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***Journal of Scientific and Engineering Research*, 2018, 5(9):133-136**

Research Article

ISSN: 2394-2630
CODEN(USA): JSERBR

On Reflection of Conic Sections According to Basic Functions

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Abstract In this paper, the symmetrical situation is of conic sections given according to a simple function as the axis of reflection $y = g(x)$ is studied. Some kinds of reflections are explored. If simple functions are one-to-one, how are the conical sections reflected when in this case?
The case in which the domain of given conic section $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ is reflecting according to the axis $y = g(x)$ is discussed in detail. The reflection of the conic section relative to $y = x$ is $Ay^2 + Bxy + Cx^2 + Dy + Ex + F = 0$ is a well-known. The reflection of some conic sections relative to simple function $y = g(x)$ is defined as $A\left(k\left(y\right)\right)^2 + B\left(k\left(y\right)\right)g\left(x\right) + C\left(g\left(x\right)\right)^2 + Dk\left(y\right) + E\left(g\left(x\right)\right) + F = 0$ in [1].

Keywords line function, exponential function, simple function, reflection, symmetric function, Implicit function.

1. Introduction

Let g: ℝ→ℝ, $y = g(x)$ be a function. Then the equation from $y = g(x)$ the axis of reflection is as $x = k(y)$. From these two equations we get the function $y = h(x)$ obtained from the two equations $y = g(x)$ and $y = f(x)$ is called the reflected function. That is,

$$y = f(x) \xLeftrightarrow{y=g(x)\Leftrightarrow x=k(y)} g\left(x\right) = f(k(y)) \Leftrightarrow y = h\left(x\right).$$

In [1], some specific reflection properties were also given.

So far we are dealing with explicitly given functions $y = f\left(x\right)$. But frequently the dependence of endogenous variable y on exogenous variable x can be given in a form $F\left(x, y\right) = k$. If for x this equation determines a corresponding value of y, we say that the endogenous variable y is an *implicit function* of exogenous variable x. The general conic equation is a special implicit function.

The symmetry of $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$, whit respect to the reflecting function $y = x$ is given below;

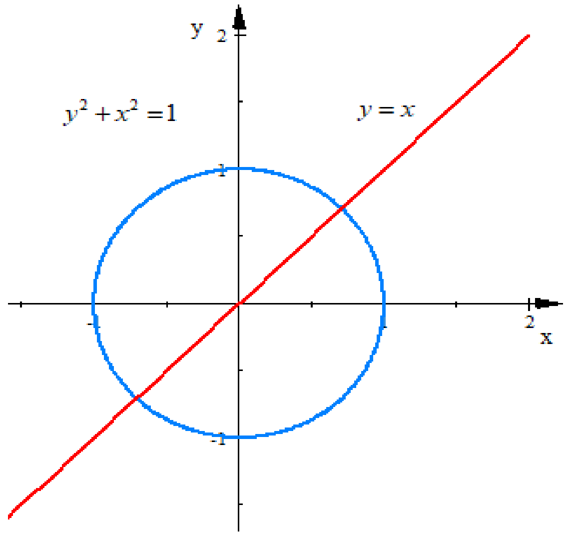
$$Ay^2 + Bxy + Cx^2 + Dy + Ex + F = 0.$$

Example 1. We can describe this circle with the relation

$$x^2 + y^2 = 1$$

that is, the circle of radius 1 centered at the origin is the set of all points $\left(x, y\right)$ such

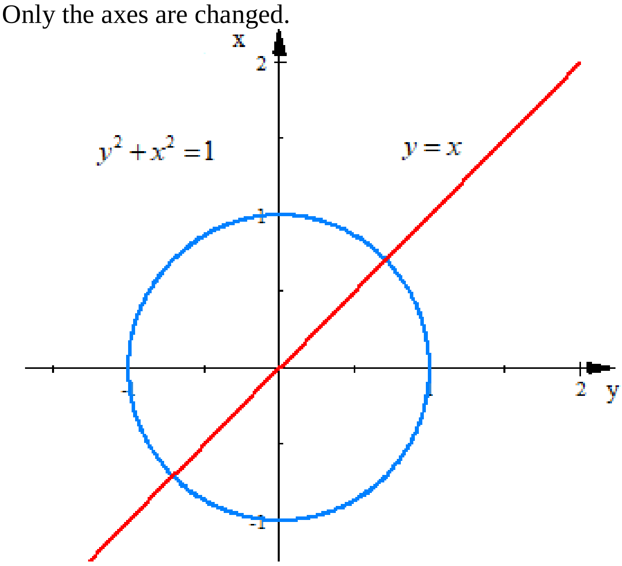
$$x^2 + y^2 = 1$$



The reflecting function according to axis of reflection $y=x$ is given below;

$$y^2+x^2=1.$$

This is the same equation. Only the axes are changed.



2. Reflections and Images of Conic Sections

Let g: ℝ→ℝ, be the axis of reflection $y=g(x)$ function. Then it is obtained

$$A(k(y))^2+B(k(y))g(x)+C(g(x))^2+Dk(y)+E(g(x))+F=0 \text{ [2].}$$

For some simple functions with the following lemma, symmetry axis and reflected states are given.

Lemma . Let $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$. Then,

i. For $p\neq 0, m\neq 0$, a conic section reflected according to a axis of reflection $g(x)=mx+n$ is

$$A\left(\frac{y-n}{m}\right)^2+B\left(\frac{y-n}{m}\right)(mx+n)+C(mx+n)^2+D\left(\frac{y-n}{m}\right)+E(mx+n)+F=0.$$

ii. For $a\in(0,1)\cup(1,+\infty)$, a conic section reflected according to a axis of reflection $g(x)=a^x$ is

$$A(\log_a y)^2+B(\log_a y)a^x+C(a^x)^2+D(\log_a y)+Ea^x+F=0.$$

iii. A conic section reflected according to axis of reflection $g(x)=\sin x$ is

$$A\left(\sin^{-1} y\right)^2 + B\left(\sin^{-1} y\right)\left(\sin x\right) + C\left(\sin^{-1} y\right)^2 + D\left(\sin^{-1} y\right) + E\left(\sin x\right) + F = 0.$$

iv. For $a \in (0,1) \cup (1,+\infty)$, a conic section reflected according to a axis of reflection $g(x) = \log_a x$ is

$$A\left(a^y\right)^2 + B\left(a^y\right)\left(\log_a x\right) + C\left(\log_a x\right)^2 + D\left(a^y\right) + E\left(\log_a x\right) + F = 0.$$

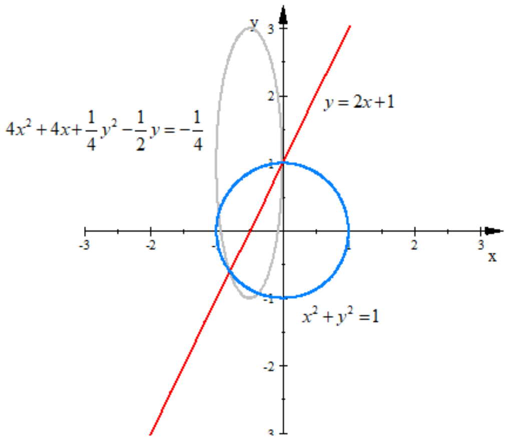
Proof. The proofs are clear.

We give a few examples below.

Example 2. Let us consider the circle

$$x^2 + y^2 = 1$$

that is, the circle of radius 1 centered at the origin is the set of all points (***x, y***) such that $x^2 + y^2 = 1$. Then, according to the axis of reflection $y = 2x + 1$ according the reflecting function is given below;

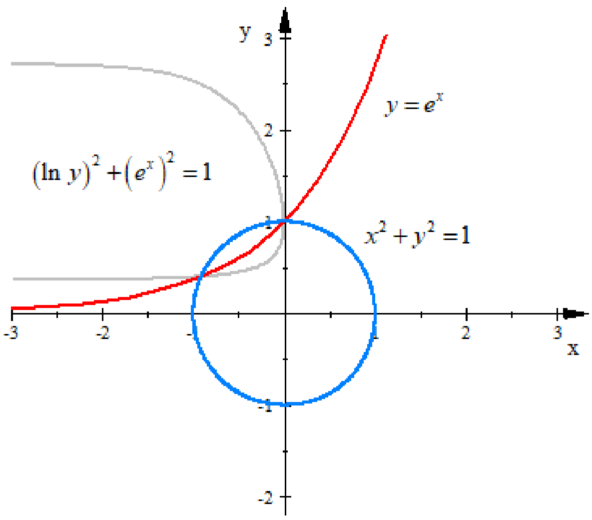


$$\left(\frac{y-1}{2}\right)^2 + (2x+1)^2 = 1 \Leftrightarrow 4x^2 + 4x + \frac{1}{4}y^2 - \frac{1}{2}y = -\frac{1}{4}.$$

Example 3. Again, Let

$$x^2 + y^2 = 1$$

Then, according to the axis of reflection $y = e^x$ the reflecting function is given below;



$$(\ln y)^2 + (e^x)^2 = 1.$$

Theorem . Let g: ℝ→ℝ be basic function with one to one, onto. Then, the reflection of the general conic section according to this axis is

$$A\left(k\left(y\right)\right)^{2}+B\left(k\left(y\right)\right)g\left(x\right)+C\left(g\left(x\right)\right)^{2}+Dk\left(y\right)+E\left(g\left(x\right)\right)+F=0.$$

Proof . Since g is one to one ,onto

$$y=g\left(x\right)\Leftrightarrow x=g^{-1}\left(y\right)$$

If the above equation is used in $Ax^2+Bxy+Cy^2+Dx+Ey+F=0$ then required equation is obtained.

3. Results and Discussions

According to a simple function, the reflection of a conic section may not be a conic section. While the conic sections have two unknown equations with the second order, their reflections sometimes turn into a closed function. It is a hard problem to know that which function to which axis of reflection gives a conic section. The focal points and the main axes of the conic sections can be calculated.

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