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Research Article

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Model of Optimum Time of Product Delivery in Conditions of Uncertainty

Maryna Iurchenko

Ph. D. in Physics and Mathematics, Department of Information Systems in Economics, Chernihiv National University of Technology, Chernihiv, Ukraine

Abstract Enterprises spend significant sums on the maintenance of stocks; therefore, the effective management of them is of strategic importance. In this paper, the problem of stochastic modeling of the enterprise commodity stocks management taking into account probabilistic character of time of the order is considered. The mathematical model, presented in the article, makes it possible to determine the optimal delivery time for a new shipment of a certain quantity of products with known demand and known numerical characteristics of the random value of the delivery time, taking into account the minimization of the cost function. Unlike traditional models, we managed to consider the stochastic nature of the delivery time of products in conditions of probabilistic nature of the order execution time. The implementation of this model under specific conditions is considered; the results are illustrated in the form of tables and graphs.

Keywords Optimization model, stochastic parameters, cost management, mathematical model, time of product supply

Introduction

In a context of increased competition on consumer markets, enterprise sustainability is ensured through the implementation of strategies aimed at maximal satisfaction of consumer demand (not only on goods, but also on services associated with these goods), as well as at reducing all types of trading costs. The formation of such strategies determines the need for improvement of various systems of the enterprise, including the system of inventory management. The improvement should be on the basis of rationalization and optimization of commodity flows and should use mathematical methods and models adequate to the formulation of these tasks.

The task of inventory management is one of the most important components of the logistics system today. The importance of such problems has led to the development and improvement of special mathematical methods, which include the theory of mass service. It should be noted that now much attention is paid to the study of a special class of models that are not considered in the analysis of mass service systems, but significantly affect the performance of the system as a whole. The distinctive feature of these models is considering the process of stock realization as a random process. At present, most of the practical problems are solved taking into account the deterministic nature of the input parameters, but in fact, they have to deal with situations where the input parameters are stochastic in nature and the use of classical methods is difficult due to the difficulty of obtaining analytic relations. For this reason, the development and improvement of the mathematical apparatus of stochastic optimization modeling may help to avoid cumbersome calculations and adapt the model under consideration to practical needs.

The theory of inventory management is an important area of managerial activity in many enterprises in different fields, both of the production of goods and of the provision of services. These issues are particularly relevant for rational and efficient organization of optimizing the level of stocks and their use, reducing their level, as well as minimizing the working capital invested in these stocks.

As shown in [1], modeling the process of inventory management involves taking into account the influence of a large number of system parameters and the choice of one of many options for setting the problem of inventory management. Note that the classification of these variants, in spite of numerous works, is realized only partially [2,3]. Some authors use iterative schemes [4], methods of approximation [4,5], and detailed schemes, algorithms or programs are not given in the research . A detailed analysis of literature has shown that the authors focus on simple problems with a large number of initial assumptions or to a more abstract formulation, neglecting accurate mathematical proofs. However, the solution of such problems is associated with significant computational and mathematical difficulties, and such methods are not sufficiently investigated and the optimal solution in the studied models is obtained in no more than one third of the cases.

Despite attempts to create a generalized model, the inventory management schemes, that are currently investigated in the literature, can be applied to a small class of tasks that arise during practical activities. Often, enterprise management is forced to adapt existing models to specific operating conditions.

As shown by the analysis of literary sources, the choice of mathematical apparatus is significantly influenced by various characteristics of demand, which may either be deterministic or probabilistic.

In turn, the probabilistic demand may be stationary, in which the function of the probability density of demand does not change with time, or non-stationary, in which the function of the probability density of demand varies.

In [5, 6], it is shown that the main features of the classification of inventory management models are demand, inventory control parameters, constraints, and management strategies. If at least one of the listed parameters is random, the model is stochastic, otherwise it is deterministic.

To date, there are two basic models of inventory management: a fixed-volume model (Q-model) and a fixed order period model (P-model). However, these models are characterized by a large complexity of use. In particular, Q-model involves constant control of the balance of stocks.

In practice, there is often uncertainty due to inaccuracy or incompleteness of information on demand, supply and other parameters of the logistics system. In many cases, the influence of random factors is crucial, so when constructing a mathematical model, random factors must be taken into account, in contrast to the deterministic case. For solving problems with uncertainty, methods of set theory and simulation modeling are proposed by authors [5,6]. The author of the paper [5] attempted to use the method of interval analysis in systems with incomplete information on demand, as well as presented the numerical results. It should be noted that the distinguishing feature is the fact that the authors mentioned above search for the solution mainly by methods of numerical analysis. Moreover, in the majority of the statements for the description of random characteristics, the Poisson process is used.

As shown in [6], uncertain factors can in turn be divided into stochastic and complete ones. Stochastic uncertainty can be represented by a random variable or a random process, and full uncertainty is not considered as a random object in principle. The type of uncertainty determines the choice of a mathematical model.

The purpose of this research is to construct a stochastic model and to find an analytical expression that allows determining the optimal time of delivery of products taking into account the probabilistic nature of the time of execution of the order.

Materials and Methods

At present, most of the practical tasks in this direction are solved taking into account the deterministic nature of the input parameters. Many authors use methods of mathematical modeling, theory of mass maintenance, optimal control for solving this class of problems, but the final results have not been widely used due to the complexity of the interpretation of the obtained analytical relations. In practice, one has to deal with situations where some of the parameters are stochastic and the use of classical methods is complicated due to the difficulties of computational nature. Consequently, the further development of optimization stochastic models of inventory management and obtaining the analytical solution of this class of problems will increase the efficiency of managerial decisions in practice.

In this paper, methods of mathematical analysis, financial analysis, optimization theory, mathematical programming, probability theory, methods of optimization and simulation modeling are used to construct a stochastic model with random time of order execution and deterministic demand. The determinism of demand is explained by the law of large numbers.

Main Results

Consider a stochastic model with uncertainty over the order execution time t, which is the main factor in the occurrence of expenses for inventory management. The task is to reduce the effect of uncertainty in t on the execution of a new order significantly. Assume that the demand P in this model is deterministic, which is largely due to the effect of the law of large numbers. We also assume the moment β of running out of stock of the product of volume V is known from statistical data.While constructing the model, we assume that there are some non-zero probabilistic delays or premature receipt of the ordered product.

Let$y$be the real moment of arrival of goods, so

$$y = t_0 + \delta t, \tag{1}$$

where t_0 is the expected moment of arrival and δt is a random variable that characterizes the deviation of the real moment from the expected one.– a random variable characterizing the moment of deviation of the actual delivery time from what was planned. Assume that the random variable δt has a normal distribution with mathematical expectation $M = 0$ and the mean square deviation $\sigma > 0$. Accordingly, the time of the actual arrival of goods y also has a normal distribution. This assumption is a prerequisite for conducting analytical studies of the model and, by virtue of the features of the law of normal distribution, reflects most of the practical situations.

We assume that the cost function Z takes into account the storage costs considered as lost profit. The cost of storing goods in volume V after delivery until the actual moment β of running out of the goods in the case when the delivery of the goods occurred before the scheduled time y $(y < \beta)$ is:

$$C = \gamma V(\beta - y), \tag{2}$$

where $\gamma = const$ is the cost of storing a unit of product per day.

In case of incomplete demand satisfaction$y > \beta$, the deficit of the goods arises at the time interval from the moment of the actual completion of the goods β to the moment of delivery in volume V:

$$S = \frac{V}{\beta}\theta(y - \beta), \tag{3}$$

where $\theta = const$isthe profit derived from the sale of a unit of goods and the coefficient $\frac{V}{\beta}$ characterizes the volume of goods sold during the day.

Taking into account (2) and (3), the total cost can be represented as follows:

$$C + S = \begin{cases} V\gamma(\beta - y), & \beta > y \\ \frac{V}{\beta}\theta(y - \beta), & y > \beta' \end{cases} \tag{4}$$

As it is shown in [6], it is convenient to use a mathematical expectation as a reliable characteristics of total expenses. In the model under consideration it has the following form:

$$F(t_0) = \int_{-\infty}^{\beta - t_0} \gamma V(\beta - t_0 - \delta t)\mu(\delta t)d(\delta t) + \int_{\beta - t_0}^{\infty} \frac{V}{\beta}\theta(t_0 + \delta t - \beta)\mu(\delta t)d(\delta t), \tag{5}$$

where $\mu(\delta t)$ is the distribution density of a continuous random variable δt, which characterizes the deviation from the delivery period.

The presented problem in the M-formulation reduces to the finding of the moment of the delivery t_0, in which the mathematical expectation of total costs will be minimal, that is:

$$F(t_0) \to min. \tag{6}$$

Transforming the first term of expression (5), we obtain

$$F_1(t_0) = \int_{-\infty}^{\beta - t_0} \gamma V(\beta - t_0 - \delta t)\mu(\delta t)d(\delta t) = \tag{7}$$

$$= \int_{-\infty}^{\beta - t_0} \gamma V \beta \mu(\delta t) d(\delta t) - \int_{-\infty}^{\beta - t_0} \gamma V t_0 \mu(\delta t) d(\delta t) - \int_{-\infty}^{\beta - t_0} \gamma V \delta t \mu(\delta t) d(\delta t)$$

$$= \gamma V \beta \int_{-\infty}^{\beta - t_0} \mu(\delta t) d(\delta t) - \gamma V t_0 \int_{-\infty}^{\beta - t_0} \mu(\delta t) d(\delta t) - \gamma V \int_{-\infty}^{\beta - t_0} \delta t \mu(\delta t) d(\delta t)$$

$$F_1(t_0) = \gamma V \beta \Phi\left(\frac{\beta - t_0}{\sigma}\right) - \gamma V t_0 \Phi\left(\frac{\beta - t_0}{\sigma}\right) - \gamma V \int_{-\infty}^{\beta - t_0} \delta t \mu(\delta t) d(\delta t), \tag{8}$$

where

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{z^2}{2}} dz$$

is a distribution function of a standard normal random variable.
Integrating by parts the third term of the expression (7), we get

$$\gamma V \int_{-\infty}^{\beta - t_0} \delta t \mu(\delta t) d(\delta t) =$$

$$= \gamma V \left[\delta t \int_{-\infty}^{\beta - t_0} \mu(\delta t) d(\delta t) - \int_{-\infty}^{\beta - t_0} \left(\int \mu(\delta t) d(\delta t) \right) d(\delta t) \right] = \tag{9}$$

$$= \gamma V \left(\frac{\beta - t_0}{\sigma}\right) \cdot \Phi\left(\frac{\beta - t_0}{\sigma}\right) - \gamma V \int_{-\infty}^{\beta - t_0} \Phi(\delta t) d(\delta t)$$

Taking into account (9), let us present the expression (8) in the following form:

$$F_1(t_0) = \gamma V \beta \Phi\left(\frac{\beta - t_0}{\sigma}\right) - \gamma V t_0 \Phi\left(\frac{\beta - t_0}{\sigma}\right) - \gamma V \left(\frac{\beta - t_0}{\sigma}\right) \Phi\left(\frac{\beta - t_0}{\sigma}\right) + + \gamma V \int_{-\infty}^{\beta - t_0} \Phi(\delta t) d(\delta t) \tag{10}$$

Similarly, transforming the second term of (5), we obtain:

$$F_2(t_0) = \frac{V}{\beta} \theta t_0 \left(1 - \Phi\left(\frac{\beta - t_0}{\sigma}\right)\right) + \frac{V}{\beta} \theta \int_{\beta - t_0}^{\infty} \delta t \mu(\delta t) d(\delta t) - - \frac{V}{\beta} \theta \left(1 - \Phi\left(\frac{\beta - t_0}{\sigma}\right)\right) \tag{11}$$

Transforming the integral

$$\int_{\beta - t_0}^{\infty} \frac{V}{\beta} \theta \delta t \mu(\delta t) d(\delta t),$$

we obtain

$$\int_{\beta - t_0}^{\infty} \frac{V}{\beta} \theta \delta t \mu(\delta t) d(\delta t) = \frac{V}{\beta} \theta \left(1 - \Phi\left(\frac{\beta - t_0}{\sigma}\right)\right) - \int_{\beta - t_0}^{\infty} \Phi(\delta t) d(\delta t). \tag{12}$$

Hence, taking into account (12), the expression (5) can be written as follows:

$$F(t_0) = F_1(t_0) + F_2(t_0) =$$
$$= \gamma V \beta \Phi\left(\frac{\alpha - t_0}{\sigma}\right) - \gamma V t_0 \Phi\left(\frac{\beta - t_0}{\sigma}\right) - \gamma V \left(\frac{\beta - t_0}{\sigma}\right) \Phi\left(\frac{\beta - t_0}{\sigma}\right) +$$
$$+ \gamma V \int_{-\infty}^{\beta - t_0} \Phi(\delta t) d\delta t + \frac{V}{\beta} \theta t_0 \left(1 - \Phi\left(\frac{\beta - t_0}{\sigma}\right)\right) +$$
$$+ \frac{V}{\beta}\theta \left[\left(1 - \left(\frac{\beta - t_0}{\sigma}\right)\Phi\left(\frac{\beta - t_0}{\sigma}\right)\right) - \int_{\beta - t_0}^{\infty} \Phi(\delta t) d\delta t\right] - \frac{V}{\beta}\theta\beta\left(1 - \Phi\left(\frac{\beta - t_0}{\sigma}\right)\right)$$

or

$$F(t_0) = \gamma V \int_{-\infty}^{\beta - t_0} \Phi(\delta t) d(\delta t) + \frac{V}{\beta}\theta t_0 + \frac{V}{\beta}\theta - \frac{V}{\beta}\theta \int_{\beta - t_0}^{\infty} \Phi(\delta t) d(\delta t). \tag{13}$$

Finding the derivative

$$F'(t_0) = \gamma V \left(-\Phi\left(\frac{\beta - t_0}{\sigma}\right)\right) + \frac{V}{\beta}\theta + \frac{V}{\beta}\theta\left(-\Phi\left(\frac{\beta - t_0}{\sigma}\right)\right)$$

and equating it to zero, we obtain the moment of time at which the minimum condition for (6) is fulfilled:

$$t_0 = \beta - \sigma \Phi^{-1}\left(\frac{\theta}{\gamma\beta + \theta}\right), \tag{14}$$

where $\Phi(x)$ is a standard normal distribution function, β is the moment of running out of goods, γ is a cost of storage of a unit of the product, θ is a unit sales price.

From formula (14) it follows that the optimal moment $t_0$is determined by the parameter β, and also depends on the parameters of the normal distribution of the random variable δt, which determine the deviation from the intended delivery period of the new consignment. Table 1 calculates $t_0$for various γ for the values $\theta = 1000$ and $\theta = 500$ units, $\beta = 10$ days, $M = 0, \sigma = 1$.

Table 1: Dependence of the optimal delivery time on the cost of storage

γ	5	10	15	20	25	30	35	40	45	50
t_0	8.332	8.665	8.876	9.032	9.158	9.264	9.354	9.434	9.505	9.569

Accordingly, Figure 1 provides a graphical dependence.

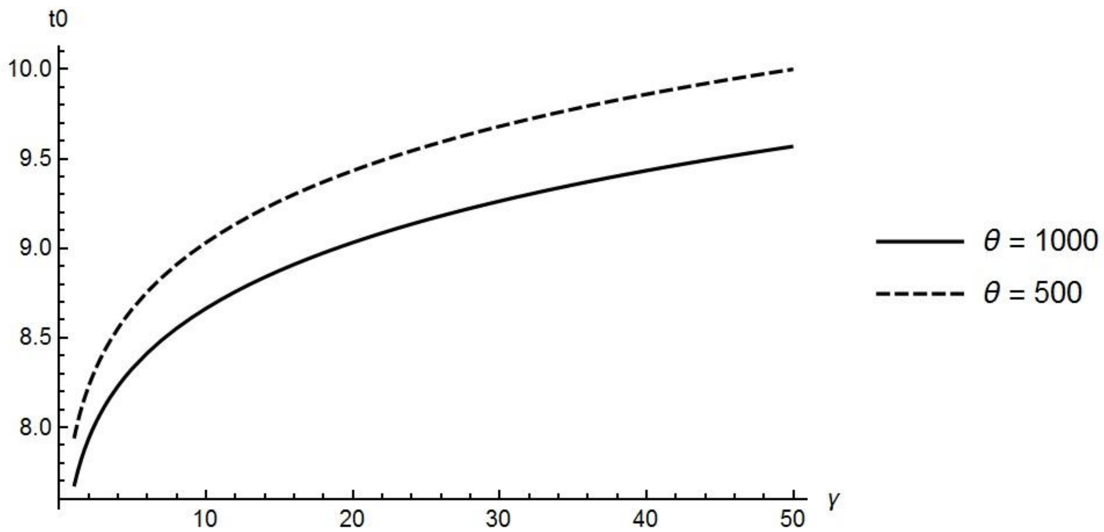

Figure 1: Dependence of the optimal delivery time on the unit storage cost

Conclusions and Perspectives of Further Research

The presented mathematical model allows to determine the day of delivery of a new batch of a certain quantity of a product under known demand and the known numerical characteristics of the random value of delivery time. The obtained analytical expression for optimal time allows to minimize the expected total costs for storage of goods. Unlike traditional models, we managed to take into account the stochastic nature of the time of delivery of products in terms of the probabilistic nature of the time of execution of the order.

The task of this kind is of practical importance. Its solution allows to increase the efficiency of managerial decisions when performing the management function of enterprises related to any economic branches.

References

[1]. Brodetskiy, G.L. (2004). Metody stokhasticheskoy optimizatsii [Methods of stochastic optimization]. Moscow: REA [in Russian].

[2]. Kopytov, E., Greenglaz, L., Muravyov, L., Puzinkevich, E. (2007). Modelling of two strategies in inventory control system with random lead time and demand. Computer Modelling and New Technologies, 1, 21-30 [in English].

[3]. Schreibfeder, D. (2006). Effektivnoe upravlenie zapasami [Effective inventory management]. Moscow: Alpina Business [in Russian].

[4]. Aggoun, L. (2001). On a stochastic inventory model with deteriorating items. IJMMS, 25(3), 197-203 [in English].

[5]. Chang, C.-T., Ouyang, L.-Y., Teng, J.-T., Cheng, M.-C. (2010). Optimal ordering policies for deteriorating items using a discounted cash-flow analysis when a trade credit is limited to order quantity. Computers & Industrial Engineering, 59, 770-777 [in English].

[6]. Hung, K.C. (2011). Continuous review inventory models under time value of money and crashable lead time consideration. Yugoslav Journal of Operations Research, 21(2), 293-306 [in English].