



Block Coded Modulation System Employing Concatenated Block Codes

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Abstract Concatenation can be used to obtain block-coded modulation [BCM] schemes of various long block lengths using short block codes. Both the inner and the outer codes correspond to a BCM scheme. The inner block code is considered to be a virtual expanded channel signal constellation by the outer block code. The scheme considers systematic concatenation of general (non-linear) block codes of a fixed block length 3. Codes obtained are of length 3^q , where q is the number of stages each of block length 3. Encoding and soft decoding schemes for these codes are discussed in this paper.

Keywords Systematic concatenation, block-coded modulation (BCM), block codes, block length

Introduction

Coded modulation schemes are used for efficient signaling in band-limited channels [1-2]. General block codes are used in coded modulation schemes known as block coded modulation (BCM) [3]. Optimal improper trellis can be obtained for a general block code of length 3 [4].

Concatenated codes [5-6] were used for obtaining binary codes, by combining other binary codes. Concatenation is also used with coded modulation, and various schemes have been proposed in the literature that uses a combination of coded modulation schemes.

Generally, the motivation behind concatenation is to obtain a resultant code which is different from the component codes and has various desirable properties. In this paper, concatenation is used to obtain block codes of various long lengths using block codes of shorter lengths, and to simplify the code search, encoding and the decoding complexity of general block codes. Concatenation provides a trade-off between time (α length of the block code n) and space (α the number of signals in the virtual channel signal constellation).

In this research work, use is made of the concatenation of general block codes each of short block length 3 to obtain block codes of larger block lengths 3^q , where q is the number of stages each of block length 3. Both the inner and the outer codes correspond to a BCM scheme. The inner block code is considered to be a virtual expanded channel signal constellation by the outer block code. The code search, encoding and soft decoding complexity of these codes is reduced as compared to directly using a code of long length.

Systematic Concatenation of BCM Codes to Obtain a General Block Code of Long Length

Block codes each of length 3 are considered for concatenation. If the concatenation scheme uses two BCM schemes 0_0 and 0_1 , then the resulting block code will be block length 9. In general, for a q -stage concatenated scheme, $q > 2$, using scheme $0_0, 0_1, \dots, 0_{q-1}$, the block code of length 3^q , will be obtained. For a concatenation scheme, the number of code words and hence the number of data words, depend on the selection of the individual BCM schemes, whether redundancy is added in space, or in both space and time.



The code search is simplified, since instead of searching for a block code of length n , where $n = 3^q$ and $q > 2$, it is necessary to search for codes of length 3. At each stage, a block code can be obtained using the schemes described in [3].

For the block code of stage 0_0 , the BCM scheme uses the actual signal constellation selected for the application. The block code at stage i , $0 < i \leq (q-1)$, uses the virtual channel signal constellation provided by the stage $(i-1)$. The code at the 0_{q-1} stage has to be selected to give sufficient number of code words and the required d_{min} for the concatenated BCM scheme. The codes at the stages $0_0, 0_1, \dots, 0_{q-2}$ are selected to provide a virtual channel signal constellation with proper number of signals and a proper Euclidean distance distribution, for the succeeding stage. In this manner, the complexity of the code search is reduced in a concatenated scheme.

The block code for a concatenated BCM-BCM (i.e., transmitting signal mode-receiving signal mode) scheme is represented as $(B', S', n, |C|, d_{min})$.

Where:

B' – is the base signal constellation;

S' - is the actual expanded channel signal constellation;

n – is the block length ($=3^q$);

$|C|$ - is the number of code words and

d_{min} – is the minimum Euclidean distance between the code words.

For each stage in the concatenated scheme, the block codes each of length 3 are represented as

$$(|V|, d_{min})_0 (|V|, d_{min})_1 \dots (|V|, d_{min})_{q-2} (|C|, d_{min})_{q-1}.$$

Where:

$|V|$ - is the number of code words at a stage, that is the number of signals in the virtual channel signal constellation provided for the succeeding stage and d_{min} - is the minimum Euclidean distance between the code words at a stage that is the minimum Euclidean distance provided by the virtual channel signal constellation for the succeeding stage.

Concatenated BCM-BCM schemes can be classified into the following three types depending on the nature of the block codes.

Type 1 codes: The BCM scheme resulting in the virtual channel signal constellation is such that the cardinality of the set of Euclidean distances between signals of the virtual channel signal constellation is relatively small. The BCM scheme using the virtual channel signal constellation, uses Euclidean distances and soft decoding.

Type 2 codes: The BCM scheme resulting in the virtual channel signal constellation is such that the cardinality of the set of Euclidean distances between signals of the virtual channel signal constellation is relatively small. The BCM scheme using the virtual channel signal constellation, uses Hamming distances and hard decoding.

Type 3 codes: The BCM scheme resulting in the virtual channel signal constellation is such that the cardinality (or angular/spatial spread) of the set of Euclidean distances between signals of the virtual channel signal constellation is not relatively small. The BCM scheme using the virtual channel signal constellation, uses Euclidean distances and soft decoding. This paper only discusses type 3 codes, since we are interested in long-distance communications.

Block Encoder for Concatenated BCM Schemes

Concatenation simplifies the implementation of the block encoder, as encoding can proceed in stages. Each stage has to deal with 3-tuples of signals from the signal constellation provided by the previous stage. The block diagram for the block encoder of a concatenated BCM-BCM scheme is shown in Figure 1. Starting from stage 0_{q-1} , the encoder, which is basically a map from the data word to the code word, outputs a 3-tuple of symbols from the virtual channel signal constellation. Since each virtual channel signal constellation is also a block code, each symbol is again block encoded by the succeeding stage, this continues till at first stage finally the symbols are mapped to the signals of the actual expanded channel signal constellation.

Soft Decoder for Concatenated BCM Schemes

Soft decoding of concatenated BCM schemes can be performed in stages. A reduced tree based soft decoder discussed in [3] can be used, or the optimal minimal trellis can be obtained as described in [4] and the Viterbi



algorithm can be used. Independent of the scheme used, the general block diagram of a soft decoder is shown in Figure 2.

As soon as the soft decoder at stage O_0 outputs a decoded code word of length 3, the soft decoding of stage O_1 can start; this proceeds till from stage O_{q-1} , the final decoded code word is obtained.

Simultaneously each stage can proceed with the soft decoding of the subsequent received words. In this way the decoding complexity is reduced. Decoding at various stages can be done in parallel, and having block codes of length 3 optimizes the soft decoder at each stage depending on the code, the Viterbi algorithm performs soft decoding using the trellis. There is no propagation of decoding errors in this scheme.

Examples

Considers a concatenated BCM-BCM scheme using the 2-PSK signal constellation as the actual signal constellation A three-stage concatenation of BCM scheme will consist of $(4, 8.0)_{0---$ and $(16, 16.0)_{1---$ represented as $(---, 2\text{-PSK}, 27, 256, 32.0)$ code. The code has redundancy in both space and time. The code diagram of the encoder is shown in Figure 3. A three- stage soft decoder for this concatenated BCM scheme is shown in Figure 4. The soft decoder at stage O_0 , will have to perform soft decoding 9 times, of the received symbols of length 3 and the maximum number of stages at this stage can be 4. The soft decoder at O_1 will have to perform soft decoding 3 times of the symbols received from stage O_0 of length 3 and the maximum number of states at this stage can be 16. The soft decoder at O_2 will have to perform soft decoding once of the symbols received from stage O_1 of length 3 and the maximum number of states at this stage can be 256. This soft decoding scheme is used in the concatenated scheme instead of a single soft decoder of length 27 and having at most 256 states.

Results

The tables included here give some example codes for concatenated BCM schemes. Results are tabulated for codes with block length of 3, code words are obtained at various d_{min}^2 . Using these code words obtained, as a virtual channel signal constellation, further over a block length of 3 again code words are obtained. Comparisons of these code words obtained with uncoded schemes are tabulated. See Figs. 5A and 5B which provide us with some very bright illumination on the concepts of concatenated and non-concatenated BCM-BCM communication schemes.

sampling technique as sample for the study. The instrument used in collecting data for this study was a self-constructed questionnaire that is 15 items. A test-retest method of reliability was used: the instrument was administered on 20 students who were not among the sample for the study on two occasions with an interval of two weeks. The Pearson Product Moment Correlation yielded a coefficient (r) 0.75.

Table 1: Actual expanded channel signal constellation is 8PSK. Virtual Channel signal constellation is 44-point BCM with $d_{min}^2 = 2.59$. Rate = Constellation rate; CG = Concatenated gain.

Serial No.	d_{min}^2	No. of code words	Comparison With QPSK		Comparison With Binary PSK	
			Rate	CG dB	Rate	CG dB
1	3.18	20160	0.79	2.01	1.59	0.99
2	4.00	16120	0.78	3.01	1.55	0.00
3	5.18	8916	0.73	4.01	1.46	1.12
4	6.00	2456	0.63	4.77	1.25	1.76
5	7.18	1336	0.58	5.55	1.15	2.54
6	8.00	761	0.53	6.02	1.06	3.01
7	9.18	427	0.49	6.61	0.97	3.61
8	10.00	240	0.44	6.99	0.88	3.98



Table 2: Actual expanded channel signal constellation is 8PSK. Virtual channel signal constellation is 8PSK. Virtual Channel signal constellation is 64-point BCM with $d_{min}^2=2.00$.

Serial No.	d_{min}^2	No. of code words	Comparison With QPSK		Comparison With Binary PSK	
			Rate	CG dB	Rate	CG dB
1	4.00	131072	0.94	3.01	1.89	0.00
2	6.00	8192	0.72	4.77	1.44	1.76
3	8.00	4096	0.67	6.02	1.33	3.01
4	10.0	512	0.50	6.99	1.00	3.98

Table 3: Actual expanded channel signal constellation is 7PSK. Virtual Channel signal constellation is 57-point BCM with $d_{min}^2 = 3.00$.

Serial No.	d_{min}^2	No. of code words	Comparison With QPSK		Comparison With Binary PSK	
			Rate	CG dB	Rate	CG dB
1	3.20	37806	1.07	0.28	1.69	-0.96
2	3.80	32918	1.05	1.03	1.68	-0.22
3	4.50	22140	1.01	1.76	1.60	0.51
4	5.30	3991	0.84	2.47	1.33	1.22
5	6.05	3111	0.81	3.05	1.29	1.80
6	7.00	1469	0.74	3.68	1.17	2.43
7	8.10	680	0.66	4.31	1.05	3.06
8	9.05	411	0.61	4.80	0.96	3.55
9	10.00	247	0.56	5.23	0.88	3.98

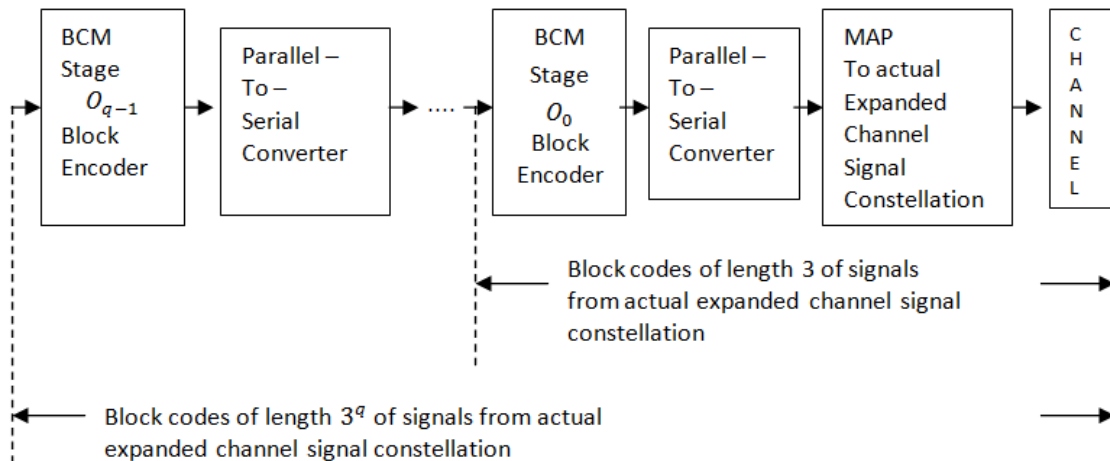


Figure 1: Block Diagram of Encoder for concatenated BCM scheme

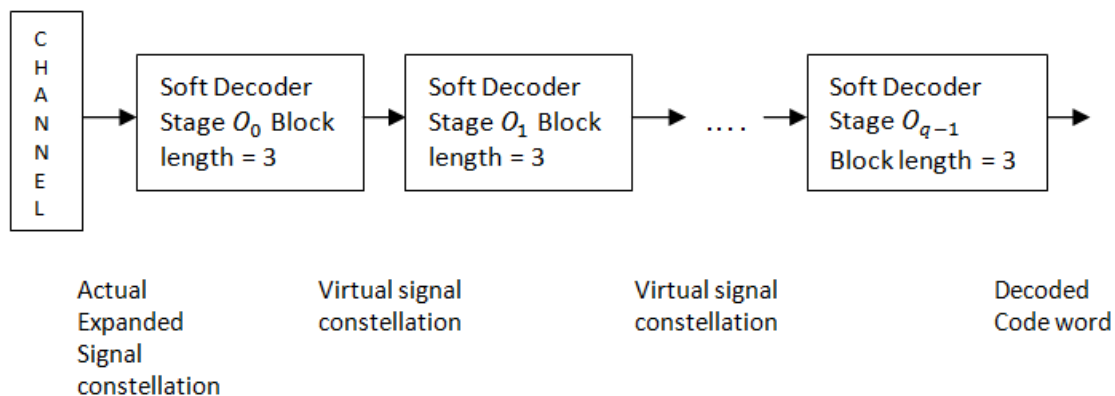


Figure 2: Block Diagram of Soft Decoder for concatenated BCM scheme

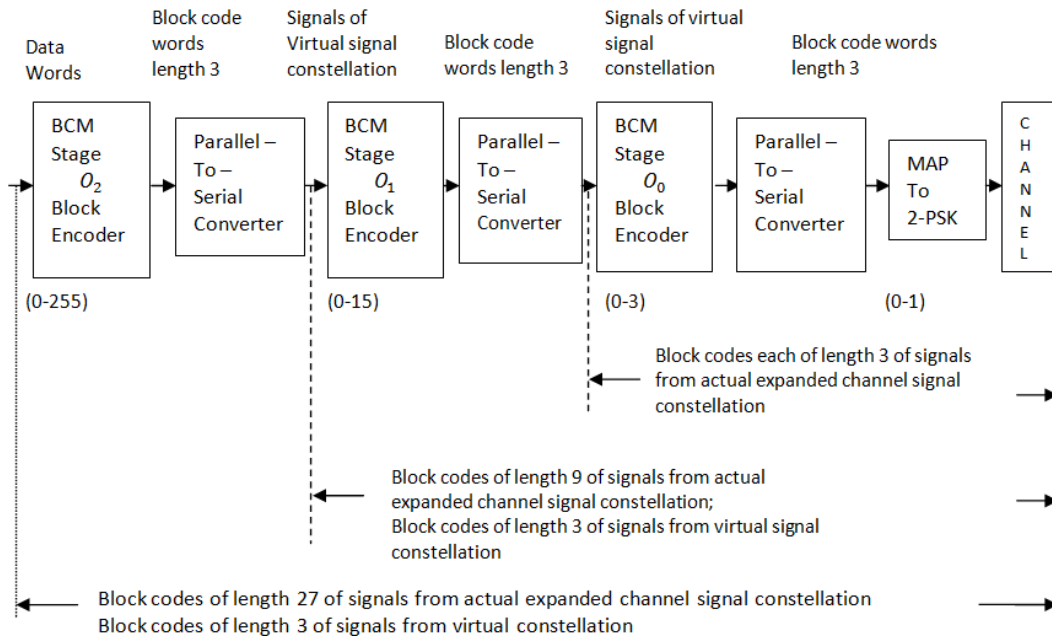


Figure 3: Block Diagram of Encoder for Concatenated BCM Scheme of the Given Example

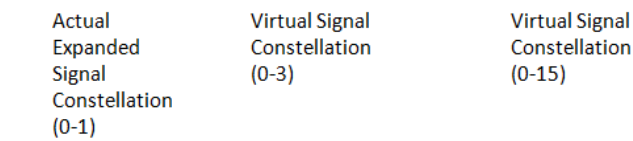
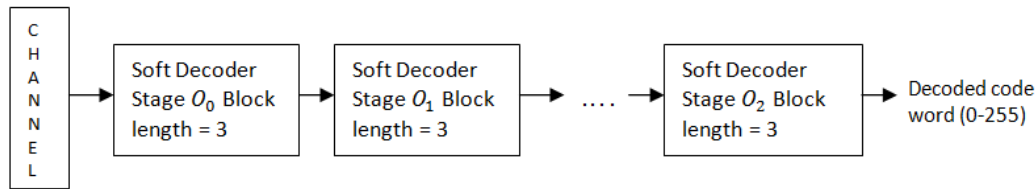


Figure 4: Block Diagram of Soft Decoder for Concatenated BCM Scheme of the Given Example

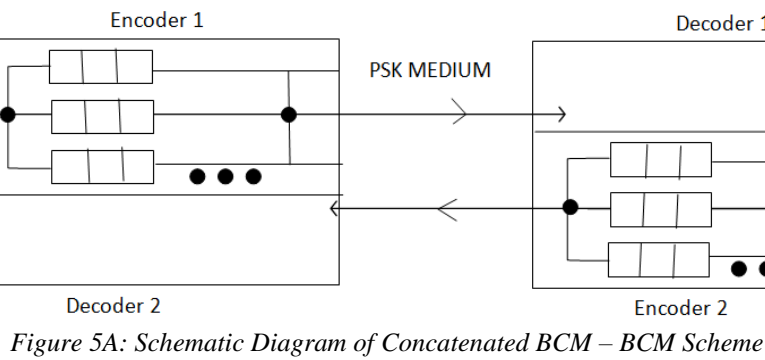


Figure 5A: Schematic Diagram of Concatenated BCM – BCM Scheme

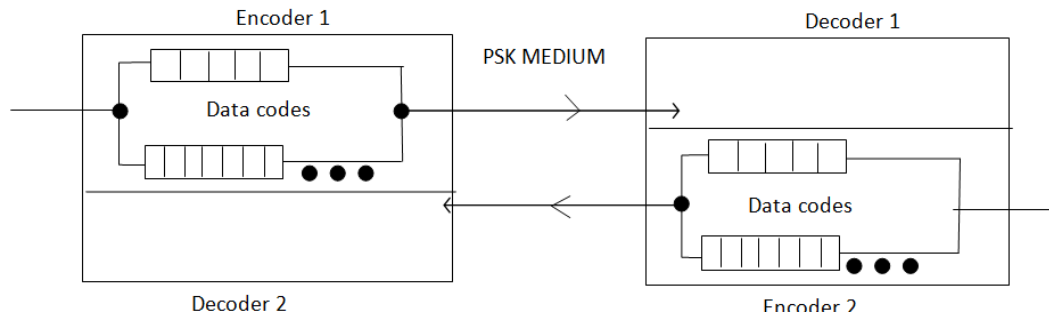


Figure 5B: Schematic Diagram of the Non - Concatenated BCM – BCM Scheme



Conclusion

A scheme is presented to use block codes of short block lengths to obtain codes of larger block lengths, using concatenation. Both the inner and the outer codes correspond to a BCM scheme. The inner block code is considered as a virtual expanded channel signal constellation by the outer block code. The scheme considers all block codes of a fixed block length 3. The code search, encoding and soft decoding complexity of these codes is reduced as compared to directly using a block code of long length. When a communication channel is established between a BCM encoder and a BCM decoder, equilibrium functional values of concatenation gain and the constellation rate are eventually reached, with the speed of communication being enhanced by parallelism. In the case of non-concatenated BCM-BCM communication scheme, the data block codes are of various long lengths each greater than 3 which makes encoding and decoding more time-consuming and more tasking. Whereas, in the case of the concatenated BCM-BCM communication scheme being proposed in this paper, the data block codes are each of word length 3, which makes data encoding and decoding easier and faster. Therefore, in conclusion, the concatenated BCM-BCM communication scheme is proposed for use in modern digital communication systems.

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