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Research Article

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On Study Generalized R^h- Trirecurrent Affinely Connected Space

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Abstract In the present paper, a Finsler space F_n whose Cartan's fourth curvature tensor R^i_{jkh} satisfies the following condition $R^i_{jkh|\ell|m|n} = c_{\ell mn} R^i_{jkh} + d_{\ell mn}(\delta^i_k g_{jh} - \delta^i_h g_{jk})$, $R^i_{jkh} \neq 0$,where $c_{\ell mn}$ and $d_{\ell mn}$ are non-zero covariant vector field of third order, respectively. Thies space satisfies the condition of affinely connected space called generalized R^h-trirecurrent affinely connected space.

Keywords generalized R^h-trirecurrent space, H – Ricci tensor

Introduction

H.D. Pande and B. Single [4] discussed the recurrence in an affinely connected space. P.K. Dwivedi [7] worked out the role of P^*-reducible space in affinely space. A.A.M. Saleem [2] obtained some results when the C^h-generalized birecurrent and C^h-special generalized birecurrent are affinely connected space. A.A.A. Muhib [1] obtained some results when R^h-generalized trirecurrent and R^h-special generalized trirecurrent are affinely connected space. M.A.A. Ali [5] obtained certain identities in a K^h-birecurrent affinely connected space. N.S.H. Hussein [6] obtained certain identities in a K^h-recurrent affinely connected space. A.M.A. Al-Qashbari [3] obtained certain identities in a R^h-recurrent affinely connected space. W.H.A. Hadi [9] obtained certain identities in a R^h-birecurrent affinely connected space.

The metric tensor g_{ij} given by [8]

(1.1) $g_{ij} = \dot{\partial}_i y_j = \dot{\partial}_j y_i$.

The covariant derivative of the vector y^i andthe metric tensor g_{ij}, vanish identically, i. e.

(1.2) a) $y^i_{|k} = 0$ and b) $g_{ij|k} = 0$.

The vector y_i and the koronelter delta δ^i_k also satisfy the following relations

(1.3) a) $\delta^i_k y^k = y^i$, b) $\delta^i_k y_i = y_k$ and c) $\delta^i_i = 0$

The vectors y^i and y_i satfies the following relation

(1.4) $y^i y_i = F^2$

The metric tensor g_{ij} and δ^i_k satisfy the following relation

(1.5) $\delta^i_k g_{ij} = g_{jk}$.

The vector y^j and the metric tensor g_{ij} also satisfy the following:

(1.6) $g_{ij} y^j = y_i$.

The h-covariant derivative, commute with partial differentiation with respect to y^j according to

(1.7) $\dot{\partial}_j(X^i_{|k}) - (\dot{\partial}_j X^i)_{|k} = X^r(\dot{\partial}_j \Gamma^{*i}_{rk}) - (\dot{\partial}_r X^i) P^r_{jk}$.

The curvature tensor $R^{\ i}_{jk\,h}$ satisfies the relation

(1.8) $R^i_{jk\,h}\, y^j = H^i_{kh}$.

The torsion tensor $P^{\ i}_{jk}$ satisfies the relation

(1.9) $P^i_{jk}\, y^j = 0$.

In view of Euler's theorem on homogeneous functions we have the following relations

(1.10) a) $H^i_{jk\,h}\, y^j = H^i_{kh}$ and b) $H^i_{jk}\, y^j = H^i_k$.

The associate curvature tensor $H_{ijk\,h}$ of the curvature tensor $H^i_{jk\,h}$ is given by

a) $H_{ijk\,h} = g_{rj}\; H^r_{ik\,h}$ and b) $H^i_{jk\,h} = \dot{\partial}_j H^i_{kh}$.(1.11)

The H-Ricci tensor H^i_{kh}, the divetion tensor H^i_h and curvature vector H_k, the curvature scalar H satisfy the relations

(1.12) a) $H^i_{kh} = \dot{\partial}_k H^i_h$, b)$H_k = H^i_{ki}$ and c)$(n-1)H = H_k\, y^k$.

Where H_{jk} and$H$ are called *H-Ricci tensor* and *the curvature scalar [7]*, respectively ,the tensor $H_{kj\,h}$ defined by

(1.13) $H_{j\,h.k} = g_{ih}\; H^i_{jk}$.

The H-Ricci tensor H_{kj} and the curvature scalar H_k defined by

a)$H_{jk} = \dot{\partial}_j H_k$ and b)$H_{jk} = H^i_{jki}$, .(1.14)

2. An R^h-Generalized Trirecurrent Space

Let us consider a Finsler space F_n whose Cartan's fourth curvature tensor $R^i_{jk\,h}$ satisfies *by the condition*

(2.1) $R^i_{jk\,h|l|m|n} = c_{lmn}\, R^i_{jk\,h} + d_{lmn}\,(\,\delta^i_k\, g_{jh} - \delta^i_h\, g_{jk}\,)$, $R^i_{jk\,h} \neq 0$

where c_{lmn} and d_{lmn} are non-zero covariant vectors field of third order .

The space and the tensor satisfy the condition (2.1) are called *generalized* R^h*- trirecurrent space* and *generalized h- trirecurrent tensor*, respectively. We shall denote them briefly by G R^h-TR F_n *and* G h – TR , Transvecting the condition (2.1) by y^j , using (1.2a), (1.8), (1.6) and (1.3a), we get

(2.2) $H^i_{kh|l|m|n} = c_{lmn}\, H^i_{kh} + d_{lmn}\,(\,\delta^i_k\, y_h - \delta^i_h\, y_k\,)$.

Transvecting (2.2) by y^k, using (1.2a), (1.10b), (1.3a) and (1.4a), we get

(2.3) $H^i_{h|l|m|n} = c_{lmn}\, H^i_h + d_{lmn}\,(\,y^i y_h - \delta^i_h\, F^2)$.

Thus, we may conclude

Theorem 2.1. *In GR^h-TR F_n , the h-covariant derivative of third order for the h(v)-torsion tensor H^i_{kh} and the deviation tensor H^i_h given by (2.2) and (2.3), respectively.*

Differentiating (2.2) partially with respect to y^j, using (1.11b) and (1.1), we get

(2.4) $$\dot{\partial}_j\left(H^i_{kh|l|m|n}\right) = \left(\dot{\partial}_j c_{lmn}\right) H^i_{kh} + c_{lmn}\; H^i_{jk\,h} + \left(\dot{\partial}_j d_{lmn}\right)\left(\,\delta^i_k\, y_h - \delta^i_h\, y_k\,\right) + d_{lmn}\,\left(\delta^{\ i}_k\, g_{jh} - \delta^i_h\, g_{jk}\,\right) .$$

Using the commutation formula exhibited by (1.7) for $\left(H^i_{kh|l|m}\right)$ in (2.4), we get

(2.5) $$\dot{\partial}_j\,(H^i_{kh|l|m})_{|n} + H^r_{kh|l|m}\left(\dot{\partial}_j \Gamma^{*i}_{rn}\right) - H^i_{rh|l|m}\left(\,\dot{\partial}_j \Gamma^{*r}_{kr}\,\right) - H^i_{rk|l|m}\left(\,\dot{\partial}_j \Gamma^{*r}_{hn}\,\right) - H^i_{kh|l|r}\left(\dot{\partial}_j \Gamma^{*r}_{mn}\,\right) - H^i_{kh|r|m}\left(\dot{\partial}_j \Gamma^{*r}_{ln}\,\right) - \dot{\partial}_r\left(H^i_{kh|l|m}\right) P^r_{jn} = \left(\dot{\partial}_j c_{lmn}\,\right) H^i_{kh} + c_{lmn}\, H^i_{jk\,h} + \left(\dot{\partial}_j d_{lmn}\,\right)\left(\,\delta^i_k\, y_h - \,\delta^i_h\, y_k\right) + d_{lmn}\,\left(\,\delta^i_k\, g_{jh} - \delta^i_h\, g_{jk}\,\right) .$$

Again applying the commutation formula exhibited by (1.7) for $\left(H^i_{kh|l}\right)$ in (2.5), we get

(2.6) $$[\left\{\dot{\partial}_j\left(H^i_{kh|l}\right)\right\}_{|m} + H^r_{kh|l}\left(\,\dot{\partial}_j \Gamma^{*i}_{rm}\,\right) - H^i_{rh|l}\left(\,\dot{\partial}_j \Gamma^{*r}_{km}\right) - H^i_{rk|l}\left(\,\dot{\partial}_j \Gamma^{*r}_{hm}\right) - H^r_{kh|r}\left(\dot{\partial}_j \Gamma^{*r}_{ml}\right) - \dot{\partial}_r\left(H^i_{kh|l}\right) P^r_{jm}\,]_{|n} + H^r_{kh|l|m}\left(\,\dot{\partial}_j \Gamma^{*i}_{rn}\,\right) - H^i_{rh|l|m}\left(\,\dot{\partial}_j \Gamma^{*r}_{kn}\,\right) - H^i_{rk|l|m}\left(\,\dot{\partial}_j \Gamma^{*r}_{hn}\,\right) - H^i_{kh|r|m}\left(\,\dot{\partial}_j \Gamma^{*r}_{ln}\,\right) - H^i_{kh|l|r}\left(\,\dot{\partial}_j \Gamma^{*r}_{mn}\,\right) - [\left\{\dot{\partial}_r\left(H^i_{kh|l}\right)\right\}_{|m} + H^s_{kh|l}\left(\,\dot{\partial}_r \Gamma^{*i}_{sm}\,\right)$$

$$- H^{i}_{sh|l}(\dot{\partial}_r \Gamma^{*s}_{km}) - H^{i}_{sk|l}(\dot{\partial}_r \Gamma^{*s}_{hm}) - H^{i}_{kh|s}(\dot{\partial}_r \Gamma^{*s}_{lm}) - \dot{\partial}_s(H^{i}_{kh|l}) P^{s}_{rm}\,]\, P^{r}_{jn}$$

$$= (\dot{\partial}_j c_{lmn}) H^{i}_{kh} + c_{lmn} H^{i}_{jkh} + (\dot{\partial}_j d_{lmn})(\delta^i_k y_h - \delta^i_h y_k) + d_{lmn}(\delta^i_k g_{jh} - \delta^i_h g_{jk}) \quad .$$

Again applying the commutation formula exhibited by (1.7) for (H^i_{kh}) in (2.6), we get

(2.7) $$[(\dot{\partial}_j H^{i}_{kh})_{|l|m} + \{H^{r}_{kh}(\dot{\partial}_j \Gamma^{*i}_{rl}) - H^{i}_{rh}(\dot{\partial}_j \Gamma^{*r}_{kl}) - H^{i}_{rk}(\dot{\partial}_j \Gamma^{*r}_{hl}) - (\dot{\partial}_r H^{i}_{kh}) P^{r}_{jl}\}_{|m}$$
$$+ H^{r}_{kh|l}(\dot{\partial}_j \Gamma^{*i}_{rm}) - H^{i}_{rh|l}(\dot{\partial}_j \Gamma^{*r}_{km}) - H^{i}_{rk|l}(\dot{\partial}_j \Gamma^{*r}_{hm}) - H^{i}_{kh|r}(\dot{\partial}_j \Gamma^{*r}_{lm}) - \{(\dot{\partial}_r H^{i}_{kh})_{|l}$$
$$+ H^{s}_{kh}(\dot{\partial}_r \Gamma^{*i}_{sl}) - H^{i}_{sh}(\dot{\partial}_r \Gamma^{*s}_{kl}) - H^{i}_{sk}(\dot{\partial}_r \Gamma^{*s}_{hl}) - (\dot{\partial}_s H^{i}_{kh}) P^{s}_{rl}\} P^{r}_{jm}\,]_{|n} + H^{r}_{kh|l|m}(\dot{\partial}_j \Gamma^{*i}_{rn})$$
$$- H^{i}_{rh|l|m}(\dot{\partial}_j \Gamma^{*r}_{kn}) - H^{i}_{rk|l|m}(\dot{\partial}_j \Gamma^{*r}_{hn}) - H^{i}_{kh|r|m}(\dot{\partial}_j \Gamma^{*r}_{ln}) - H^{i}_{kh|l|r}(\dot{\partial}_j \Gamma^{*r}_{mn})$$
$$- [\{(\dot{\partial}_j H^{i}_{kh})_{|l} + H^{r}_{kh}(\dot{\partial}_r \Gamma^{*i}_{sl}) - H^{i}_{sh}(\dot{\partial}_r \Gamma^{*r}_{sl}) - H^{i}_{ks}(\dot{\partial}_r \Gamma^{*s}_{hl}) - (\dot{\partial}_s H^{i}_{kh}) P^{s}_{rl}\}_{|m}$$
$$+ H^{s}_{kh|l}(\dot{\partial}_r \Gamma^{*i}_{sm}) - H^{i}_{sh|l}(\dot{\partial}_r \Gamma^{*s}_{km}) - H^{i}_{sk|l}(\dot{\partial}_r \Gamma^{*s}_{hm}) - H^{i}_{kh|s}(\dot{\partial}_r \Gamma^{*s}_{lm}) - \{(\dot{\partial}_s H^{i}_{kh})_{|l}$$
$$+ H^{t}_{kh}(\dot{\partial}_s \Gamma^{*i}_{lt}) - H^{i}_{th}(\dot{\partial}_s \Gamma^{*t}_{kl}) - H^{i}_{kt}(\dot{\partial}_s \Gamma^{*t}_{hl}) - (\dot{\partial}_t H^{i}_{kh}) P^{t}_{sl}\} P^{s}_{rm}\,]\, P^{r}_{jn}$$
$$= (\dot{\partial}_j c_{lmn}) H^{i}_{kh} + c_{lmn} H^{i}_{jkh} + (\dot{\partial}_j d_{lmn})(\delta^i_k y_h - \delta^i_h y_k) + d_{lmn}(\delta^i_k g_{jh} - \delta^i_h g_{jk}) \quad .$$

Using (1.1b) in (2.7), we get

(2.8) $$H^{i}_{jkh|l|m|n} + [\{H^{r}_{kh}(\dot{\partial}_j \Gamma^{*i}_{rl}) - H^{i}_{rh}(\dot{\partial}_j \Gamma^{*r}_{kl}) - H^{i}_{rk}(\dot{\partial}_j \Gamma^{*r}_{hl}) - (H^{i}_{krh}) P^{r}_{jl}\}_{|m}$$
$$+ H^{r}_{kh|l}(\dot{\partial}_j \Gamma^{*i}_{rm}) - H^{i}_{rh|l}(\dot{\partial}_j \Gamma^{*r}_{km}) - H^{i}_{rk|l}(\dot{\partial}_j \Gamma^{*r}_{hm}) - H^{i}_{kh|r}(\dot{\partial}_j \Gamma^{*r}_{lm}) - \{H^{i}_{rkh|l}$$
$$+ H^{s}_{kh}(\dot{\partial}_r \Gamma^{*i}_{sl}) - H^{i}_{sh}(\dot{\partial}_r \Gamma^{*s}_{kl}) - H^{i}_{ks}(\dot{\partial}_r \Gamma^{*s}_{hl}) - (H^{i}_{skh}) P^{s}_{rl}\} P^{r}_{jm}\,]_{|n} + H^{r}_{kh|l|m}(\dot{\partial}_j \Gamma^{*i}_{rn})$$
$$- H^{i}_{rh|l|m}(\dot{\partial}_j \Gamma^{*r}_{kn}) - H^{i}_{rk|l|m}(\dot{\partial}_j \Gamma^{*r}_{hn}) - H^{i}_{kh|r|m}(\dot{\partial}_j \Gamma^{*r}_{ln}) - H^{i}_{kh|l|r}(\dot{\partial}_j \Gamma^{*r}_{mn})$$
$$- [\{H^{i}_{rkh|l} + H^{s}_{kh}(\dot{\partial}_r \Gamma^{*i}_{sl}) - H^{i}_{sh}(\dot{\partial}_r \Gamma^{*s}_{kl}) - H^{i}_{ks}(\dot{\partial}_r \Gamma^{*s}_{hl}) - H^{i}_{skh} P^{s}_{rl}\}_{|m}$$
$$+ H^{s}_{kh|l}(\dot{\partial}_r \Gamma^{*i}_{sm}) - H^{i}_{sh|l}(\dot{\partial}_r \Gamma^{*s}_{km}) - H^{i}_{sk|l}(\dot{\partial}_r \Gamma^{*s}_{hm}) - H^{i}_{kh|s}(\dot{\partial}_r \Gamma^{*s}_{lm}) - \{H^{i}_{skh|l}$$
$$+ H^{t}_{kh}(\dot{\partial}_s \Gamma^{*i}_{lt}) - H^{i}_{th}(\dot{\partial}_s \Gamma^{*t}_{kl}) - H^{i}_{tk}(\dot{\partial}_s \Gamma^{*t}_{hl}) - H^{i}_{tkh} P^{t}_{sl}\} P^{s}_{rm}\,]\, P^{r}_{jn}$$
$$= (\dot{\partial}_j c_{lmn}) H^{i}_{kh} + c_{lmn} H^{i}_{jkh} + (\dot{\partial}_j d_{lmn})(\delta^i_k y_h - \delta^i_h y_k) + d_{lmn}(\delta^i_k g_{jh} - \delta^i_h g_{jk}) \quad .$$

This shows that

$$H^{i}_{jkh|l|m|n} = c_{lmn} H^{i}_{jkh} + d_{lmn}(\delta^i_k g_{jh} - \delta^i_h g_{jk})$$

if and only if

(2.9) $$[\{H^{r}_{kh}(\dot{\partial}_j \Gamma^{*i}_{rl}) - H^{i}_{rh}(\dot{\partial}_j \Gamma^{*r}_{kl}) - H^{i}_{rk}(\dot{\partial}_j \Gamma^{*r}_{hl}) - (H^{i}_{rkh}) P^{r}_{jl}\}_{|m} + H^{r}_{kh|l}(\dot{\partial}_j \Gamma^{*i}_{rm})$$
$$- H^{i}_{rh|l}(\dot{\partial}_j \Gamma^{*r}_{km}) - H^{i}_{rk|l}(\dot{\partial}_j \Gamma^{*r}_{hm}) - H^{i}_{kh|r}(\dot{\partial}_j \Gamma^{*r}_{lm}) - \{H^{i}_{rkh|l} + H^{s}_{kh}(\dot{\partial}_r \Gamma^{*i}_{sl})$$
$$- H^{i}_{sh}(\dot{\partial}_r \Gamma^{*s}_{kl}) - H^{i}_{ks}(\dot{\partial}_r \Gamma^{*s}_{hl}) - H^{i}_{skh} P^{s}_{rl}\} P^{r}_{jm}\,]_{|n} + H^{r}_{kh|l|m}(\dot{\partial}_j \Gamma^{*i}_{rn}) - H^{i}_{rh|l|m}(\dot{\partial}_j \Gamma^{*r}_{kn})$$
$$- H^{i}_{rk|l|m}(\dot{\partial}_j \Gamma^{*r}_{hn}) - H^{i}_{kh|r|m}(\dot{\partial}_j \Gamma^{*r}_{ln}) - H^{i}_{kh|l|r}(\dot{\partial}_j \Gamma^{*r}_{mn}) - [\{H^{i}_{rkh|l} + H^{s}_{kh}(\dot{\partial}_r \Gamma^{*i}_{sl})$$
$$- H^{i}_{sh}(\dot{\partial}_r \Gamma^{*s}_{kl}) - H^{i}_{ks}(\dot{\partial}_r \Gamma^{*s}_{hl}) - (\dot{\partial}_s H^{i}_{kh}) P^{s}_{rl}\}_{|m} + H^{s}_{kh|l}(\dot{\partial}_r \Gamma^{*i}_{sm}) - H^{i}_{sh|l}(\dot{\partial}_r \Gamma^{*s}_{km})$$
$$- H^{i}_{sk|l}(\dot{\partial}_r \Gamma^{*s}_{hm}) - H^{i}_{kh|s}(\dot{\partial}_r \Gamma^{*s}_{lm}) - \{H^{i}_{skh|l} + H^{t}_{kh}(\dot{\partial}_s \Gamma^{*i}_{lt}) - H^{i}_{th}(\dot{\partial}_s \Gamma^{*t}_{kl})$$
$$- H^{i}_{kt}(\dot{\partial}_s \Gamma^{*t}_{hl}) - H^{i}_{tkh} P^{t}_{sl}\} P^{s}_{rm}\,] P^{r}_{jn} = (\dot{\partial}_j c_{lmn}) H^{i}_{kh} + (\dot{\partial}_j d_{lmn})(\delta^i_k y_h - \delta^i_h y_k) \quad .$$

Thus, we may conclude

Theorem 2.2. *In GR^h- TRF_n , Berwald curvature tensor H^i_{jkh} is generalized trirecurrent tensor if and only if (2.9) hold good.*

Transvecting (2.8) by g_{ip} , using (1.2b), (1.11a), (1.13) and (1.5), we get

$$H_{jpkh|l|m|n} + g_{ip}[\{H^{r}_{kh}(\dot{\partial}_j \Gamma^{*i}_{rl}) - H^{i}_{rh}(\dot{\partial}_j \Gamma^{*r}_{kl}) - H^{i}_{rk}(\dot{\partial}_j \Gamma^{*r}_{hl}) - H^{i}_{rkh} P^{r}_{jl}\}_{|m}$$
$$+ H^{r}_{kh|l}(\dot{\partial}_j \Gamma^{*i}_{rm}) - H^{i}_{rh|l}(\dot{\partial}_j \Gamma^{*r}_{km}) - H^{i}_{rk|l}(\dot{\partial}_j \Gamma^{*r}_{hm}) - H^{i}_{kh|r}(\dot{\partial}_j \Gamma^{*r}_{lm}) - \{H^{i}_{rkh|l}$$
$$- H^{t}_{kh}(\dot{\partial}_r \Gamma^{*i}_{sl}) - H^{i}_{th}(\dot{\partial}_r \Gamma^{*t}_{kl}) - H^{i}_{kt}(\dot{\partial}_r \Gamma^{*t}_{hl}) - H^{i}_{tkh} P^{t}_{rl}\} P^{r}_{jm}\,]_{|n} + g_{ip}[H^{r}_{kh|l|m}(\dot{\partial}_j \Gamma^{*i}_{rn})$$
$$- H^{i}_{rh|l|m}(\dot{\partial}_j \Gamma^{*r}_{kn}) - H^{i}_{rk|l|m}(\dot{\partial}_j \Gamma^{*r}_{hn}) - H^{i}_{kh|r|m}(\dot{\partial}_j \Gamma^{*r}_{ln}) - H^{i}_{kh|l|r}(\dot{\partial}_j \Gamma^{*r}_{mn})]$$
$$- g_{ip}[\{H^{i}_{rkh|l} - H^{s}_{kh}(\dot{\partial}_r \Gamma^{*i}_{sl}) - H^{i}_{sh}(\dot{\partial}_r \Gamma^{*s}_{kl}) - H^{i}_{ks}(\dot{\partial}_r \Gamma^{*s}_{hl}) - (\dot{\partial}_s H^{i}_{kh}) P^{s}_{rl}\}_{|m}$$

$$- H^s_{kh|l}\left(\dot{\partial}_r \Gamma^{*i}_{sm} \right) - H^i_{sh|l}\left(\dot{\partial}_r \Gamma^{*s}_{km} \right) - H^i_{sk|l}\left(\dot{\partial}_r \Gamma^{*s}_{hm}\right) - H^i_{kh|s}\left(\dot{\partial}_r \Gamma^{*s}_{lm}\right) - \{ H^{ii}_{skh|l}$$
$$- H^t_{kh}\left(\dot{\partial}_s \Gamma^{*i}_{lt} \right) - H^i_{th}\left(\dot{\partial}_s \Gamma^{*t}_{kl} \right) - H^i_{kt}\left(\dot{\partial}_s \Gamma^{*t}_{hl} \right) - H^i_{tkh}\, P^t_{sl} \}\, P^s_{rm} \,]\, P^r_{jn} = \left(\dot{\partial}_j c_{lmn}\right) H_{kp.h} -$$

$$+ c_{lmn}\, H_{jpk\,h} + \left(\dot{\partial}_j d_{lmn}\right)\left(g_{kp}\, y_h - g_{hp}\, y_k \right) + d_{lmn}\left(g_{kp}\, g_{jh} - g_{hp}\, g_{jk} \right) \quad .$$

This shows that

$$H_{jpk\,h|l|m|n} = c_{lmn}\, H_{jpk\,h} + d_{lmn}\left(g_{kp}\, g_{jh} - g_{hp}\, g_{jk} \right)$$

if and only if

(2.10) $$g_{ip}\,[\,\{ H^r_{kh}\left(\dot{\partial}_j \Gamma^{*i}_{rl}\right) - H^i_{rh}\left(\dot{\partial}_j \Gamma^{*r}_{kl}\right) - H^i_{rk}\left(\dot{\partial}_j \Gamma^{*r}_{hl}\right) - H^i_{rkh} P^r_{jl} \}_{|m} + H^r_{kh|l}\left(\dot{\partial}_j \Gamma^{*i}_{rm} \right)$$
$$- H^i_{rh|l}\left(\dot{\partial}_j \Gamma^{*r}_{km}\right) - H^i_{rk|l}\left(\dot{\partial}_j \Gamma^{*r}_{hm}\right) - H^i_{kh|r}\left(\dot{\partial}_j \Gamma^{*r}_{lm}\right) - \{H^i_{rkh|l} - H^t_{kh}\left(\dot{\partial}_r \Gamma^{*i}_{sl}\right) - H^i_{th}\left(\dot{\partial}_r \Gamma^{*t}_{kl}\right)$$
$$- H^i_{kt}\left(\dot{\partial}_r \Gamma^{*t}_{hl}\right) - H^i_{tkh} P^t_{rl} \}\, P^r_{jm} \,]_{|n} + g_{ip}\,[H^r_{kh|l|m}\left(\dot{\partial}_j \Gamma^{*i}_{rn}\right) - H^i_{rh|l|m}\left(\dot{\partial}_j \Gamma^{*r}_{kn}\right)$$
$$- H^i_{rk|l|m}\left(\dot{\partial}_j \Gamma^{*r}_{hn}\right) - H^i_{kh|r|m}\left(\dot{\partial}_j \Gamma^{*r}_{ln} \right) - H^i_{kh|l|r}\left(\dot{\partial}_j \Gamma^{*r}_{mn} \right)] - g_{ip}\,[\{ H^i_{rkh|l} - H^s_{kh}\left(\dot{\partial}_r \Gamma^{*i}_{sl}\right)$$
$$- H^i_{sh}\left(\dot{\partial}_r \Gamma^{*s}_{kl}\right) - H^i_{ks}\left(\dot{\partial}_r \Gamma^{*s}_{hl}\right) - \left(\dot{\partial}_s H^i_{kh}\right) P^s_{rl} \}_{|m} - H^s_{kh|l}\left(\dot{\partial}_r \Gamma^{*i}_{sm}\right) - H^i_{sh|l}\left(\dot{\partial}_r \Gamma^{*s}_{km}\right)$$
$$- H^i_{sk|l}\left(\dot{\partial}_r \Gamma^{*s}_{hm}\right) - H^i_{kh|s}\left(\dot{\partial}_r \Gamma^{*s}_{lm}\right) - \{ H^{ii}_{skh|l} - H^t_{kh}\left(\dot{\partial}_s \Gamma^{*i}_{lt} \right) - H^i_{th}\left(\dot{\partial}_s \Gamma^{*t}_{kl} \right)$$
$$- H^i_{kt}\left(\dot{\partial}_s \Gamma^{*t}_{hl} \right) - H^i_{tkh}\, P^t_{sl} \}\, P^s_{rm} \,]\, P^r_{jn} = \left(\dot{\partial}_j c_{lmn}\right) H_{kp.h} + \left(\dot{\partial}_j d_{lmn}\right)\left(g_{kp}\, y_h - g_{hp}\, y_k\right) -$$

Thus, we may conclude

Theorem 2.3. *In GR^h- TRF_n , the associative curvature tensor $H_{jpk\,h}$ of Berwald curvature tensor $H^i_{jk\,h}$ is generalized trirecurrent tensor if and only if (2.10) holds good.*

Contracting the indices I and h in (2.8), using (1.14b), (1.12c), (1.3b), (1.4) and (1.5), we get

$$H_{jk|l|m|n} + [\,\{H^r_{kp}\left(\dot{\partial}_j \Gamma^{*p}_{rl}\right) - H_r\left(\dot{\partial}_j \Gamma^{*r}_{kl}\right) - H^p_{rk}\left(\dot{\partial}_j \Gamma^{*r}_{pl}\right) - H_{kr} P^r_{jl} \}_{|m} + H^r_{kp|l}\left(\dot{\partial}_j \Gamma^{*p}_{rm}\right)$$
$$- H_{r|l}\left(\dot{\partial}_j \Gamma^{*r}_{km}\right) - H^p_{rk|l}\left(\dot{\partial}_j \Gamma^{*r}_{pm}\right) - H_{k|r}\left(\dot{\partial}_j \Gamma^{*r}_{lm}\right) - \{ H_{rk|l} + H^s_{kp}\left(\dot{\partial}_r \Gamma^{*p}_{sl}\right) - H_s\left(\dot{\partial}_r \Gamma^{*s}_{kl}\right)$$
$$- H^p_{ks}\left(\dot{\partial}_r \Gamma^{*s}_{pl}\right) - H_{ks} P^s_{rl} \}\, P^r_{jm} \,]_{|n} + H^r_{kp|l|m}\left(\dot{\partial}_j \Gamma^{*p}_{rn}\right) - H_{r|l|m}\left(\dot{\partial}_j \Gamma^{*r}_{km}\right) - H^p_{rk|l|m}\left(\dot{\partial}_j \Gamma^{*r}_{pn}\right)$$
$$- H_{k|r|m}\left(\dot{\partial}_j \Gamma^{*r}_{ln}\right) - H_{k|l|r}\left(\dot{\partial}_j \Gamma^{*r}_{mn}\right) - [\{H_{rk|l} + H^s_{kp}\left(\dot{\partial}_r \Gamma^{*p}_{sl}\right) - H_s\left(\dot{\partial}_r \Gamma^{*s}_{kl}\right) - H^p_{ks}\left(\dot{\partial}_r \Gamma^{*s}_{pl}\right)$$
$$- H_{ks} P^s_{rl} \}_{|m} + H^s_{kp|l}\left(\dot{\partial}_r \Gamma^{*p}_{sm}\right) - H_{s|l}\left(\dot{\partial}_r \Gamma^{*s}_{km}\right) - H^p_{sk|l}\left(\dot{\partial}_r \Gamma^{*s}_{pm}\right) - H_k\left(\dot{\partial}_r \Gamma^{*s}_{lm}\right)$$
$$- \left\{ H_{ks|l} - H^t_{kp}\left(\dot{\partial}_s \Gamma^{*p}_{lt} \right) - H_t\left(\dot{\partial}_s \Gamma^{*t}_{kl} \right) - H^p_{tk}\left(\dot{\partial}_s \Gamma^{*t}_{pl} \right) - H_{kt} P^t_{sl} \right\} P^s_{rm} \,]\, P^r_{jn}$$
$$= \left(\dot{\partial}_j c_{lmn}\right) H_k + c_{lmn} H_{jk} + (1-n)\left(\dot{\partial}_j d_{lmn}\right) y_k + (1-n) d_{lmn}\, g_{jk} \quad .$$

This shows that

(2.11) $$H_{jk|l|m|n} = c_{lmn}\, H_{jk} + (1-n)\, d_{lmn}\, g_{jk}$$

If and only if

(2.12) $$[\{H^r_{kp}\left(\dot{\partial}_j \Gamma^{*p}_{rl}\right) - H_r\left(\dot{\partial}_j \Gamma^{*r}_{kl}\right) - H^p_{rk}\left(\dot{\partial}_j \Gamma^{*r}_{pl}\right) - H_{kr} P^r_{jl}\}_{|m} + H^r_{kp|l}\left(\dot{\partial}_j \Gamma^{*p}_{rm}\right) - H_{r|l}\left(\dot{\partial}_j \Gamma^{*r}_{km}\right)$$
$$- H^p_{rk|l}\left(\dot{\partial}_j \Gamma^{*r}_{pm}\right) - H_{k|r}\left(\dot{\partial}_j \Gamma^{*r}_{lm}\right) - \{ H_{rk|l} + H^s_{kp}\left(\dot{\partial}_r \Gamma^{*p}_{sl}\right) - H_s\left(\dot{\partial}_r \Gamma^{*s}_{kl}\right) - H^p_{ks}\left(\dot{\partial}_r \Gamma^{*s}_{pl}\right)$$
$$- H_{ks} P^s_{rl} \} P^r_{jm} \,]_{|n} + H^r_{kp|l|m}\left(\dot{\partial}_j \Gamma^{*p}_{rn}\right) - H_{r|l|m}\left(\dot{\partial}_j \Gamma^{*r}_{km}\right) - H^p_{rk|l|m}\left(\dot{\partial}_j \Gamma^{*r}_{pn}\right) - H_{k|r|m}\left(\dot{\partial}_j \Gamma^{*r}_{ln}\right)$$
$$- H_{k|l|r}\left(\dot{\partial}_j \Gamma^{*r}_{mn}\right) - [\{H_{rk|l} + H^s_{kp}\left(\dot{\partial}_r \Gamma^{*p}_{sl}\right) - H_s\left(\dot{\partial}_r \Gamma^{*s}_{kl}\right) - H^p_{ks}\left(\dot{\partial}_r \Gamma^{*s}_{pl}\right) - H_{ks} P^s_{rl} \}_{|m}$$
$$+ H^s_{kp|l}\left(\dot{\partial}_r \Gamma^{*p}_{sm}\right) - H_{s|l}\left(\dot{\partial}_r \Gamma^{*s}_{km}\right) - H^p_{sk|l}\left(\dot{\partial}_r \Gamma^{*s}_{pm}\right) - H_k\left(\dot{\partial}_r \Gamma^{*s}_{lm}\right) - \{H_{ks|l} - H^t_{kp}\left(\dot{\partial}_s \Gamma^{*p}_{lt}\right)$$
$$- H_t\left(\dot{\partial}_s \Gamma^{*t}_{kl} \right) - H^p_{tk}\left(\dot{\partial}_s \Gamma^{*t}_{pl} \right) - H_{kt} P^t_{sl} \}\, P^s_{rm} \,]\, P^r_{jn} = \left(\dot{\partial}_j c_{lmn}\right) H_k \quad .$$

The equation (2.11) shows that the H–Ricci tensor H_{jk} can't vanish, because the vanishing of it would implies $d_{lmn} = 0$, if and only if (2.12) hold good, a contradiction.

Thus, we may conclude

Theorem 2.4. *In GR^h- TRF_n , the H-Ricci tensor H_{jk} can't vanish if and only if (2.12) holds good.*

3. On Generalized R^h-Trirecurrent-Affinely Connected Space

In this section, we shall introduce new definition for GR^h- $TRF_{nnspace\ and\ tensor\ brifly\ by\ G\ R}$, whose also possess the properties of an affinely connected space.

Definition 3.1. A Finsler space $F_{nnspace\ and\ tensor\ brifly\ by\ G\ R}$, whose coefficient connection parameter $G^{\ i}_{jk}$ is independent of y^i is called *an affinely connected space (Berwald space).* Thus, an affinely connected space is characterized by any one of the following eguivalent equations

(3.1) a) $G^{\ i}_{jkh} = 0$ and b) $C_{ijk\,|h} = 0$.

The coefficients connection parameters Γ^{*i}_{kh} of Cartan and $G^{\ i}_{kh}$ of Berwald coincide in affinely connected space and they are independent of directional argument [9], i.e.

(3.2) a) $\dot\partial_j G^{\ i}_{kh} = 0$ and b) $\dot\partial_j \Gamma^{*i}_{kh} = \text{o}$.

Definition 3.2. The generalized R^h-trirecurrent space which possess the properties of an affinely connected space [i.e.satisfies any one of the equations (3.1a), (3.1b), (3.2a) and (3.2b)] will be called it *a generalized R^h-trirecurrent affinely connected space* and denoted briefly by *GR^h-TR - affinely connected space.*

Remark 3.1. It will be sufficient to call Cartan's third curvature tensor R^i_{jkh} which possess the property of GR^h-TR- affinely connected space as *generalized h-trirecurrent tensor (briefly by Gh-TR)* .

Let us consider GR^h- TR - affinely connected space.

In view of the theorem 2.1 and definition 3.2, we may conclude

Theorem 3.1. In generalized R^h-recurrent affinely connected space, the generalized R^h- birecurrent affinely connected space is GR^h- TR- affinely connected space.

Using (3.2b) in (2.8), we get

(3.3)
$$H^i_{jkh|l|m|n} - \{(H^i_{rkh}P^r_{jl})_{|m} + \left(H^i_{krh_{|l}} - H^i_{tkh}P^t_{rl}\right)P^r_{jm}\}_{|n} - \{(H^i_{rkh|l} - H^i_{skh}P^s_{rl})_{|m}$$
$$- (H^i_{skh|l} - H^i_{tkh}P^t_{s})P^s_{rm}\}P^r_{jn} = (\dot\partial_j c_{lmn})H^i_{kh} + c_{lmn}H^i_{jkh} + (\dot\partial_j d_{lmn})(\delta^i_k y_h - \delta^i_h y_k)$$
$$+ d_{lmn}\,(\delta^i_k g_{jh} - \delta^i_h g_{jk}) \quad .$$

This shows that

$$H^i_{jkh|l|m|n} = c_{lmn}H^i_{jkh} + d_{lmn}\,(\delta^i_k g_{jh} - \delta^i_h g_{jk})$$

If and only if

(3.4)
$$\{-(H^i_{rkh}P^r_{jl})_{|m} - (H^i_{krh_{|l}} - H^i_{tkh}P^t_{rl})P^r_{jm}\}_{|n} - \{(H^i_{rkh|l} - H^i_{skh}P^s_{rl})_{|m}$$
$$- (H^i_{skh|l} - H^i_{tkh}P^t_{sl})P^s_{rm}\}P^r_{jn} = (\dot\partial_j c_{lmn})H^i_{kh} + (\dot\partial_j d_{lmn})(\delta^i_k y_h - \delta^i_h y_k) \quad .$$

Transvecting (3.3) by g_{ip} , using (1.2b), (1.5), (1.11a) and (1.13), we get

$$H_{jpkh|l|m|n} - \{(H_{rpkh}P^r_{jl})_{|m} + (H_{kprh_{|l}} - H_{tpkh}P^t_{rl})P^r_{jm}\}_{|n} - \{(H_{rpkh|l} - H_{spkh}P^s_{rl})_{|m}$$
$$- (H_{spkh|l} - H_{tpkh|l}P^t_{sl})P^s_{rm}\}P^r_{jn} = (\dot\partial_j c_{lmn})H_{kph} + c_{lmn}H_{jpkh}$$
$$+ (\dot\partial_j d_{lmn})(g_{kp}\,y_h - g_{hp}\,y_k) + d_{lmn}\,(g_{kp}\,g_{jh} - g_{hp}\,g_{jk}) \quad .$$

This shows that

$$H_{jpkh|l|m|n} = c_{lmn}H_{jpkh} + d_{lmn}\,(g_{kp}\,g_{jh} - g_{hp}\,g_{jk})$$

if and only if

(3.5)
$$\{-(H_{rpkh}P^r_{jl})_{|m} - (H_{kprh_{|l}} - H_{rpkh}P^t_{rl})P^r_{jm}\}_{|n} - \{(H_{rpkh|l} - H_{spkh}P^s_{rl})_{|m}$$
$$- (H_{spkh|l} - H_{tpkh}P^t_{sl}P^s_{rm}\}P^r_{jn} = (\dot\partial_j c_{lmn})H_{kph} + (\dot\partial_j d_{lmn})(g_{kp}\,y_h - g_{hp}\,y_k) \quad .$$

Thus, we may conclude

Theorem 3.2. *In GR^h-TR- affinely connected space*, Berwald curvature tensor H^i_{kjh} *and its associative curvature tensor H_{jpkh} are generalized trirecurrent tensor if and only if (34) and (3.5),respectively hold good.*

Transvecting (3.3) by y^j , using (1.2a), (1.10a), (1.9) and (1.6), we get

(3.6)
$$H^i_{kh|l|m|n} = y^j(\dot\partial_j c_{lmn})H^i_{kh} + c_{lmn}H^i_{kh} + y^j(\dot\partial_j d_{lmn})(\delta^i_k y_h - \delta^i_h y_k)$$
$$+ d_{lmn}\,(\delta^i_k y_h - \delta^i_h y_k) \quad .$$

This shows that

(3.7) $H^i_{kh|l|m|n} = c_{lmn} H^i_{kh} + d_{lmn} (\delta^i_k y_h - \delta^i_h y_k)$

if and only if

(3.8) $(\dot{\partial}_j c_{lmn}) H^i_{kh} + (\dot{\partial}_j d_{lmn})(\delta^i_k y_h - \delta^i_h y_k)$, since $y^j \neq 0$.

Transvecting (3.6) by g_{ir} , using (1.2b), (1.13) and (1.5), we get

$$H_{kr.h|l|m|n} = y^j (\dot{\partial}_j c_{lmn}) H_{kr.h} + c_{lmn} H_{kr.h} + y^j (\dot{\partial}_j d_{lmn})(g_{kr} y_h - g_{hr} y_k) + d_{lmn} (g_{kr} y_h - g_h y_k) \quad .$$

This shows that

(3.9) $H_{kr.h|l|m|n} = c_{lmn} H_{kr.h} + d_{lmn} (g_{kr} y_h - g_h y_k)$

if and only if

(3.10) $y^j (\dot{\partial}_j c_{lmn}) H_{kr.h} + y^j (\dot{\partial}_j d_{lmn})(g_{kr} y_h - g_{hr} y_k) = 0$.

Transvecting (3.6) by y^k , using (1.2a), (1.10b), (1.3a) and (1.4), we get

$$H^i_{h|l|m|n} = y^j (\dot{\partial}_j c_{lmn}) H^i_h + c_{lmn} H^i_h + y^j (\dot{\partial}_j d_{lmn})(y^i y_h - \delta^i_h F^2) + d_{lmn} (y^i y_h - \delta^i_h F^2)$$

This shows that

(3.11) $H^i_{h|l|m|n} = c_{lmn} H^i_h + d_{lmn} (y^i y_h - \delta^i_h F^2)$

if and only if

(3.12) $y^j (\dot{\partial}_j c_{lmn}) H^i_h + y^j (\dot{\partial}_j d_{lmn})(y^i y_h - \delta^i_h F^2) = 0$.

Thus, we may conclude

Theorem 3.3. *In GR^h-TR- affinely connected space, the h-covariant derivative of third order for the h(v)-torsion tensor H^i_{kh} , its associative tensor H_{krh} and the deviation tensor H^i_h given by (3.7) ,(3.9) and (3.11) if and only if (3.8), (3.10) and (3.12), respectively hold.*

Contracting the indices i and h in (3.3), using (1.14b), (1.3a), (1.3b) and (1.3c), we get

$$H_{jk|l|m|n} + \{(-H_{rk} P^r_{jl})_{|m} - \left(H_{kr|l} - H_{tk} P^t_{rl} \right) P^r_{jm} \}_{|n} - \{(H_{rk|l} - H_{sk} P^s_{rl})_{|m} - (H_{sk|l} - H_{tk} P^t_{sl}) P^s_{rm} \} P^r_{jn} = (\dot{\partial}_j c_{lmn}) H_k + c_{lmn} H_{jk} + (1-n)(\dot{\partial}_j d_{lmn}) y_k + (1-n) d_{lmn} g_{jk} \quad .$$

This shows that

(3.13) $H_{jk|l|m|n} = c_{lmn} H_{jk} + (1-n) d_{lmn} g_{jk}$

if and only if

(3.14) $\{-(H_{rk} P^r_{jl})_{|m} - (H_{kr|l} - H_{tk} P^t_{rl})P^r_{jm} \}_{|n} - \{(H_{rk|l} - H_{sk} P^s_{rl})_{|m} -(H_{sk|l} - H_{tk} P^t_{sl}) P^s_{rm} \} P^r_{jn} = (\dot{\partial}_j c_{lmn}) H_k + (1-n)(\dot{\partial}_j d_{lmn}) y_k$.

Contracting the indices i and h in (3.6), using (1.12b) , (1.3b) and (1.3c), we get

(3.15) $H_{k|l|m|n} = y^j (\dot{\partial}_j c_{lmn}) H_k + c_{lmn} H_k + (1-n)(\dot{\partial}_j d_{lmn}) y_k y^j + (1-n) d_{lmn} y_k$.

This shows that

(3.16) $H_{k|l|m|n} = c_{lmn} H_k + (1-n) d_{lmn} y_k$

if and only if

(3.17) $y^j (\dot{\partial}_j c_{lmn}) H_k + (1-n)(\dot{\partial}_j d_{lmn}) y_k y^j = 0$.

Transvecting (3.15) by y^k , using (1.2a), (1.12c) and (1.4), we get

$$H_{|l|m|n} = y^j (\dot{\partial}_j c_{lmn}) H + c_{lmn} H - y^j (\dot{\partial}_j d_{lmn}) F^2 - d_{lmn} F^2.$$

This shows that

(3.18) $H_{|l|m|n} = c_{lmn} H + d_{lmn} F^2$

if and only if

(3.19) $y^j (\dot{\partial}_j c_{lmn}) H - (\dot{\partial}_j d_{lmn}) F^2 y^j = 0$.

The equations (3.13), (3.16) and (3.18) show that the H-Ricci tensor H_{jk}, the curvature vector H_k and the scalar curvature H, can't vanish, because the vanishing of any one of them would imply $d_{lmn} = 0$, if and only if (3.14), (3.17) and (3.19), respectively, hold, a contradiction.
Thus, we may conclude

Theorem 3.4. *In GR^h- TR- affinely connected space, the H-Ricci tensor H_{jk}, the curvature vector H_k and the scalar curvature H, are non-vanishing if and only if (3.14), (3.17) and (3.19), respectively hold.*

Differentiating (2.1), Partially with respect to y^j, using (1.14a) and (1.1), we get

(3.20) $\dot{\partial}_j\left(H_{k|l|m|n}\right) = \left(\dot{\partial}_j c_{lmn}\right)H_k + c_{lmn}H_{jk} + (1-n)\left(\dot{\partial}_j d_{lmn}\right)y_k + (1-n)\, d_{lmn}\, g_{jk}$.

Using the commutation formula exhibited by (1.7), for $(H_{k|l|m})$ in (3.20), we get

(3.21) $(\dot{\partial}_j H_{k|l|m})_{|n} - H_{r|l|m}\left(\dot{\partial}_j \Gamma^{*r}_{kn}\right) - H_{k|r|m}\left(\dot{\partial}_j \Gamma^{*r}_{ln}\right) - H_{k|l|r}\left(\dot{\partial}_j \Gamma^{*r}_{mn}\right) - \left(\dot{\partial}_r H_{k|l|m}\right)P^r_{jn}$
$= \left(\dot{\partial}_j c_{lmn}\right)H_k + c_{lmn}H_{jk} + (1-n)\left(\dot{\partial}_j d_{lmn}\right)y_k + (1-n)d_{lmn}\, g_{jk}$.

Again using the commutation formula exhibited by (1.7), for $(H_{k|l})$ in (3.21), we get

(3.22) $\left\{\left(\dot{\partial}_j H_{k|l}\right)_{|m} - H_{r|l}\left(\dot{\partial}_j \Gamma^{*r}_{km}\right) - H_{k|r}\left(\dot{\partial}_j \Gamma^{*r}_{lm}\right) - \left(\dot{\partial}_r H_{k|l}\right)P^r_{jm}\right\}_{|n} - H_{r|l|m}\left(\dot{\partial}_j \Gamma^{*r}_{kn}\right)$
$- H_{k|r|m}\left(\dot{\partial}_j \Gamma^{*r}_{ln}\right) - H_{k|l|r}\left(\dot{\partial}_j \Gamma^{*r}_{mn}\right) - \{\left(\dot{\partial}_r H_{k|l}\right)_{|m} - H_{s|l}\left(\dot{\partial}_r \Gamma^{*s}_{km}\right) - H_{k|s}\left(\dot{\partial}_r \Gamma^{*s}_{lm}\right)$
$- \left(\dot{\partial}_s H_{k|l}\right) P^s_{rm}\, \} P^r_{jn} = \left(\dot{\partial}_j c_{lmn}\right)H_k + c_{lmn}H_{jk} + (1-n)\left(\dot{\partial}_j d_{lmn}\right)y_k$
$+ (1-n)\, d_{lmn}\, g_{jk}$.

Again, using the commutation formula exhibited by (1.7), for (H_k) in (3.22), we get

(3.23) $[\left\{\left(\dot{\partial}_j H_k\right)_{|l} - H_r\left(\dot{\partial}_j \Gamma^{*r}_{kl}\right) - \left(\dot{\partial}_r H_k\right) P^r_{jl}\right\}_{|m} - H_{r|l}\left(\dot{\partial}_j \Gamma^{*r}_{km}\right) - H_{k|r}\left(\dot{\partial}_j \Gamma^{*r}_{ml}\right) -$
$\left\{\left(\dot{\partial}_r H_k\right)_{|l} - H_t\left(\dot{\partial}_r \Gamma^{*t}_{kl}\right) - \left(\dot{\partial}_t H_k\right) P^t_{rl}\right\} P^r_{jm}\,]_{|n} - H_{r|l|m}\left(\dot{\partial}_j \Gamma^{*r}_{kn}\right) - H_{k|l|r}\left(\dot{\partial}_j \Gamma^{*r}_{mn}\right) -$
$H_{k|r|m}\left(\dot{\partial}_j \Gamma^{*r}_{ln}\right) - [\left\{\left(\dot{\partial}_r H_k\right)_{|l} - H_t\left(\dot{\partial}_r \Gamma^{*t}_{kl}\right) - \left(\dot{\partial}_t H_k\right) P^t_{rl}\right\}_{|m} - H_{s|l}\left(\dot{\partial}_r \Gamma^{*s}_{km}\right) -$
$H_{k|s}\left(\dot{\partial}_r \Gamma^{*s}_{ml}\right) - \left\{\left(\dot{\partial}_s H_k\right)_{|l} - H_t\left(\dot{\partial}_s \Gamma^{*t}_{kl}\right) - \left(\dot{\partial}_t H_k\right) P^t_{sl}\right\} P^s_{mr}\,] P^r_{jn} = \left(\dot{\partial}_j c_{lmn}\right)H_k +$
$c_{lmn}H_{jk} + (1-n)\left(\dot{\partial}_j d_{lmn}\right)y_k + (1-n)d_{lmn}\, g_{jk}$.

Using (1.14a), (3.13) and (3.2b) in (3.23), we get

(3.24) $\left\{-\left(H_{rk}P^r_{jl}\right)_{|m} - \left(H_{rk|l} - H_{tk}\, P^t_{rl}\right)P^r_{jm}\right\}_{|n} - \{(H_{rk|l} - H_{tk}\, P^t_{rl}) - (H_{sk|l} -$
$H_{tk}\, P^t_{sl})\, P^s_{rm}\}P^r_{jn} = \left(\dot{\partial}_j c_{lmn}\right)H_k - (1-n)\left(\dot{\partial}_j d_{lmn}\right)y_k$

if

(3.25) $\left\{-\left(H_{rk}P^r_{jl}\right)_{|m} - \left(H_{rk|l} - H_{tk}\, P^t_{rl}\right)P^r_{jm}\right\}_{|n}$
$-\left\{\{H_{rk|l} - H_{tk}\, P^t_{rl})_{|n} - \left(H_{sk|l} - P^t_{sl}\right)P^s_{rm}\}\right\}P^r_{jn} = 0$,

then (3.24) implies

(3.26) $\left(\dot{\partial}_j c_{lmn}\right)H_k - (1-n)\left(\dot{\partial}_j d_{lmn}\right)y_k = 0$.

Transvecting (3.26) by y^k, using (1.12c) and (1.4), we get

$$\left(\dot{\partial}_j c_{lmn}\right) H - \left(\dot{\partial}_j d_{lmn}\right) F^2 = 0$$

which can be written as

(3.27) $\dot{\partial}_j d_{lmn} = \frac{\left(\dot{\partial}_j c_{lmn}\right) H}{F^2}$.

If the covariant tensor field c_{lmn} is independent of y^i, (3.27) shows that the covariant tensor field d_{lmn} is also independent of y^i .

Conversely, if the tensor d_{lmn} is independent of y^i, we hav $(\dot{\partial}_j c_{lmn})H = O$, in view of theorem 2.2, the equation $(\dot{\partial}_j c_{lmn})\, H = 0$ implies $\dot{\partial}_j c_{lmn} = 0$, i.e. the covariant tensor field c_{lmn} is also independent of y^i .
Thus, we may conclude

Theorem 3.5. In GR^h-TR- affinely connected space, the covariant tensor field d_{lmn} is independent of the directional argument y^i if the covariant tensor field c_{lmn} is independent of the directional argument y^i if and only if (3.14) holds good [provided (2.5) holds good].

4. Conclusion

(4.1) The $generalized\ R^h$ - trirecurrent space is called generalized R^h- trirecurrent affinely connected if it satisfies any one of the conditions (3.1a), (3.1b), (3.2a) and (3.2b).

(4.2) In generalized R^h- recurrent affinely connected space, the generalized R^h- birecurrent affinely connected space is G $R^h - T\ R -$ affinely connected space.

(4.3) In R^h- trirecurrent affinely connected space, then Berwald curvature tensor $H^i_{jk\,h}$ and its associative curvature tensor $H_{jk\,h}$ generalized trirecurrent If and ony if (3.4) and (3.5) respectively holds good.

(4.4) In generalized R^h- trireurrent affinely connected space, the h – covariant drivative of third order h(v) – for the torsion tensor H^i_{kh} ,its associating tensor $H_{kj\,h}$ and the deviation tensor H^i_k , the curvature vector H_k , the curvature scalar H and the tensor $H_{kp.h}$ are given by (3.7) and (3.11) if and only if (3.8), (3.10) and (3.12) respectively hold

(4.5) In GR^h- TR-affinely connected space the H – Ricci tensor H_{jk} , the curvature vector H_k and the scalar curvature H are non – vanishing if and ony if (3.14) ,(3.17) and (3.19) respectively hold .

(4.6) In GR^h- TR-affinely connected space, the covariat tensor field d_{lmn} is independent of the directional orgument y^i if the covariant tensor c_{lmn} is independent of the directional orgument y^i if and only if (3.14) holds good [provided (3.25) holds good].

5. Recommendations

Authors recommend the need for the continuing research and development in Finsler space due to its vital applying importance in other fields.

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