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Research Article

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On the Symmetric Conditions of Simple Functions

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Abstract In this study, the symmetrical state is of simple function given according a simple function $y=g(x)$ the axis of reflection. Some kind of reflections are explored. The special conditions under which the reflecting function is one-to-one are studied. If the reflecting function is one-to-one, it is determined how the results will emerge. We know simple functions as;

$f(x)=px+q, g(x)=a^x, h(x)=x^n, \ldots$ where $p,q,n\in\mathbb{R}, a\in(0,1)\cup(1,+\infty)$

Are explained by using graphical examples of their reflections

The case in which the domain of given simple function $y=f(x)$ is reflecting according to the axis reflection $y=g(x)$ is discussed in detail. If $y=f(x)$ is the reflecting function and $y=g(x)$ is the axis of reflection, the n the reflected function $y=h(x)$ is given as follows.

Keywords line function, exponential function, simple function, reflection, symmetric function.

1. Introduction

The $y=f(x)$ function, which is one-to-one and overlapped, can be projected in relation to the $y=x$ axis of reflection, which is also a one-to-one and overlapping, to obtain the inverse function. By extending this known process, the situation has been generalized by working with known simple functions.

A relation β between two sets $\varnothing\neq X$ and $\varnothing\neq Y$ is simply a subset of the Cartesian product $X\times Y$, i.e., a collection of ordered pairs (x,y).

$\forall\beta\subseteq X\times Y=\{(x,y)|x\in X, y\in Y\}$.

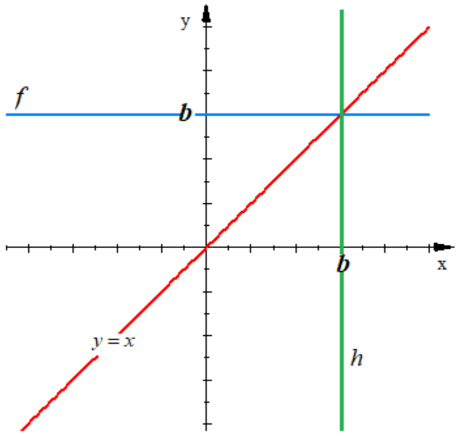
Let X and Y be sets. A function $\beta: X\to Y$ is a special kind of relation between X and Y. Namely, it is a relation $\beta\subseteq X\times Y$ satisfying the following conditions:

i. For $\forall(x,y_1),(x,y_2)\in\beta$, $y_1=y_2$.

ii. For $\forall x\in X$, $\underbrace{\exists!}_{\text{only one}}$ $y\in Y: (x,y)\in\beta$.

Example 1. Let us find the symmetry of b for a fixed real number, $f(x)=b$ reflecting function to $y=x$ -axis of reflection according the reflecting function is given below;

$$f=\{(x,b)|x\in\mathbb{R}\}\subseteq\mathbb{R}^2.$$



$$h=\{(b,y) \mid y\in \mathbb{R}\} \subseteq \mathbb{R}^2.$$

f is a function from $\mathbb{R}$ to $\mathbb{R}$. However, h is not the function.

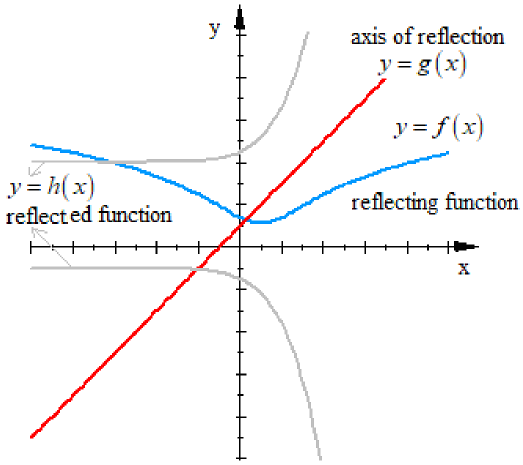
In the next study the reflection was taken axially, other simple functions are taken.

2. Reflections and Images of Simple Functions

Let $f,g:\mathbb{R}\to\mathbb{R}$, $y=f(x)$ $y=g(x)$ be two functions and equation from $y=g(x)$ axis of reflection is obtained the equation $x=k(y)$. From the two equations $y=h(x)$

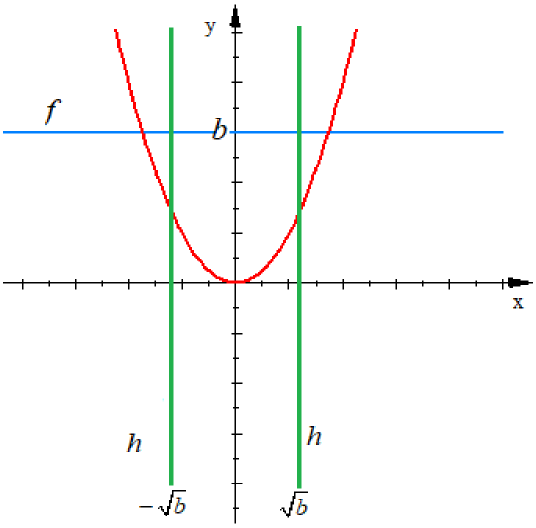
The function $y=h(x)$ obtained from two equations is called the function which is reflected by function $y=g(x)$ and $y=f(x)$ .Then,

$$y=f(x) \overset{y=g(x)\Leftrightarrow x=k(y)}{<\!\!=\!=\!=\!=\!=\!=\!=\!\!>} g(x)=f(k(y)) \Leftrightarrow y=h(x).$$



Let $A(a,b)$ be point and $g=\{(x,g(x)) \mid x\in\mathbb{R}\}\subseteq\mathbb{R}^2$ axis of reflection .The reflection of $A(a,b)$ is $A'(k(b),g(a))$ reflected point.

Example 1. Let us find the symmetry of b for a fixed real number, $f(x)=b$ reflecting to $y=x^2$ -axis of reflection according the reflecting function is given below.

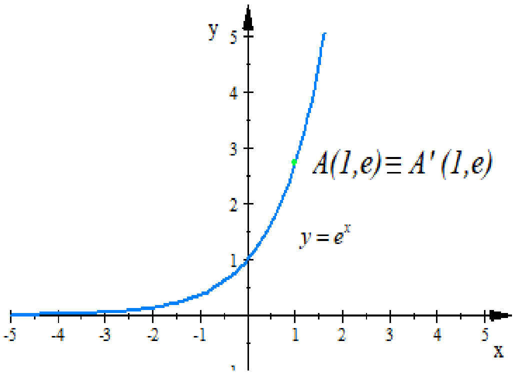


$$h=\left\{\left(\pm\sqrt{b},y\right)\middle|\,y\in\mathbb{R}\right\}\subseteq\mathbb{R}^2$$

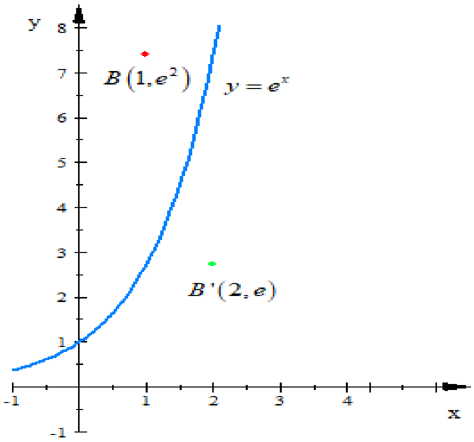
it is not unique .reflected cannot be the function although reflecting is a function

Example 2. Let $A(1,e)$ be point and $g=\left\{\left(x,e^x\right)\middle|\,x\in\mathbb{R}\right\}\subseteq\mathbb{R}^2$ axis of reflection. Then

The reflected point is $A'(1,e)$ according to $y=e^x$. The two points are the same.



If the reflection of point $B\left(1,e^2\right)$ is $B'(2,e)$ according to $y=e^x$. Then ,its image;



$$y=e^x \overset{B\left(1,e^2\right)}{<\!\!==\!\!>} B'(2,e).$$

For some of the simple functions with the following lemma, symmetry axis and reflected states are given.

Lemma . Let $f, g, h : \mathbb{R} \to \mathbb{R}$ be functions defined as follows. Then,

i. For $p \neq 0, m \neq 0$, the function of $f(x) = px + q$ is reflected according to $g(x) = mx + n$ axis of reflection;

$$h(x) = \frac{m^2}{p}x + \frac{mn - mq + np}{p}.$$

ii. For $p \neq 0, a \in (0,1) \cup (1,+\infty)$, the function of $f(x) = px + q$ is reflected according to $g(x) = a^x$ $g(x) = mx + n$ axis of reflection;

$$h(x) = a^{\frac{a^x - q}{p}}.$$

iii. For $p \neq 0$, the function of $f(x) = px + q$ is reflected according to $g(x) = \sin x$ axis of reflection;

$$h(x) = \sin\left(\frac{\sin x - q}{p}\right).$$

iv. For $p \neq 0, a \in (0,1) \cup (1,+\infty)$, the function of $f(x) = px + q$ is reflected according to $g(x) = \log_a x$ axis of reflection;

$$h(x) = \log_a\left(\frac{\log_a x - q}{p}\right).$$

Proof

i. There reflection state of a line equation according to a line;

If $y = px + q$ and $y = mx + n$ equations of two line then

$$x = \frac{y-n}{m},\ mx + n = p\left(\frac{y-n}{m}\right) + q$$

$$mx + n - q = \frac{p}{m}(y - n)$$

$$\frac{m^2 x + mn - mq}{p} = y - n$$

$$y = h(x) = \frac{m^2}{a}x + \frac{mn - mq + np}{a}.$$

ii. The reflection state of a line equation according to exponential function;

If a equation of line $y = px + q$ and $y = a^x$ equation of exponential then

$$x = \log_a y$$

$$a^x = p(\log_a y) + q \Rightarrow \frac{a^x - q}{p} = \log_a y \Rightarrow h(x) = a^{\frac{a^x - q}{p}}.$$

iii. The reflection state of a line function according to the sine function.

If a equation of line $y = px + q$ and equation trigonometric $y = \sin x$ then,

$$x = \sin^{-1} y$$

$$\sin x = p(\sin^{-1} y) + q$$

$$\frac{\sin x - q}{p} = \sin y \Rightarrow y = \sin\left(\frac{\sin x - q}{p}\right).$$

iv. The reflection state of a line function according to the equation of logarithmic.

If a equation of line $y = px + q$ and logarithmic $y = \log_a x$ then,

$$x = a^y,\ \log_a x = pa^y + q$$

$$(\log_a x) - q = pa^y$$

$$a^y = \frac{\log_a x - q}{p} \Rightarrow y = \log_a\left(\frac{\log_a x - q}{p}\right).$$

Proposition . Let $f, g, h : \mathbb{R} \to \mathbb{R}$ be basic functions and

i. If $f(x) = px + q$, for any $p \neq 0$. Then,

$$h(x) = g\left(\frac{g(x) - q}{p}\right).$$

ii. If $f(x) = a^x$ for any $a \in (0,1) \cup (1, +\infty)$. Then,

$$h(x) = g(\log_a g(x))$$

Proof.

This $y = g(x)$ axis of reflection is generated new function from basic functions.

i. If $f(x) = px + q$, $y = g(x)$ axis of reflection is generated new function as follows;

$$y = g(x) \Leftrightarrow x = g^{-1}(y)$$

$$g(x) = p(g^{-1}(y)) + q$$

$$\frac{g(x) - q}{p} = g^{-1}(y) \Leftrightarrow h(x) = g\left(\frac{g(x) - q}{p}\right).$$

ii. If $f(x) = a^x$, $y = g(x)$ axis of reflection is generated new function as follows;

$$y = g(x) \Leftrightarrow x = g^{-1}(y)$$

$$g(x) = a^{g^{-1}(y)}$$

$$\log_a g(x) = g^{-1}(y) \Leftrightarrow h(x) = g(\log_a g(x)).$$

Similar proposition can be given for other simple functions .In general, the following they are is given.

Theorem. Let $f, g : \mathbb{R} \to \mathbb{R}$ be basic functions with reversible properties. Then reflected function is $h : \mathbb{R} \to \mathbb{R}$ as follows;

$$h(x) = g\left(f^{-1}(g(x))\right).$$

Proof

If f, g functions that can be inverted then

$$y = g(x) \Leftrightarrow x = g^{-1}(y)$$

If the above equation is used in $y = f(x)$ thus,

$$g(x) = f\left(g^{-1}(y)\right) \Leftrightarrow h(x) = g\left(f^{-1}(g(x))\right).$$

3. Results and Discussions

Reflection of a function with respect to the axis of reflection appears to be not unique. If the reflection is a single double function, the reflected number changes. The following are open discussions.

i. If the function and the reflected state are known, the n can the reflection axis be found? Is this axis unique?
ii. On the contrary, can the function be calculated if the reflected function and reflection n axis are known? Is this function unique?
iii. What are the known simple functions, reflected by which axis of symmetry?

Further' more the combinatorial properties in this regard have been left to investigate.

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