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Research Article

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A Semi-Analytic Method for Solving Partial Differential Equations with Non-Local Boundary Conditions

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Abstract This paper deals with the non-local boundary and initial value problems for two-dimensional two-sided partial differential equation model by using the semi analytic method. Tested examples and the obtained results demonstrate efficiency of the proposed method.

Keywords two-dimensional, two-sided, partial differential equation, non-local boundary condition problem

Introduction

Adomian decomposition method can solve large classes of linear and nonlinear differential equations and it is much simpler in computation and quicker in convergence than any other method available in the open literature [1,2].

There are many literatures developed concerning Adomian decomposition method [3-6] and the related modification to investigate various scientific model [7- 10]. E. Babolian et al. introduced the restart method to solve the equation f (x) = 0 [11], and the integral equations [12]. H. Jafari et.al used a correction of decomposition method for ordinary and nonlinear systems of equations and show that the correction accelerates the convergence [131, 14]

In this paper, we present computationally efficient numerical method for solving the partial differential equation with boundary integral conditions:

$$D_t\Psi(x,y,t)-D_{+xx}\Psi(x,y,t)-D_{-xx}\Psi(x,y,t)-D_{+yy}\Psi(x,y,t)-D_{-yy}\Psi(x,y,t)+\Psi(x,y,t)=k(x,y,t) \quad (1)$$

with the initial condition

$$\Psi(x,y,0)=f(x,y), 0\le x\le T, 0\le y\le T$$

and the non-local boundary conditions

$$\int_0^1\int_0^1 \Psi(x,y,t)dxdy=g_1(t), 0<t\le T$$

$$\int_0^1\int_0^1 \mathrm{P}(x,y)\Psi(x,y,t)dxdy=g_2(t), 0<t\le T$$

Where f, g_1, g_2, ψ and h are known functions. T is given constant. In the present work, we apply the modified Adomian's decomposition method for solving eq.(1) and compare the results with exact solution. The paper is organized as follows: In section 2 the two-dimensional partial differential equations with boundary integral conditions and its solution by modified decomposition method is presented. In section 3 an example is solved numerically using the modified decomposition method. Finally, we present conclusion about solution of the two-dimensional partial differential equation.

Numerical Method

In this section, we present modified decomposition method for solving two-dimensional two-sided partial differential equations with boundary integral given in eq.(1). In this method we assume that:

$$\Psi(x,y,t)=\sum_{n=0}^{\infty}\Psi_n(x,y,t)$$

eq.(1) can be rewritten:

$$L_t\Psi(x,y,t)=L_{+xx}\Psi(x,y,t)+L_{-xx}\Psi(x,y,t)+L_{+yy}\Psi(x,y,t)+L_{-yy}\Psi(x,y,t)-\Psi(x,y,t)+k(x,y,t) \quad (2)$$

Where

$$L_t(\cdot)=\frac{\partial}{\partial t}(\cdot)\ ,\ L_{xx}=\frac{\partial^2}{\partial x^2} \quad \text{and} \quad L_{yy}=\frac{\partial^2}{\partial y^2}$$

The inverse $$L^{-1}=\int_0^t(\cdot)dt \quad (3)$$

Take L^{-1} on both sides of eq (2) we have

$$L^{-1}(L_t\Psi((x,y,t)))=L^{-1}(L_{+xx}(\Psi(x,y,t))+L_{-xx}(\Psi(x,y,t)))+L^{-1}(L_{+yy}(\Psi(x,y,t))+L_{-yy}(\Psi(x,y,t)))-$$
$$L^{-1}(\Psi(x,y,t))+L^{-1}(k(x,y,t))$$

Then, we can write,

$$\Psi(x,y,t)=\Psi(x,y,0)+L_t^{-1}\left(L_{+xx}\left(\sum_{n=0}^{\bowtie}\Psi_n\right)+L_{-xx}\left(\sum_{n=0}^{\bowtie}\Psi_n\right)\right)+L_t^{-1}\left(L_{+yy}\left(\sum_{n=0}^{\bowtie}\Psi_n\right)+L_{-yy}\left(\sum_{n=0}^{\bowtie}\Psi_n\right)\right)-$$
$$L_t^{-1}(\Psi(x,y,t))+L_t^{-1}(k(x,y,t)) \quad (4)$$

The modified decomposition method was introduced by Wazwaz [6]. This method is based on the assumption that the function $\gamma(x,y)$ can be divided into two parts, namely $\gamma_1(x,y)$ and $\gamma_2(x,y)$. Under this assumption we set

$$\gamma(x,y)=\gamma_1(x,y)+\gamma_2(x,y)$$

Then the modification

$$u_0=\gamma_1$$

$$\Psi_1=\gamma_2+L_t^{-1}\left(L_{+xx}\Psi_0\right)+L_t^{-1}\left(L_{-xx}\Psi_0\right)+L_t^{-1}\left(L_{+yy}\Psi_0\right)+L_t^{-1}\left(L_{-yy}\Psi_0\right)-L_t^{-1}\left(\Psi_0\right)$$

$$\Psi_{n+1}=L_t^{-1}\left(L_{+xx}\left(\sum_{n=0}^{\infty}\Psi_n\right)+L_{-xx}\left(\sum_{n=0}^{\infty}\Psi_n\right)\right)+L_t^{-1}\left(L_{+yy}\left(\sum_{n=0}^{\infty}\Psi_n\right)+L_{-yy}\left(\sum_{n=0}^{\infty}\Psi_n\right)\right)-L_t^{-1}\left(\sum_{n=0}^{\infty}\Psi_n\right),$$

n>1

Numerical Example

Consider two-dimensional two-sided partial differential equation with non-local boundary condition for the equation (1):

$$D_t\Psi-D_{+xx}\Psi-D_{-xx}\Psi-D_{+yy}\Psi-D_{-yy}\Psi+\Psi=2t+t^2+x+y$$

subject to the initial condition

$\Psi(x,y,0)=x^2+y^2$ $\qquad x,y\in(0,1),\quad 0\le t\le T$

and the non-local boundary conditions

$\int_0^1 \int_0^1 (x^2 + y^2 + t^2)\, dxdy = t^2 + \frac{2}{3}$, $\quad 0 \le t \le T$

$\int_0^1 \int_0^1 x^2y^2(x^2 + y^2 + t^2)\, dxdy = \frac{t^2}{9} + \frac{2}{15}$, $\quad 0 \le t \le T$

We apply the above proposed method; we obtain:

$\Psi_0(x, y, t) = x^2 + y^2 + t^2$

$\Psi_1(x, y, t) = 0$

$\Psi_2(x, y, t) = 0$

$\Psi_3(x, y, t) = 0$

Then the series form is given by:

$$\Psi(x, y, t) = \Psi_0(x, y, t) + \Psi_1(x, y, t) + \Psi_2(x, y, t) + \Psi_3(x, y, t)$$
$$= x^2 + y^2 + t^2$$

This is the exact solution:

$\Psi(x, y, t) = x^2 + y^2 + t^2$

Table 1 shows the analytical solutions for partial differential equation with boundary integral condition obtained for different values and comparison between exact solution and analytical solution.

Table 1: Comparison between exact solution and analytical solution for example

x = y	**t**	**Exact Solution**	**Modified Adomian Decomposition Method**	$\vert \Psi_{ex} - \Psi_{MADM} \vert$
0	2	4.0	4.0	0.0000
0.1	2	4.02	4.02	0.0000
0.2	2	4.08	4.08	0.0000
0.3	2	4.18	4.18	0.0000
0.4	2	4.32	4.32	0.0000
0.5	2	4.50	4.50	0.0000
0.6	2	4.72	4.72	0.0000
0.7	2	4.98	4.98	0.0000
0.8	2	5.28	5.28	0.0000
0.9	2	5.62	5.62	0.0000
1	2	6.0	6.0	0.0000

Conclusion

In this paper, we have applied the modified decomposition method for the solution of the two-dimensional two-sided the partial differential equation with ono-local boundary condition. This algorithm is simple and easy to implement. The obtained results confirmed a good accuracy of the method.

References

[1]. G. Adomian, Nonlinear Stochastic Operator Equations, Academic Press, Florida, 1986.

[2]. G. Adomian and R. Rach, On linear and nonlinear integro-differential equations, J. Math. Anal. Appl. 113 (1986), no. 1, 199–201.

[3]. Adomian G. and Rach R., Noise terms in decomposition solution series, Computers Math. Appl., 24 (1992), 61-74.

[4]. Adomian G., Solving Frontier Problems of Physics: The Decomposition Method, Kluver Academic Publishers, Boston, 1994.

[5]. Cannon J.R and Van der Hoek J., Diffusion equation subject to the specification of mass, J. Math. Anal. Appl., 115, (1986), 517-529.

[6]. Wazwaz A. M, J., Necessary conditions for appearance of noise terms in decomposition solution series Math. Anal. Appl., 5 (1997), 265-274.

[7]. A.M. Wazwaz, A new algorithm for calculating Adomian polynomials for nonlinear operators, Appl Math Comput, 111(1) (2000), 53-69.

[8]. A.M. Wazwaz, A reliable modification of Adomian decomposition method, Appl Math Comput, 102, (1999), pp. 77–86.

[9]. E. Babolian and S. H. Javadi, Restarted Adomian Method for Algebraic Equations, App. Math. And Computation, 146 (2003), 533- 541.

[10]. E. Babolian, S. H. Javadi, H. Sadeghi, Restarted Adomian Method for Integral Equations, App. Math. And Computation, 167 (2003), 1150-1155.

[11]. Hossein Jafari, Varsha Daftardar-Gejji, Revised Adomian decomposition method for Solving a system of nonlinear equations, App. Math. And Computation, 175 (2006), 1-7.

[12]. Hossein Jafari, Varsha Daftardar-Gejji, Revised Adomian decomposition method for Solving systems of Ordinary and ractional Differential Equations, App. Math. And Computation, 181 (2006), 598-608.