Available online www.jsaer.com

Journal of Scientific and Engineering Research, 2019, 6(12):230-234

Research Article

ISSN: 2394-2630
CODEN(USA): JSERBR

Variation of FEM Based Solutions of Rectangular Plate Deformations

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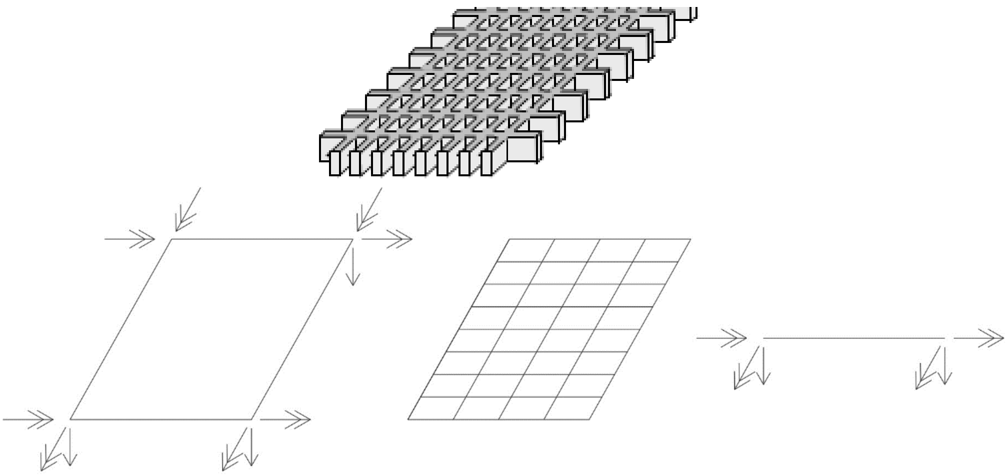
Abstract In this study, an algorithm was developed to determine deformation values of rectangular plates subjected to external load in the direction perpendicular to the plane as finite grid method. A group of problems, have different boundary conditions and loading conditions, determined have been solved. Similar problems were solved by using well-known finite element method such as Sap2000 and Ansys Workbench software programs are compared with the finite grid method. The error rates in all the analyses are determined taking into account the results obtained from the analytical methods. In many cases analytical solution models, Sap2000 and Ansys Workbench have some difficulties about mathematical modeling. On the other hand the finite grid method provides an advantage take considering about flexible external loading and boundary conditions. As a result, with the acceptable error rates obtained from the analyses made are taken into account, it is concluded that the method can be classified as a powerful preferable method for complex plate bending problems with thickness variations.

Keywords FEM, finite grid method, plate bending problems

Introduction

A broad range of the engineering problems has been solved by computer-based methods such as finite element and boundary element methods [1-3]. However closed form solutions for plate problems have been published for a limited number of cases. Owing to its convenience in solution of plate bending problems some numerical and approximate methods, such as finite element, finite difference, boundary element, framework methods finite grid methods in applied mechanics have been developed to overcome such problems [4-9]. After all, within limitations of simplified formulation as Wilson [10] indicated, plate bending is an extension of one dimensional beam theory. In present study the rectangular plates through the lattice analogy at which the discrete elements are connected at finite nodal points is represented by one dimensional beam elements. These individual element matrices are used to form the system load and stiffness matrices for plates. The matrix displacement method based on stiffness-matrix approach is suitable to solve gridworks with arbitrary load and boundary conditions. A combination of finite element method, lattice analogy and matrix displacement analysis of grid works was used to obtain a finite grid solution [11-13]. By using any convenient numbering scheme to collect all displacements for each nodal point in a convenient sequence the stiffness and geometric stiffness matrices of the system for any type of grids can be generated. The system cannot truly be equal to the continuous structure but solutions adequate for engineering purposes can be found with greater ease. In this form, plates are idealized as a grillage of beams of a given geometry satisfying given boundary conditions as shown in Figure 1.



a **b** **c**

Figure 1: The idealized discrete system a) the elements are connected at finite nodal points of a rectangular thin plate in flexure, b) Parallel sets of one-dimensional elements replaced by the continuous surface c) Typical node displacements and forces in a grid plane as a beam in local coordinates

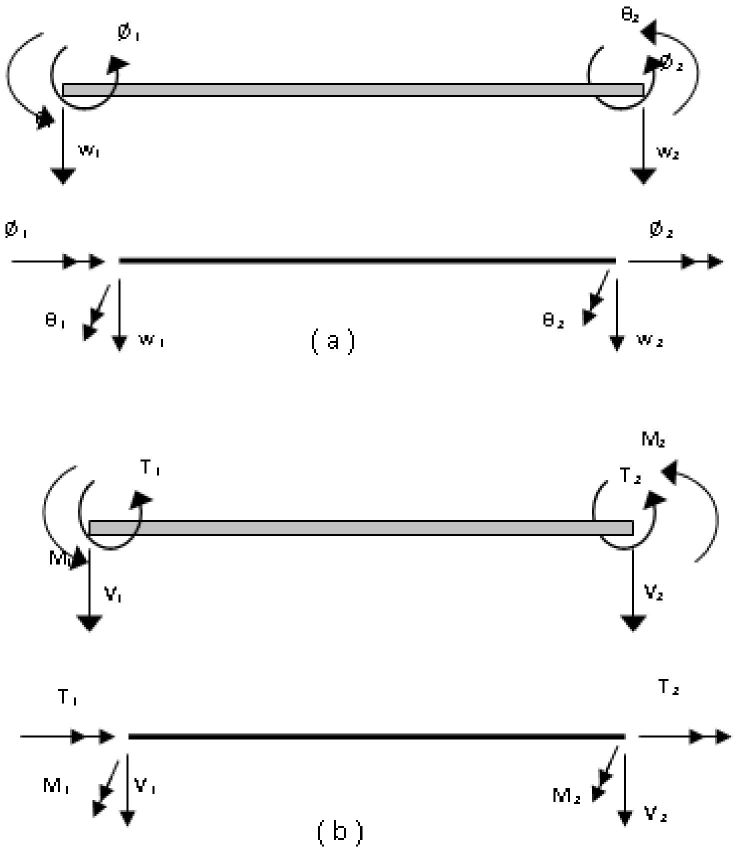


Figure 2: The idealized discrete finite element of with 3 DOF's at each node (a) generalized displacements, (b) loads applied at nodes

In Figure 2 the plate element through the lattice analogy at which the discrete elements are connected at finite nodal points represented by one dimensional elements have 3 degree of freedom (two rotations and one translation) at each node. These elements of the framework method replace a continuous surface by an idealized discrete system represent a two-dimensional plate is the essential of the finite grid model. Because of plane rigid intersection, the elements can resist torsion as well as bending moment and shear. Then finite element based matrix methods is used to determine stiffness and geometric stiffness matrices of one-dimensional beam elements resting on elastic foundations by exact shape functions. These individual element matrices are used to form the system load and stiffness matrices for plates.

Materials and Methods

The maximum deflection values of thin rectangular plates were calculated by finite grid method and other solution methods as case study. The results were compared with the analytical solutions. The maximum deflection values of 8 different types of samples in the literature were calculated with various loading and boundary conditions taking into account. An algorithm developed in Matlab using the stiffness and load matrices to determine Finite grid method solution [14]. The finite element method was calculated using Sap2000 and Ansys Workbench package programs [15-16], and the exact solutions were calculated using the data presented by Timoshenko [17]. Some plate bending problems with different loading and boundary conditions were solved in 14 types of different analyzes. Selected types of samples and analysis to be applied are given in Table 1and Table 2 respectively.

Table 1: Boundary and Loading Conditions of Examples

Code	Boundary conditions	Loading Conditions	ratio (b/a)
SSSS-P-1	All edges are simple supported	Concentrated Load at Centre	1
SSSS-P-2			2
CCCC-P-1	All edges are fixed	Concentrated Load at Centre	1
CCCC-P-2			2
SSSS-Q-1	All edges are simple supported	Uniform Distributed Load	1
SSSS-Q-2			2
CCCC-Q-1	All edges are fixed	Uniform Distributed Load	1
CCCC-Q-2			2
SSSF-Q-1	3-edges are simple supported the other free	Uniform Distributed Load	1
SSSF-Q-2			2
SSCC-Q-1	2-opposite edges are fixed the others simple	Uniform Distributed Load	1
SSCC-Q-2			2
CCFF-Q-1	2-opposite edges are fixed the others free	Uniform Distributed Load	1
CCFF-Q-2			2

The normalized deflection for concentrated loading P, and uniform loading Q cases are formulated as;

$$w_{\max,g} = w_{\max} \frac{D}{Pa^2}$$

$$w_{\max,g} = w_{\max} \frac{D}{Qa^4}$$

$$D = \frac{2h^3E}{3(1-v^2)}$$

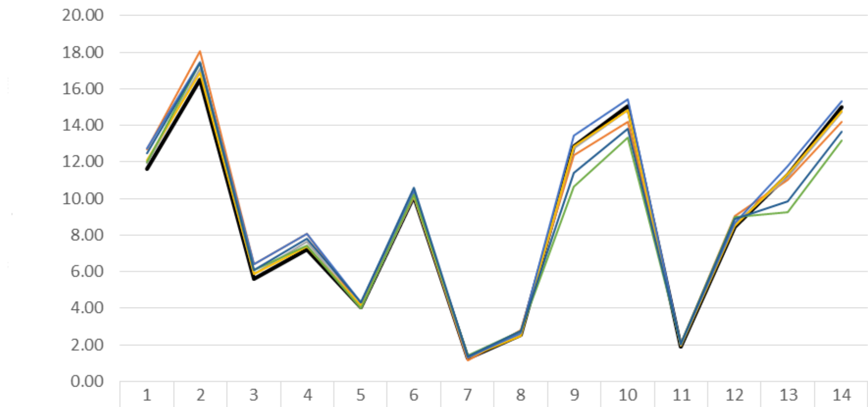
Where a is the small dimension of rectangular plate and D is flexural rigidity of the plate.

Table 2: Types of Analysis

Analysis Code	**Analysis Type**
Ref	Analytical Solution (Timoshenko, 1959)
FEM #1	Sap2000 solution by 4 subdivisions
ANSYS #1	Ansys Workbench solution by 4 subdivisions
FGM #1	Finite Grid Solution by 4 subdivisions
FEM #2	Sap2000 solution by 8 subdivisions
ANSYS#2	Ansys Workbench solution by 200 subdivisions
FGM#2	Finite Grid Solution by 8 subdivisions

Results & Discussion

In the analysis performed according to the data given in Table 1 and Table 2, the modulus of elasticity was considered to be 87360 kN / m^2, poisson ratio ν= 0.3, plate thickness h= 0.05 m, plate sizes 1x1 m for b / a = 1 and 2x1 m for b / a = 2. The maximum deformation values given in Figure 3 are as normalized maximum deflection values.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Ref	11.60	16.52	5.60	7.22	4.06	10.13	1.26	2.54	12.86	15.07	1.92	8.44	11.30	15.00
FEM#1	12.69	18.07	6.41	8.09	4.05	10.17	1.15	2.83	12.37	14.21	2.08	9.06	11.04	14.17
FEM#2	11.94	17.09	5.90	7.66	4.06	10.14	1.32	2.60	12.73	14.85	1.96	8.63	11.21	14.76
ANSYS#1	12.11	16.86	5.85	7.35	4.16	10.14	1.31	2.50	12.88	14.86	1.96	8.52	11.36	14.76
ANSYS#2	12.75	17.47	6.42	8.06	4.29	10.41	1.33	2.62	13.43	15.41	2.01	8.66	11.77	15.31
FGM#1	12.00	17.43	6.10	7.44	4.01	10.23	1.40	2.76	10.64	13.32	2.04	8.97	9.25	13.19
FGM#2	12.47	17.41	6.09	7.78	4.33	10.58	1.37	2.75	11.41	13.82	2.06	8.89	9.84	13.65

Figure 3: The normalized maximum deflection values of analysis types for rectangular plates with various loading and boundary conditions

The normalized maximum deflection values with the reference results solved by analytical method shown in Fig.3. When the average error rates calculated in 14 samples were taken into consideration, the lowest error rate was ANSYS with 1.96% and the largest error rate was FGM with 8.14%. According to the analysis made, when the number of mesh size is doubled, the error rate in finite grid method partially decreases and the error rate in Sap 2000 decreases too much. According to the Ansys analysis, the error rate increases with the increasing number of mesh size from 8 to 200. When these results are taken into consideration, in the case of finite element solutions with Sap 2000, there are difficulties in assigning boundary conditions. It can be concluded that the finite grid method as an approximate numerical method has an acceptable error rate considering that it provides practical and modeling facilities for thin plate solutions.

Conclusion

The solution method of the finite grid technique as grid work analogy based on a treatment of view of use the strain energy functions to obtain shape functions and stiffness matrices used to develop a more general simplified numerical approach for complicated plate bending problems. The finite grid solution based on the matrices of one- dimensional beam properties for plate bending problems verified with a high degree of accuracy. The finite grid solution as a combination of finite element method, lattice analogy and matrix displacement analysis of grid works is a useful tool to improve the solution of plate bending problems.

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