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Research Article

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Thoughts on the Teaching of Integrable Function Proof

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Abstract According to the characteristics of independent college students, this paper summarizes the integrable function proof methods, classifies the theorems and examples, and makes the students' proofs clearer, and cultivates their habit of finding problems and summing up experience.

Keywords integrable, amplitude, integral interval

Introduction

According to the characteristics of independent college students, the author will use the teaching methods such as mind map, knowledge structure diagram and example analysis as much as possible to help students master new knowledge efficiently, and explain the topic to achieve the purpose of the students.

The integrable function class teaches a lot of proofs of integrable functions. The author uses integrable criteria to draw inferences, summarize the proof ideas, and teach some proof of integrable functions in an easy-to-understand manner.

According to the content of the mathematical analysis textbook, we express the integrable criterion as:

A necessary and sufficient condition for a function f defined on a closed interval $[a,b]$ to be integrable on $[a,b]$: $\forall \varepsilon > 0, \exists \delta > 0,$ if $\|T\| < \delta,$ then $\sum_{i=1}^{n} w_i \Delta x_i < \varepsilon.$

among them $T = \{a = x_0, x_1, \cdots, x_{n-1}, x_n = b\}$ is a partition of the closed interval $[a,b]$, The largest of the lengths of the intervals of the partition T, denoted $\|T\|$, w_i denotes the oscillation of the function f on $[x_{i-1}, x_i], \Delta x_i = x_i - x_{i-1}$.

According to the integrable criterion, we investigate $\sum_{i=1}^{n} w_i \Delta x_i < \varepsilon$, you can get three proof ideas. According to these three proofs, the author classifies the theorem and the example to help students master the proof method.

Idea 1:

For any small positive number ε ,when $w_i < \frac{\varepsilon}{b-a}$,Have: $\sum_{i=1}^{n} w_i \Delta x_i < \frac{\varepsilon}{b-a} \sum_{i=1}^{n} \Delta x_i = \frac{\varepsilon}{b-a}(b-a) = \varepsilon$ 。

Proposition 1: If a continuous function f defined on a closed interval $[a,b]$, then f is integrable on that closed interval.

Proof. Since f is continuous on the closed interval $[a,b]$, it is uniformly continuous on $[a,b]$.For $\forall \varepsilon > 0, \exists \delta > 0,$ Any two points $x', x'' \in [a,b]$,as long as $|x' - x''| < \delta$ There will be

$$|f(x')-f(x'')|<\frac{\varepsilon}{b-a}$$

So if $\|T\|<\delta$, for $\forall\Delta_i\in T$, f satisfies:

$$w_i=M_i-m_i=\sup_{x',x''\in\Delta_i}|f(x')-f(x'')|\le\frac{\varepsilon}{b-a}$$

Resulting in

$$\sum_T w_i\Delta x_i\le\frac{\varepsilon}{b-a}\sum_T\Delta x_i=\varepsilon$$

From the integrable criterion, f is integrable on $[a,b]$.

Idea 2:

1、 By f being bounded on $[a,b]$, f is bounded on $[x_{i-1},x_i]$, so that the amplitude w_i is bounded.set $w_i<k$,for $\forall\varepsilon>0$,as long as $\|T\|<\frac{\varepsilon}{k}$, There is $\sum_T w_i\Delta x_i<\sum_T w_i\|T\|<k\cdot\frac{\varepsilon}{k}<\varepsilon$。

2、 f is Bounded on $[a,b]$,so that f is bounded on $[x_{i-1},x_i]$, and so that w_i is bounded on $[x_{i-1},x_i]$,set $w_i<k$,for $\forall\varepsilon>0$,as long as $\sum_{i=1}^{n}\Delta x_i$ is small enough, there is $\sum_{i=1}^{n}w_i\Delta x_i<k\sum_{i=1}^{n}\Delta x_i<\varepsilon$。

The second point in the idea 2 is generally combined with the idea 1 to prove the integrable function, which will be described in detail in the idea 3.

Proposition 2: If f is monotonouson $[a,b]$, then f is integrable on $[a,b]$.

Proof. If $f(a)=f(b)$, then f is a constant function, obviously integrable. Let f be an increasing function, and $f(a)<f(b)$.

For any partition T of interval $[a,b]$, since f is an increasing function, the amplitude of f between each interval Δ_i to which T belongs is:

$w_i=f(x_i)-f(x_{i-1})$,

Then there is

$$\sum_T w_i\Delta x_i\le\sum_{i=1}^{n}[f(x_i)-f(x_{i-1})]\|T\|=[f(b)-f(a)]\|T\|$$

It can be seen that for $\forall\varepsilon>0$, as long as $\|T\|<\frac{\varepsilon}{f(b)-f(a)}$, then there is $\sum_T w_i\Delta x_i<\varepsilon$, so f f is integrable on $[a,b]$.

The above is proved by the idea 1 and the idea 2 respectively. Many topics are combined with the idea 1 and the idea 2 to prove.

Idea 3: Idea 1 + Idea 2

$$\sum_T w_i\Delta x_i=\sum_{T'}w_i\Delta x_i+\sum_{T''}w_i\Delta x_i$$

1） If w_i is small enough on any intervals of the partition T' ,and $\sum_{T'}\Delta x_i$ is bounded $\Rightarrow\sum_{T'}w_i\Delta x_i$ is small enough

2） w_i is bounded on any intervals of the partition T'', let $\sum_{T''}\Delta x_i$ Small enough $\Rightarrow \sum_{T''} w_i\Delta x_i$ is small enough

Example 1: Let

$$f(x)=\begin{cases}0, x=0\\ \frac{1}{n}, \frac{1}{n+1}<x\le\frac{1}{n}, n=1,2,\cdots\end{cases}$$

Then f is integrable on the interval $[0,1]$

Proof. $\forall\varepsilon>0$,since $\lim\limits_{n\to\infty}\frac{1}{n}=0$,when n is sufficiently large , there is $\frac{1}{n}<\frac{\varepsilon}{2}$,This shows that f has only a finite number of discontinuities on the interval $\left[\frac{\varepsilon}{2},1\right]$, It can be seen that f is integrable on the interval $\left[\frac{\varepsilon}{2},1\right]$, and There is a certain partition T' for $\left[\frac{\varepsilon}{2},1\right]$ such that $\sum_{T'} w_i\Delta x_i<\frac{\varepsilon}{2}$, and then the interval $\left[0,\frac{\varepsilon}{2}\right]$ and T' are merged to become a partition T for $[0,1]$.Since the amplitude of f on the interval $\left[0,\frac{\varepsilon}{2}\right]$ is less than 1,write $w_0<1$, $\sum_{T} w_i\Delta x_i=w_0\cdot\frac{\varepsilon}{2}+\sum_{T'} w_i\Delta x_i<\frac{\varepsilon}{2}+\frac{\varepsilon}{2}=\varepsilon$ is obtained,and so f is integrable on the interval $[0,1]$.

Example 2: Prove that the Riemann function

$$f(x)=\begin{cases}\frac{1}{q}, x=\frac{p}{q}, & p \text{ and } q \text{ are mutual prime}, q>p\\ 0, x=0,1, & \text{and the irrational number in } (0,1)\end{cases}$$

is integrable on the interval $[0,1]$.

Analysis: The Riemann function is known to be continuous at $x=0,1$, and at all irrational points, and that all rational points in $(0,1)$ are discontinuous, and the intuitive idea that it is integrable on $[0,1]$ is as follows:

Draw a horizontal line $y=\frac{\varepsilon}{2}$ in the image of the Riemann function.

1、 Look at the points above the line, using Idea 1 we can get: Above this line there are only a finite number of points in the function image, and the independent variables corresponding to these points can be included in a limited number of interval belonging to the partition T .When $\|T\|$ is small enough, the total length between the limited interval can be any small.

Look at the points below the line, using Idea 2we can get:The amplitude of the function between the remaining intervals in T is not greater than $\frac{\varepsilon}{2}$ .Combine Idea 1 and Idea 2 to prove that: $\sum_{T} w_i\Delta x_i<\varepsilon$

The above analysis process can be presented in the mind map at the same time as the language expression, which is more conducive to students' understanding.

Proof. Let $\varepsilon > 0$, on $[0,1]$, make the rational point $\frac{p}{q}$ of $\frac{1}{q} > \frac{\varepsilon}{2}$ only a finite number, let them be $r_1, \cdots, r_k$, Now split $[0,1]$ for $T = \{\Delta_1, \Delta_2, \cdots \Delta_n\}$, making $\|T\| < \frac{\varepsilon}{2k}$. And divide all the intervals in T into two categories: $\{\Delta_i' | i = 1,2,\cdots,m\}$ and $\{\Delta_i'' | i = 1,2,\cdots,n-m\}$. Where $\{\Delta_i'\}$ is all intervals containing points in $\{r_i | i = 1,2,\cdots,k\}$. The number of such intervals is $m \le 2k$ (when all r_i happens to be the division point of T, there is $m = 2k$); and $\{\Delta_i''\}$ is the interval between all the remaining points in T without $\{r_i\}$.

Since the amplitude of f on Δ_i' satisfies $w_i' \le \frac{1}{2}$, then there is

$$\sum_{i=1}^{m} w_i' \Delta x_i' \le \frac{1}{2} \sum_{i=1}^{m} \Delta x_i' \le \frac{1}{2} \cdot 2k \|T\| < \frac{\varepsilon}{2}$$

and the amplitude of f on Δ_i'' satisfies $w_i'' \le \frac{\varepsilon}{2}$, so there is

$$\sum_{i=1}^{n-m} w_i'' \Delta x_i'' \le \frac{\varepsilon}{2} \sum_{i=1}^{n-m} \Delta x_i'' < \frac{\varepsilon}{2},$$

and the two parts are combined to prove

$$\sum_{i=1}^{n} w_i \Delta x_i \le \sum_{i=1}^{m} w_i' \Delta x_i' + \sum_{i=1}^{n-m} w_i'' \Delta x_i'' < \varepsilon,$$

that is, f is integrable on $[0,1]$.

To sum up:

The proof of the integrable function mentioned in this paper is given in the textbook. The author will prove that some functions are integrable methods, and combine the characteristics of independent college students to visualize and organize knowledge points. To help students sort out the proof of ideas, it is beneficial to improve students' interest in learning mathematics analysis and enhance learning effects.

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