^2\partial y^3}\right)\right)$$

Where $\nabla^2=\dfrac{\partial^2}{\partial y^2}+\delta^2\dfrac{\partial^2}{\partial x^2}$

The non-dimensional parameters are

$R=\dfrac{c\,d}{\upsilon}$ is the Reynolds number, $S=\dfrac{\beta}{d^2}$ is the Visco-elastic parameter,

$P_r=\dfrac{c_p\rho\nu}{k}$ is the Prandtl number, $E=\dfrac{c^2}{c_p\rho(T_1-T_0)}$ is the Eckert parameter,

$a=\dfrac{a_1}{d_1}$, $b=\dfrac{a_2}{d_1}$, $d=\dfrac{d_2}{d_1}$ and $\delta=\dfrac{d}{\lambda}$ are geometric parameters

$E_1=-\dfrac{T\,d^3}{\lambda^3\rho\,\upsilon\,c}$, $E_2=\dfrac{m\,c\,d^3}{\lambda^3\rho\,\nu}$, $E_3=\dfrac{C\,d^3}{\lambda^2\rho\,\nu}$ are elasticity parameters.

3. Method of Solution

We seek perturbation solution in terms of small parameter δ as follows:

$$\psi=\psi_0+\delta\,\psi_1+\delta^2\psi_2+..... \quad (22)$$

$$\theta^* = \theta_0^* + \delta\ \theta_0^* + \delta^2 \theta_0^* + ... \tag{23}$$

Substituting equations (22) & (23) in equations (17) to (21) and collecting the coefficients of various powers of δ

The zeroth order equations are

$$\frac{\partial^4 \psi_0}{\partial y^4} = 0 \tag{24}$$

$$\frac{1}{P_r}\left(\frac{\partial^2 \theta_0^*}{\partial y^2}\right) + E\left(\frac{\partial^2 \psi_0}{\partial y^2}\right)^2 = 0 \tag{25}$$

The corresponding boundary conditions are

$$\frac{\partial \psi_0}{\partial y} = 0 \quad on \quad \begin{cases} y = 1 + a\ Cos\, 2\pi\,(x-t) \\ y = -d - b\ Cos\left[\,2\pi\,(x-t) + \theta\right] \end{cases} \tag{26}$$

$$\frac{\partial^3 \psi_0}{\partial y^3} = \left(E_1 \frac{\partial^3}{\partial x^3} + E_2 \frac{\partial^3}{\partial x\,\partial t^2} + E_3 \frac{\partial^2}{\partial x\,\partial t}\right)\begin{Bmatrix} h_1 \\ h_2 \end{Bmatrix} \quad at\ \ y = \begin{cases} 1 + a\ Cos\ 2\pi(x-t) \\ -d - b\ Cos\left[2\pi\,(x-t) + \theta\right] \end{cases} \tag{27}$$

$$\left.\begin{aligned} \theta_0{}^* &= 0 \quad on \quad y = h_1 \\ \theta_0{}^* &= 1 \quad on \quad y = h_2 \end{aligned}\right\} \tag{28}$$

Zeroth-order problem

On solving the equations (24) & (25) subject to the conditions (26) - (28), we get

$$\psi_0 = A_1 \frac{y^3}{6} + A_2 \frac{y^2}{2} + A_3\, y \tag{29}$$

$$\theta_0^* = -\,a_1\left[A_1 \frac{y^4}{12} + A_2 \frac{y^3}{3} + A_3 \frac{y^2}{2}\right] + G_1 y + G_2 \tag{30}$$

The first order equations are

$$\begin{aligned} &R\left(\left(\frac{\partial}{\partial t}\left(\frac{\partial^2 \psi_0}{\partial y^2}\right)\right) + \frac{\partial \psi_0}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial^2 \psi_0}{\partial y^2}\right)\right) - \frac{\partial \psi_0}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial^2 \psi_0}{\partial y^2}\right)\right)\right) = \frac{\partial^4 \psi_1}{\partial y^4} + \\ &+R\ S\left(\left(\frac{\partial}{\partial t}\left(\frac{\partial^4 \psi_0}{\partial y^4}\right)\right) + \frac{\partial \psi_0}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial^4 \psi_0}{\partial y^4}\right)\right) + \left(\frac{\partial^4 \psi_0}{\partial y^4}\right)\left(\frac{\partial^4 \psi_0}{\partial x \partial y}\right) - \left(\frac{\partial^2 \psi_0}{\partial y^2}\right)\left(\frac{\partial^4 \psi_0}{\partial x\,\partial y^3}\right) - \frac{\partial \psi_0}{\partial x}\left(\frac{\partial^5 \psi_0}{\partial y^5}\right)\right) \end{aligned} \tag{31}$$

$$\begin{aligned} &R\left(\frac{\partial \theta_0^*}{\partial t} + \frac{\partial \psi_0}{\partial y}\frac{\partial \theta_0^*}{\partial x} - \frac{\partial \psi_0}{\partial\, x}\frac{\partial \theta_0^*}{\partial y}\right) = \frac{1}{P_r}\left(\frac{\partial^2 \theta_1^*}{\partial y^2}\right) + 2E\left(\frac{\partial^2 \psi_0}{\partial y^2}\right)\left(\frac{\partial^2 \psi_1}{\partial y^2}\right) \\ &-S\ R\ E\left(\frac{\partial^2 \psi_0}{\partial y^2}\frac{\partial^3 \psi_0}{\partial t\,\partial y^2} + \frac{\partial \psi_0}{\partial y}\frac{\partial^2 \psi_0}{\partial y^2}\frac{\partial^3 \psi_0}{\partial t\,\partial y^2} - \frac{\partial \psi_0}{\partial x}\frac{\partial^2 \psi_0}{\partial y^2}\frac{\partial^3 \psi_0}{\partial y^3}\right) \end{aligned} \tag{32}$$

The corresponding boundary conditions are

$$\frac{\partial \psi_1}{\partial y} = 0 \quad on \quad \begin{cases} y = 1 + a\ Cos\, 2\pi\,(x-t) \\ y = -d - b\ Cos\left[\,2\pi\,(x-t) + \theta\right] \end{cases} \tag{33}$$

$$\frac{\partial^3\psi_1}{\partial y^3}-R\left(\frac{\partial^2\psi_0}{\partial y\,\partial t}+\frac{\partial\psi_0}{\partial y}\frac{\partial^2\psi_0}{\partial x\,\partial y}-\frac{\partial\psi_0}{\partial x}\frac{\partial^2\psi_0}{\partial y^2}\right)=0 \quad at \quad y=\begin{cases}1+a\ Cos\ 2\pi(x-t)\\ -d-b\ Cos\left[2\pi\,(x-t)+\theta\right]\end{cases} \tag{34}$$

$$\left.\begin{aligned}\theta_1^*=0 \quad on \quad y=h_1\\ \theta_1^*=0 \quad on \quad y=h_2\end{aligned}\right\} \tag{35}$$

First-order problem

On solving the equation (31) & (32) subject to the conditions (33) - (35), we obtain

$$\psi_1=R\left[\left(A_4-SA_1A_6\right)\frac{y^5}{120}+\left(A_5-SA_2A_6\right)\frac{y^4}{24}\right]+\frac{y^3}{6}B_1+\frac{y^2}{2}B_2+B_3y \tag{36}$$

$$\theta_1^*=\frac{y^8}{56}M_1+\frac{y^7}{42}M_2+\frac{y^6}{30}M_3+\frac{y^5}{20}M_4+\frac{y^4}{12}M_5+\frac{y^3}{6}M_6+\frac{y^2}{2}M_7+H_1y+H_2 \tag{37}$$

Substituting θ_0^* from (24) and θ_1^* from (36) into (22) for θ^*, we have temperature θ^* in the form

$$\begin{aligned}\theta^*=&-a_1\left[A_1\frac{y^4}{12}+A_2\frac{y^3}{3}+A_3\frac{y^2}{2}\right]+G_1y+G_2+\\ &\delta\left(\frac{y^8}{56}M_1+\frac{y^7}{42}M_2+\frac{y^6}{30}M_3+\frac{y^5}{20}M_4+\frac{y^4}{12}M_5+\frac{y^3}{6}M_6+\frac{y^2}{2}M_7+H_1y+H_2\right)\end{aligned} \tag{38}$$

The heat transfer coefficient Z at the (upper) wall is given by

$$Z=\left(\frac{\partial h_1}{\partial x}\right)\left(\frac{\partial\theta^*}{\partial y}\right) \tag{39}$$

Substituting Eq. (13) & Eq. (38) in Eq. (39), we get

$$\begin{aligned}Z=&\left(-2\pi a\ \ Sin\,2\pi\,(x-t)\right)\left(-a_1\left(A_1\frac{y^3}{3}+A_2\,y^2+A_3\,y\right)+G_1\right.\\ &\left.+\left(\delta\left(\frac{y^7}{7}M_1+\frac{y^6}{6}M_2+\frac{y^5}{5}M_3+\frac{y^4}{4}M_4+\frac{y^3}{3}M_5+\frac{y^2}{2}M_6+M_7y+H_1\right)\right)\right)\end{aligned} \tag{40}$$

Where

$$\begin{aligned}A_1=&4\pi^3(E_1+E_2)\left[a\ Sin\ 2\pi\left(x-t\right)-b\ Sin\ (2\pi\left(x-t\right)+\theta)\right]+\\ &2E_3\pi^2\left[a\ Cos\ 2\pi\left(x-t\right)-b\ Cos\ (2\pi\left(x-t\right)+\theta)\right]\end{aligned}, \qquad A_2=-\frac{A_1}{2}(h_1+h_2),$$

$$A_3=\frac{A_1h_1h_2}{2}\quad A_4=A_{1t},\ A_5=A_{2t},\ A_6=A_{1x},\ A_7=A_{2x},\ A_8=A_{3x},\ A_9=A_{3t},$$

$$\begin{aligned}B_1=&R(A_1A_6\frac{(h_1+h_2)^4}{12}+A_2A_6\frac{(h_1+h_2)^3}{3}+(A_2A_7-A_1A_8+A_3A_6+SA_1A_6)\frac{(h_1+h_2)^2}{2}\\ &+(A_3A_7+SA_2A_6)(h_1+h_2)+(A_3A_8+A_9))\end{aligned}$$

$$B_2=-\frac{1}{48}R(A_4-SA_1A_6)(h_1+h_2)(h_1^2+h_2^2)-\frac{1}{6}R(A_5-SA_2A_6)(h_1^2+h_1h_2+h_2^2)-\frac{1}{2}(h_1+h_2)B_1$$

$$B_3 = -\frac{1}{48}R(A_4 - SA_1A_6)(h_1^4 + h_2^4) - \frac{1}{12}R(A_5 - SA_2A_6)(h_1^3 + h_2^3) - \frac{1}{2}(h_1^2 + h_2^2)B_1 - \frac{1}{2}(h_1 + h_2)B_2$$

$$M_1 = -\frac{1}{36}p_r^2 ER\, A_1^2 A_6\,, \quad M_2 = -\frac{1}{6}p_r^2 ER\, A_1 A_2 A_6\,,$$

$$M_3 = \frac{p_r^2 ER}{6}(A_2^2 A_6 - A_1 A_4 - 2A_1^2 A_6 + 2A_1^2 A_8 - 2A_1 A_2 A_7 - A_1 A_3 A_6) + \frac{p_r ER}{3}(2S\, A_1^2 A_6 - A_1 A_4)$$

$$M_4 = \frac{p_r^2 ER}{6}(6A_1 A_2 A_8 - 3A_2^2 A_7 - 2A_1 A_3 A_7 - 2A_2 A_3 A_6 - 2A_1 A_5 - 2A_2 A_4) + \frac{p_r R}{6}(3A_1 G_3 - A_6 G_1) + \frac{p_r ER}{3}(8S\, A_1 A_2 A_6 - 3A_1 A_5 - A_2 A_4)$$

$$M_5 = \frac{p_r^2 ER}{2}(A_2^2 A_8 - 2A_2 A_3 A_7 - 2A_2 A_5) + \frac{p_r R}{2}(A_1 G_4 - A_7 G_1 + A_2 G_3) + 2p_r E A_1 B_1 + p_r ER\,(S\, A_1 A_4 + A_2 A_5 + 2S\, A_2^2 A_6 + S\, A_1 A_3 A_6 + S\, A_1 A_2 A_7 - S\, A_1^2 A_8)$$

$$M_6 = p_r ER\, S(A_1 A_5 + A_2 A_4 + A_2^2 A_7 + A_1 A_3 A_7 + A_2 A_3 A_6 - A_1 A_2 A_8) + p_r R(G_5 + A_3 G_3 - B_1 G_3 + A_2 G_4) - 2p_r E(A_1 B_2 + A_2 B_1)$$

$$M_7 = p_r ER\, S(A_2 A_5 + A_2 A_3 A_7) + p_r R(G_6 + A_3 G_4) - 2p_r E\, A_2 B_2$$

$$H_1 = \frac{1}{h_2 - h_1}\left(\begin{array}{l} \frac{M_1}{56}(h_1^8 - h_2^8) + \frac{M_2}{42}(h_1^7 - h_2^7) + \frac{M_3}{30}(h_1^6 - h_2^6) + \frac{M_4}{20}(h_1^5 - h_2^5) \\ + \frac{M_5}{12}(h_1^4 - h_2^4) + \frac{M_6}{6}(h_1^3 - h_2^3) + \frac{M_7}{2}(h_1^2 - h_2^2) \end{array}\right)$$

$$H_2 = -\frac{1}{2}\left(\begin{array}{l} \frac{M_1}{56}(h_1^8 + h_2^8) + \frac{M_2}{42}(h_1^7 + h_2^7) + \frac{M_3}{30}(h_1^6 + h_2^6) + \frac{M_4}{20}(h_1^5 + h_2^5) \\ + \frac{M_5}{12}(h_1^4 + h_2^4) + \frac{M_6}{6}(h_1^3 + h_2^3) + \frac{M_7}{2}(h_1^2 + h_2^2) + H_1(h_1 + h_2) \end{array}\right)$$

4. Results and Discussions

In this analysis, we analyzed effect of heat transfer coefficient Z for the different values of E_1, E_2, E_3, R, p_r, S, and E. The heat transfer coefficient Z is shown in Figures (1) to (7), we noticed that the heat transfer coefficient Z decreases in the region $0 \le y \le 0.2$ and more significant at the lower wall for higher values of E_1 and E_3 and Z increases in $0.2 \le y \le 1$ and attains the prescribed value at y =1. Z decreases in the region $0 \le y \le 0.7$ with increase in E_2 and increases in the region $0.7 \le y \le 1$. Z decreases in the region $0 \le y \le 0.6$ and increases in $0.6 \le y \le 1$ with increase in R and more significant at the walls and it is observed that Z decreases in the region $0 \le y \le 0.75$ and increases in $0.75 \le y \le 1$ with increase in p_r, S, E and more significant at the upper wall.

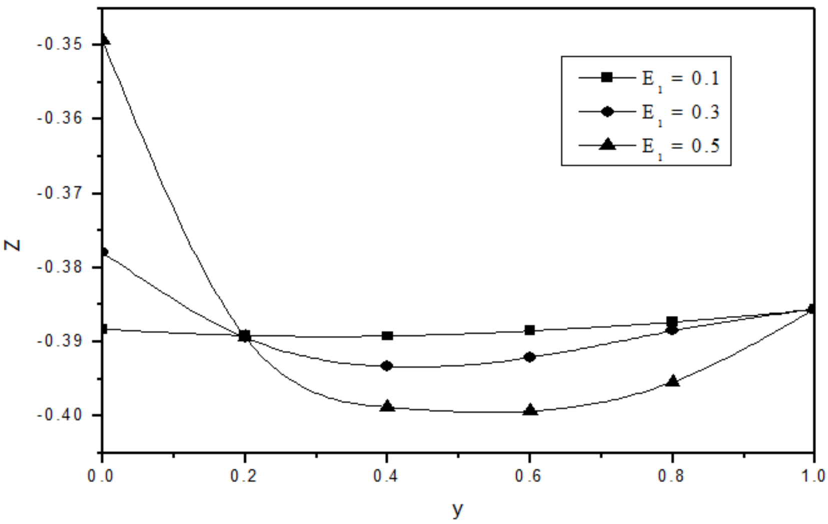

Figure 1: Effect of the rigidity of the wall E_1 on Z for
$d = 0.1$, $b = 0.1$, $\theta = \pi/3$, $a = 0.1$, $\delta = 0.01$, $R = 1$, $S = 1$, $E = 1$, $P_r = 0.7$, $E_2 = 0.2$, $E_3 = 0.3$

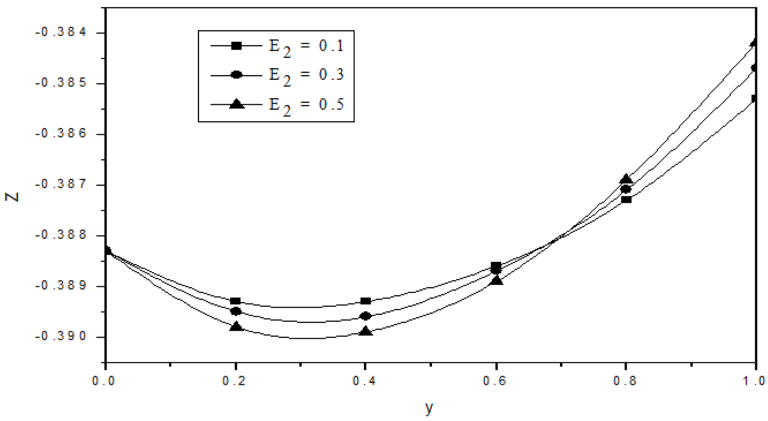

Figure 2: Effect of the stiffness of the wall E_2 on Z for
$d = 0.1$, $b = 0.1$, $\theta = \pi/3$, $a = 0.1$, $\delta = 0.01$, $R = 1$, $S = 1$, $E = 1$, $P_r = 0.7$, $E_1 = 0.1$, $E_3 = 0.3$

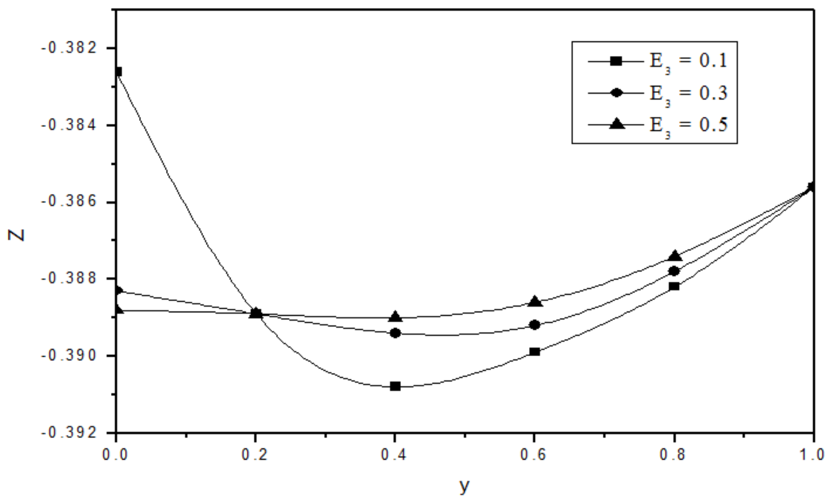

Figure 3: Effect of the damping nature of the wall E_3 on Z for
$d = 0.1$, $b = 0.1$, $\theta = \pi/3$, $a = 0.1$, $\delta = 0.01$, $R = 1$, $S = 1$, $E = 1$, $P_r = 0.7$, $E_1 = 0.1$, $E_2 = 0.2$

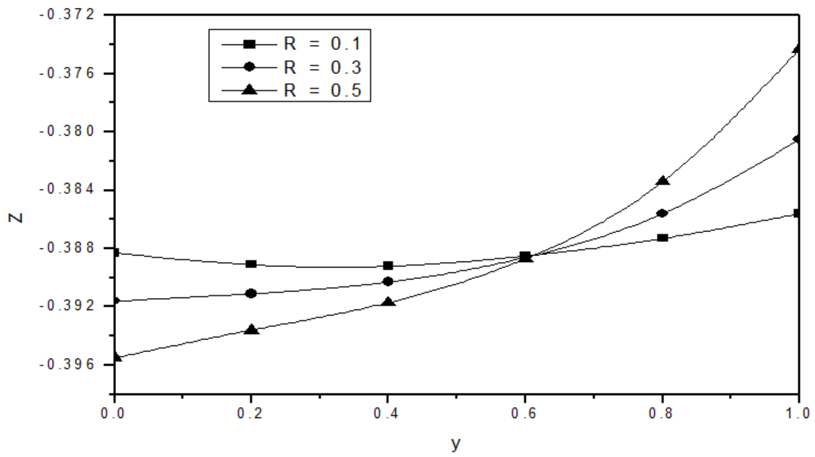

Figure 4: Effect of the Reynolds number R on Z for
$d = 0.1,\ b = 0.1,\ \theta = \pi/3,\ a = 0.1,\ \delta = 0.01,\ S = 1,\ E = 1,\ p_r = 0.7,\ E_1 = 0.1,\ E_2 = 0.2,\ E_3 = 0.3$

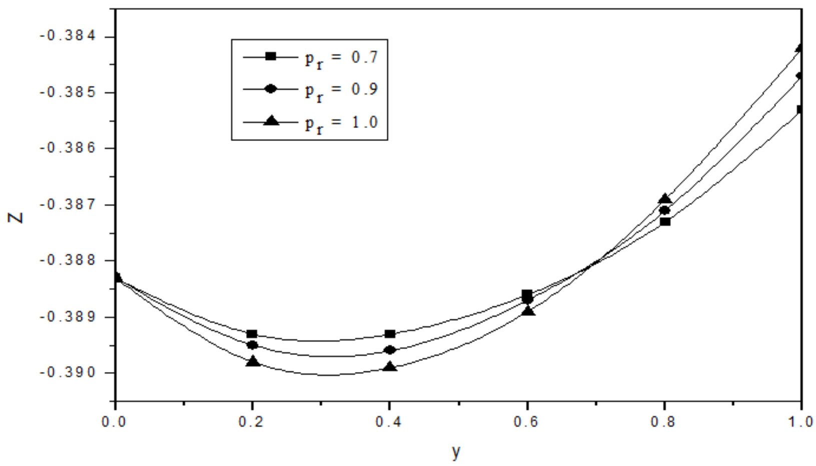

Figure 5: Effect of Prandtl number p_r on Z for
$d = 0.1,\ b = 0.1,\ \theta = \pi/3,\ a = 0.1,\ \delta = 0.01,\ S = 1,\ E = 1,\ R = 1,\ E_1 = 0.1,\ E_2 = 0.2,\ E_3 = 0.3$

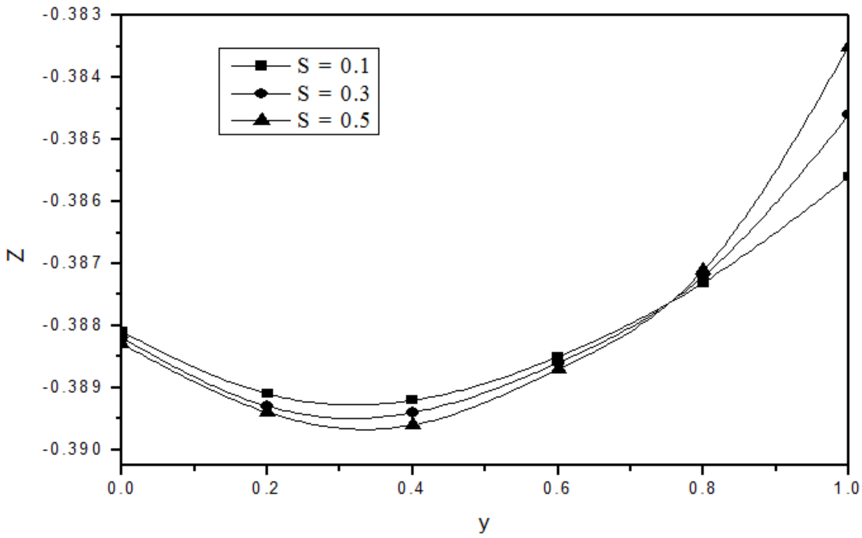

Figure 6: Effect of the visco-elastic parameter S on Z for
$d = 0.1,\ b = 0.1,\ \theta = \pi/3,\ a = 0.1,\ \delta = 0.01,\ p_r = 1,\ E = 1,\ R = 1,\ E_1 = 0.1,\ E_2 = 0.2,\ E_3 = 0.3$

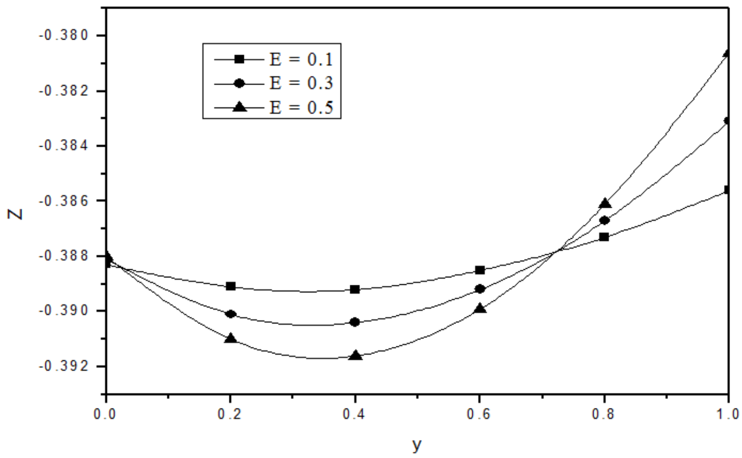

Figure 7: Effect of the Eckert number E on Z for
$d = 0.1,\ b = 0.1,\ \theta = \pi/3,\ a = 0.1,\ \delta = 0.01,\ p_r = 1,\ S = 1,\ R = 1,\ E_1 = 0.1,\ E_2 = 0.2,\ E_3 = 0.3$

5. Conclusions

In the present paper we have discussed the peristaltic transport of a Visco- Elastic Rivlin Ericksen fluid with wall properties in an asymmetric channel. The governing equations of motion are solved analytically using long wave length approximation. Furthermore, the effect of elastic parameters and phase difference on temperature distribution and heat transfer coefficient have been computed numerically and explained graphically. We conclude the following observations:

1. Heat transfer coefficient Z decreases in the region $0 \le y \le 0.2$ and increases in $0.2 \le y \le 1$ with increase in E_1 and E_3.
2. Z decreases in the region $0 \le y \le 0.7$ with increase in E_2 and increases in the region $0.7 \le y \le 1$.
3. Z decreases in the region $0 \le y \le 0.6$ and increases in $0.6 \le y \le 1$ with increase in R and more significant at the walls.
4. Z decreases in the region $0 \le y \le 0.75$ and increases in $0.75 \le y \le 1$ with increase in p_r, S, E and more significant at the upper wall.

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