

Heat Transfer and Fluid Flow from a Wavy Surface Subjected to a Variable Heat Flux in Porous Media

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Abstract In the present work we study the heat transfer along the wavy surface that's passed through porous media. Where the wavy surface is described heat flux proportional to $(1 + x)^m$ where (m is constant) the governing equation are first cast into non-dimensional form by using suitable boundary layer variables. The boundary layer equations then solved numerically by explicit finite difference method the numerical results are present for various values of exponent **m** , amplitude **a** , prandtl number **Pr** and Darcy number **Da** ..

Keywords Heat transfer, Heat flux, wavy surface and porous media

Notation

a Amplitude of the wavy surface
 K Wave number $= \frac{2\pi}{L}$
 k Thermal conductivity
 L Wave length
 m Wall heat flux exponent
 P Pressure
 Pr Prandtl number
 q Heat flux
 Re_l Reynolds number
 T Temperature
 u,v Dimensionless velocity components along (x,y)
 x,y Cartesian Coordinates
 Nu(x) local Nusselt number
 $Nu_{av}(x)$ Average Nusselt number
 Da Darcy number

Greek Symbols

α Thermal diffusivity
 θ Dimensional temperature
 ν Kinematics viscosity
 ρ Density
Superscript
 Dimensional quantity-
Subscript
 x differentiation with respect to x
 Free stream ∞
 Av Average value
 ω Wall surface

1. Introduction

The study and analysis of heat transfer in porous media has been subject of many investigations due to their frequent occurrence in industrial and technological applications .examples of some applications include geothermal reservoirs, drying of porous solids, thermal insulations, enhanced oil recovery, packed-bed catalytic reactors, and many others and wavy geometries are used in many engineering system as a means enhancing the transport performance therefore knowledge about flow and heat transfer through wavy surfaces becomes important in this context. Solar collectors, condensers in refrigerators, grain storage containers and industrial heat radiators .Surface with intentionally placed roughness elements are encountered in several heat transfer devices, such as flat plate solar collectors, and flat plate condensers in refrigerators. In addition. Roughened surfaces could be used in cooling of electrical and nuclear components where the wall heat flux is known. Shu and Pop [1] have studied laminar forced convection on along a semi-infinite horizontal flat plate. They obtained



the complete solution of forced convection thermal boundary layer on a flat plate by considering the case when a prescribed wall heat flux is given and is of form $\frac{\partial \theta}{\partial y} = -(1+x^2)^m$ Ghosh Moulic and Yao [2] studied mixed convection along a wavy surface and concluded that forced convection component of heat transfer contains two harmonics. B. Tashtoush and E. Abu-Irshaid [3] have studied the heat and fluid flow from a wavy surface subjected to a variable heat flux. Kim [4] has studied the convection of heat along a wavy surface vertical plate to non-Newtonian fluids. Ress. D.A.S, Pop [5] studied the free convection included by a vertical wavy surface with uniform heat flux in porous medium. S.V. Rahman and H.M. Badr [6] have studied the natural convection from a vertical wavy surfaces embedded in saturated porous media. A. Mahdy, R.A. Mohamed and F.M. Hady [7] have studied the influence of magnetic field on natural convection flow near a wavy cone in porous media. R.A. Mohamed, A. Mahdy and F.M. Hady [8] have studied Non-Darcy natural convection flow along a vertical wavy plate embedded in a non-Newtonian fluid saturated porous medium. We study in this work the effect of heat transfer in porous media on the fluid and the effect of amplitude of the wavy surface on heat transfer characteristic is investigated. The variation of velocity, temperature and Nusselt number as a function of Pr and variable heat flux exponent are studied.

2. Mathematical Formulation and Analysis

A wavy surface subjected to a variable heat flux $q_w = (1+x^2)^m$ is passed through the porous. Where the Newtonian fluid of constant thermal properties at uniform temperature T_∞ with no heat generation no body force acting on the system and no viscous dissipation.

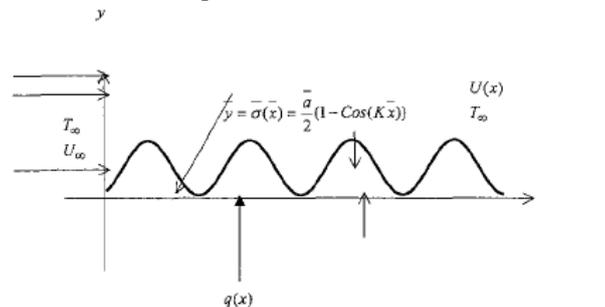


Figure 1: Physical Model and Coordinates

The governing equations are

Continuity equation

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0 \quad (1)$$

Momentum equation

$$\bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial \bar{x}} + \nu \left[\frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} \right] - \frac{\nu}{K} \bar{u} \quad (2)$$

Energy equation

$$\bar{u} \frac{\partial T}{\partial \bar{x}} + \bar{v} \frac{\partial T}{\partial \bar{y}} = \alpha \left[\frac{\partial^2 T}{\partial \bar{x}^2} + \frac{\partial^2 T}{\partial \bar{y}^2} \right] \quad (3)$$

The surface is described by $\bar{y} = \bar{\sigma}(\bar{x}) = \frac{\bar{a}}{2} (1 - \cos k\bar{x})$ is an arbitrary geometric function where \bar{a} is the amplitude of the wavy surface and $k = \frac{2\pi}{L}$ is the wave number and L is the wave length.

The boundary condition

$$\begin{aligned} \bar{y} = \bar{\sigma}(\bar{x}) & : \quad \bar{u} = \bar{v} = 0, \quad -k \frac{\partial T}{\partial \bar{x}} = (1+x^2)^m \\ \bar{y} \rightarrow \infty & : \quad \bar{u} = U(x), \quad T = T_\infty \\ \bar{x} = 0 & : \quad \bar{u} = U_\infty, \quad T = T_\infty \end{aligned} \quad (4)$$

This equations (1-4) may be transformed by using suitable dimensional variables (95) in order to subtract out the effect on the wavy surface from the boundary conditions.



The transfer motion variables are:

$$\begin{aligned} x &= \frac{\bar{x}}{L}, & a &= \frac{\bar{a}}{L}, & p &= \frac{\bar{p}}{\rho u_\infty^2}, & y &= \frac{\bar{y} - \bar{\sigma}(x)}{L Re_L^{-1/2}} \\ v &= \left[\frac{\bar{v} - \sigma_x}{U_\infty} \right] Re_L^{1/2}, & u &= \frac{\bar{u}}{U_\infty}, & \theta &= \left[\frac{T - T_\infty}{q(x)L} \right] k Re_L^{1/2} \\ \sigma(x) &= \frac{\bar{\sigma}(x)}{L}, & \sigma_x &= \frac{\partial \sigma(x)}{\partial x} \end{aligned} \quad (5)$$

In above equations, Re_L , Pr , L and U_∞ is the Reynolds number, Prandtl number, the reference length, the reference velocity. Letting $Re_L \rightarrow \infty$ and Neglecting the small order terms of magnitude after transformation, the non-transformed equations are:

The continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (6)$$

The momentum equation:

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = \frac{U(x)}{U_\infty} \frac{\partial (U(x))}{\partial x} + (1 + \sigma_x^2) \frac{\partial^2 u}{\partial y^2} - Da^{-1} u \quad (7)$$

The energy equation:

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} - \frac{2mxu\theta}{1+x^2} = \frac{1}{Pr} (1 + \sigma_x^2) \frac{\partial^2 \theta}{\partial y^2} \quad (8)$$

The boundary conditions are:

$$\begin{aligned} y = 0 & : & u = 0, & \frac{\partial \theta}{\partial y} = \frac{1}{\sqrt{(1 + \sigma_x^2)}} \\ y \rightarrow \infty & : & u = \frac{U(x)}{U_\infty}, & \theta = 0 \\ x = 0 & : & u = 1, & \theta = 0 \end{aligned} \quad (9)$$

The transformed equations (6)-(8) subject to the boundary conditions (9) which has been solved numerically using finite difference method.

The local Nusselt number is defined as

$$\frac{Nu(\bar{x})}{\sqrt{Re_{\bar{x}}}} = \frac{\sqrt{\bar{x}}}{\theta(x,0)} \quad (10)$$

And average Nusselt number, based on the average heat transfer coefficient is

$$\frac{Nu_{av}}{\sqrt{Re_{\bar{x}}}} = \frac{\sqrt{\bar{x}}}{\int_0^x \sqrt{1 + \sigma_x^2} dx} \int_0^x \frac{\sqrt{1 + \sigma_x^2}}{\theta(x,0)} dx \quad (11)$$

Where $Re_{\bar{x}}$ is the local Reynolds number.

3. Results and discussion

The resulting equations (6-8) with the boundary conditions (9) are solved by using finite difference fully implicit method. Derivatives with respect to x and y are approximated by central difference. In fig (2) shows the velocity distribution along the surface for different value of inverse Darcy number, $Pr=0.72$, $m=1$ and $a=0.13$ this figure shows that the flow accelerates along the rising portions of the surface, where the slope is positive and decelerates along the portions of the surface, where the slope is negative. The velocity varies periodically along the wavy surface. In fig (3) for different wavy surface amplitude, $Pr=0.72$, $m=1$, $Da^{-1}=0.1$, the temperature increases as the wavy surface amplitude increase. Fig (4) shows the variation of the wall heat flux exponent, $Pr=0.72$, $a=0.13$ and $Da^{-1}=0.1$, the temperature increases as the wall heat flux exponent increases. Fig (5) shows the variation of inverse Darcy number at fixed $m=1$, $a=0.13$ and $Pr=0.72$. The temperature increases as the inverse Darcy number increases. Fig (6) and fig (7) present the variations of surface temperature and Nusselt number with x respectively, for different values of Prandtl number with $m=1$, $Da^{-1}=0.1$ and $a=0.13$. It is clear that the temperature of surface and Nusselt number increases as the Prandtl number decreases. Fig (8) shows the variation of Nusselt number distribution with x for different value of inverse Darcy number, $Pr=0.72$, $m=1$ and $a=0.13$. The values of Nusselt number decreases as the inverse Darcy



number increases. Fig (9) shows the variation of Nusselt number distribution with x for different value of exponent m, Pr=0.72, a=0.13 and $Da^{-1}=0.1$. The values of Nusselt number decreases as the exponent m increases.

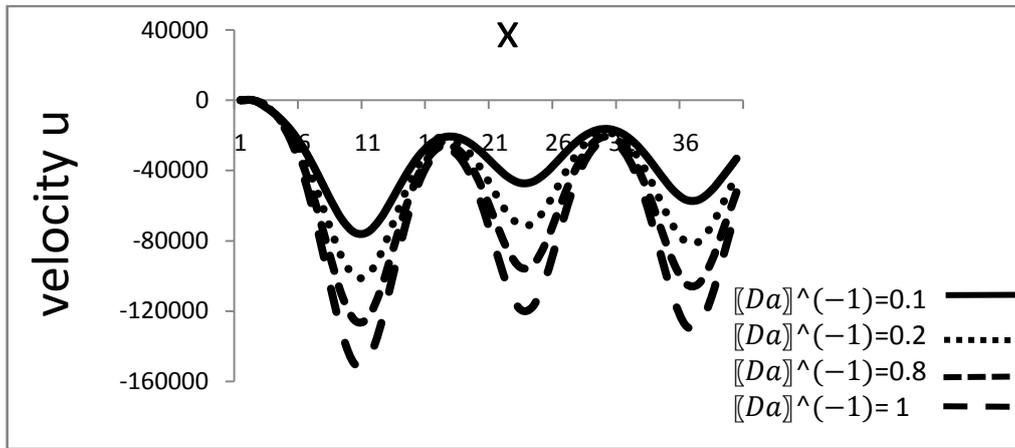


Figure 2: Velocity distribution u for different value of inverse Darcy Number Da^{-1} , $Pr=0.72$, $m=1$ and $a=0.13$

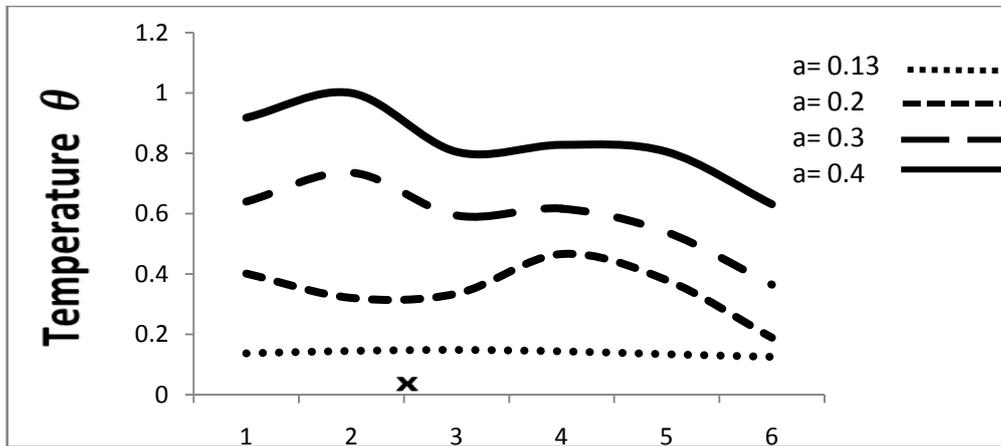


Figure 3: Surface temperature variation for $Pr=0.72$, $Da^{-1}=0.1$, $m=1$ and different values of amplitude a.

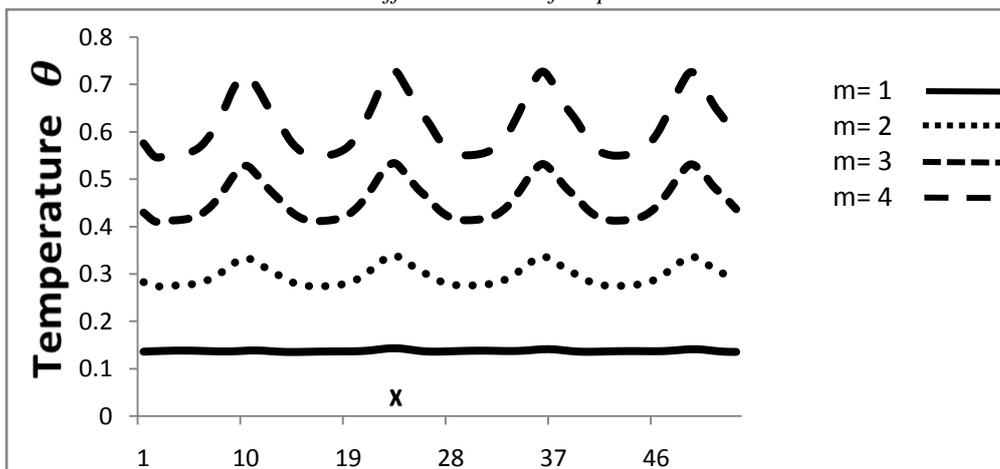


Figure 4: Surface temperature variation for $Pr=0.72$, $Da^{-1}=0.1$, $a=0.13$ and different values of exponent m.

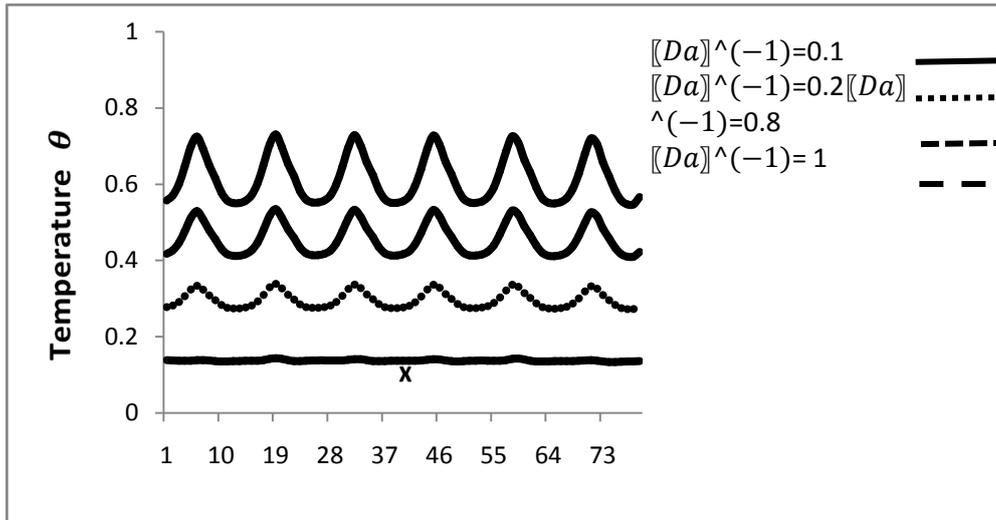


Figure 5: Surface temperature variation for $Pr=0.72$, $m=1$, $a=0.13$ and different values of inverse Darcy number Da^{-1} .

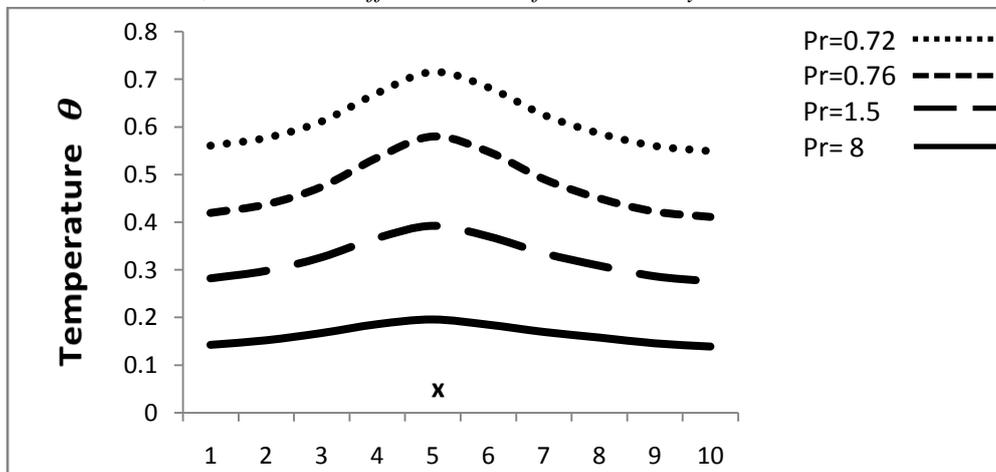


Figure 6: Surface temperature variation for $Da^{-1}=0.1$, $m=1$, $a=0.13$ and different values Prandtl number Pr

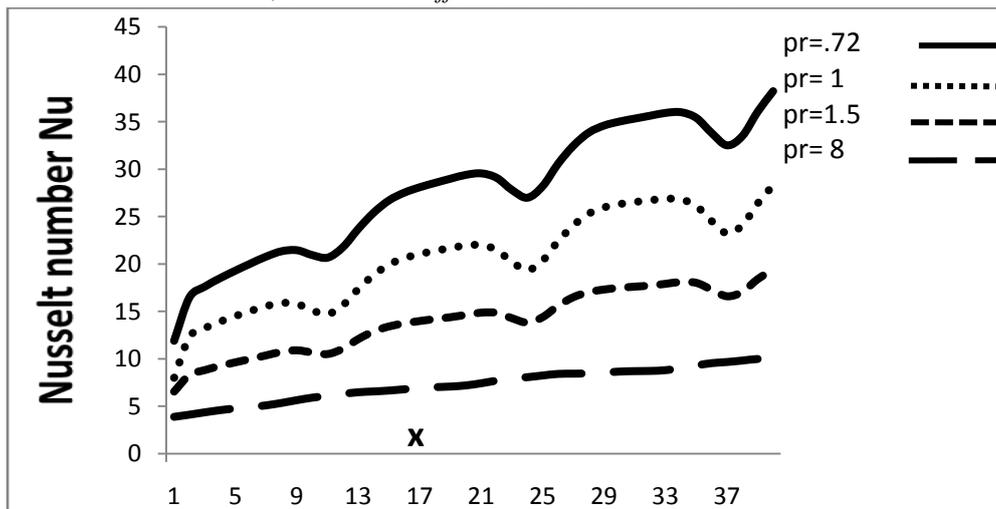


Figure 7: Nusselt number distribution for different value of Prandtl number Pr , $Da^{-1}=0.1$, $m=1$, and $a=0.13$.

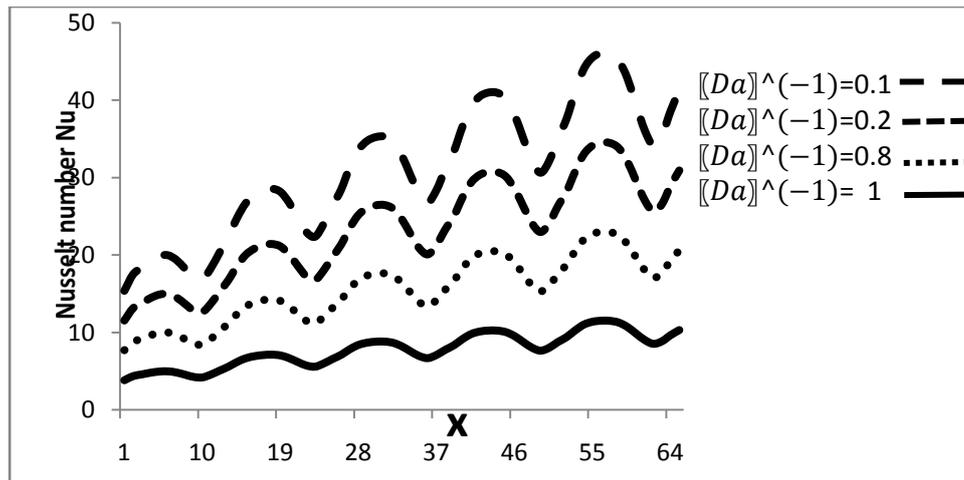


Figure 8: Nusselt number distribution for different value of inverse Darcy number Da^{-1} , $Pr=0.72$, $m=1$, and $a=0.13$

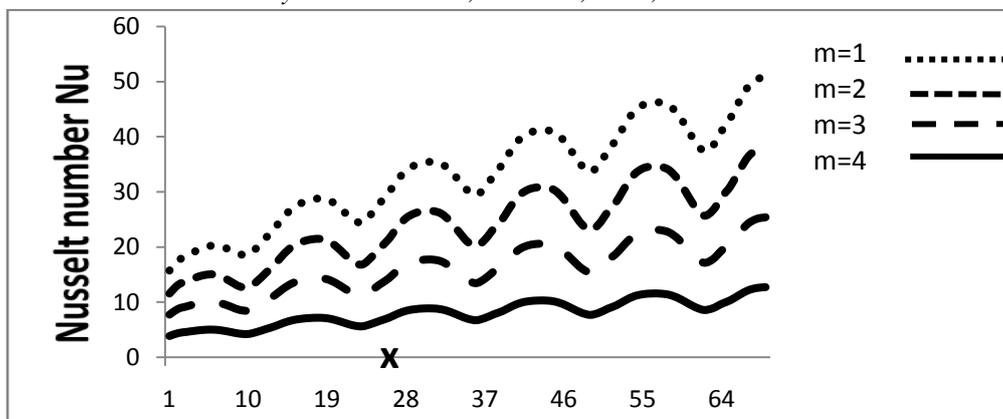


Figure 9: Nusselt number distribution for different value of exponent m , $Pr=0.72$, $Da^{-1}=0.1$, and $a=0.13$.

Conclusions

We studied the effect of heat flux in porous media and for different value of relevant physical parameters including Prandtl number Pr and Darcy number Da . The governing equations were solved numerically by finite difference fully implicit method. Detailed results for the velocity and temperature are presented and also for Nusselt number.

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