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Compressibility effects on the Rayleigh–Taylor instability growth rate between two magnetized plasmas layers

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Abstract Compressibility effects on Rayleigh–Taylor instability of two plasmas layers are investigated. The system is acted upon by an external transverse magnetic field. The solution of the relevant linearized perturbation equations have been developed by the normal mode technique. The dispersion relation is obtained and is analyzed when the transverse magnetic field is parallel or perpendicular to the wave number and is solved numerically. The results show that, in the absence of external transverse magnetic field the rate of growth is more sensitive to the change in compressibility of the lower layer, while in the presence of it and when it is parallel to the wave number, the rate of growth is more sensitive to the change in compressibility of the upper layer. In the case when external transverse magnetic field is perpendicular to the wave number, we specialize the case of $M \ll 1$ (M is the static Mach number defined using the gravity wave speed and the isothermal speed of sound), the results show that the rate of growth is more sensitive to the change in compressibility of the upper layer in the absence of (or presence) external transverse magnetic field.

Keywords Rayleigh–Taylor instability, external transverse magnetic field

1. Introduction

The Rayleigh–Taylor instability occurs [1-3] when a heavy fluid is supported by a lighter one in a gravitational field. Perturbations at this interface are amplified exponentially in time and result in finger of heavy material falling in the light one with light bubbles growing up in the heavy one when the instability reaches the nonlinear regime. The Rayleigh- Taylor instability problem manifests in several natural physical phenomena, for example, mushroom clouds like those from volcanic eruptions and atmospheric nuclear explosions [4], supernova explosions in which expanding core gas is accelerated into denser shell gas [5] and instabilities in plasma fusion reactors [6].

Several attempts to determine the effects of compressibility on the linearized RTI development have been made by a number of researchers [7-25], but the results were in disagreement with each other. Scannapieco [12], Bernstein and Book [14] and Yang and Zhang [16] found a destabilizing effect of compressibility. Blake [10], Mathews and Blumenthal [11], Sharp [15] and Li [17] reported that the compressibility has a stabilizing effect on RTI. While Plesset and Heish [8], and Livescu [18] conclude both stabilizing and destabilizing effects are possible. He showed that compressibility can be characterized by two independent parameters, the isothermal speed of sound c_T (which is function of pressure at the interface p_0) and the ratio of specific heats γ. He concluded that, For isothermal initial conditions, as p_0 decreases (more compressible flow) the compressibility effects on the growth rates are destabilized, the growth rate increases when γ decreases (more compressible fluid). Which implies that the compressibility has stabilizing and destabilizing effect on RTI? Also he cleared that the incompressible limits are independently obtained when either ($\gamma \to \infty$ or $M \to 0$) where M is the static Mach number defined using the gravity wave speed and the isothermal speed of sound.

In plasmas RTI has been studied for stratified incompressible layer in the presence of magnitude of the gravitational acceleration (g) by Goldston and Rutherford [26]. RTI has been considered in inhomogeneous plasma rotating uniformly in an external magnetic field (vertical or horizontal-direction) by Al-Khateeb and Laham [27, 28].

In actual physical situations plasmas is often compressible. Thus the study of Rayleigh- Taylor instability for compressible plasmas is more realistic.

RTI in compressible magnetized fluids plays an important role in several astrophysical compressible plasmas, e.g., accretion onto compact objects [29], flux emerging from the solar photosphere [30], or shell of young supernova remnant. The last concerns two unstable contact discontinuities: at the interface between the upper boundary of ejecta and swept-up material [31, 32] such as Tycho's supernova remnant with fingers pointing outward, or at the interface between the pulsar-driven synchrotron nebula and the inner boundary shell of ejecta [33] such as the Crab Nebula with fingers pointing inward. In laser-produced plasmas, nonlocal self-generated magnetic fields may affect the RTI growth rate for large wave numbers [34].

The topic of interaction between plasma and a rotating magnetic field have been a subject of great interest for the last several decades, which have many applications, including astrophysical problems [35-38] and technological devices [39, 40]. For example, the possibility of providing gyroscopic stability to a mass accelerated by an electromagnetic launcher with plasma armature was considered by Becherini et al. [41]. In this study, the launch mass was supposed to have a ferromagnetic sleeve rotated by an additional rotating magnetic field.

The interaction between plasma and a rotating magnetic field have been studied by several authors. For example, the theoretical works have been done by Moffat [42], Kono and Tanaka [43] and Grants and Gerebeth [44]. The interaction between plasma and a rotating magnetic field with constant angular velocity and amplitude is studied by Mauro Bologna et al. [45]. Experimental investigations of this problem have been done by Volz and Mazuruk [46] and Nagaoka et al. [47].

Keeping in mind the importance of the Rayleigh-Taylor instability under the effect of compressibility and rotating magnetic field in astrophysics (supernova and supernova remnant), e.g. in the supernovae evolution some studies show that the compressibility is one of dominant parameters in the RTI growth rate [25, 48], the present paper attempts to study the effect of compressibility on Rayleigh-Taylor instability in inhomogeneous plasma rotating uniformly in an external magnetic field in the horizontal direction of two plasmas layers.

2. The mathematical model

The equations for ideal compressible plasma as a fluid of electrons and immobile ions in the presence of an external transverse magnetic field along the $y-axis$ ($\vec{B}=B_0\vec{e}_y$) are

$$\rho\left(\frac{\partial}{\partial t}+\vec{\mathrm{U}}.\vec{\nabla}\right)\vec{\mathrm{U}}=-\vec{\nabla}p+\rho\vec{g}-\rho\left(\vec{\mathrm{U}}\times\vec{\omega}\right), \tag{1}$$

$$\frac{\partial\rho}{\partial t}+\vec{\nabla}.\rho\vec{\mathrm{U}}=0, \tag{2}$$

$$\frac{\partial p}{\partial t}=-\gamma\, p\left(\vec{\nabla}.\vec{\mathrm{U}}\right)-\left(\vec{\mathrm{U}}.\vec{\nabla}\right)p. \tag{3}$$

where $\vec{U}$ is the velocity, ρ is the density, p pressure, $\vec{g}=(0,0,-g)$ is the acceleration due to gravity directed anti-parallel to $y-$axis, $\vec{\omega}=\frac{eB_0}{m_e}\vec{e}_y$ is the plasma angular velocity

(m_e is the electron mass and e is magnitude of the electrons charge) and γ is constant.

Now, we give a small perturbation to the system, where the perturbations in the velocity $\vec{U}$, pressure p, and density ρ, respectively, written as $\vec{U}=\vec{U}_1, P=P_0(z)+P_1(x,y,z,t)$ and $\rho=\rho_0(z)+\rho_1(x,y,z,t)$, $(\rho_0=\rho_0(z),\ p_0=p_0(z), \frac{d\,p_0}{dz}=-\rho_0 g)$. Then, the linearized equations can be easily derived from Eqs. (1)-(3) in the form: $\rho_0\frac{\partial\vec{U}_1}{\partial t}=-\vec{\nabla}P_1+\rho_1\vec{g}-\rho_0(\vec{U}_1\times\vec{\omega}_y)$, (4)

$$\frac{\partial\rho_1}{\partial t}+\vec{\nabla}.\ \rho_0\vec{U}_1=0, \tag{5}$$

$$\frac{\partial p}{\partial t}=-p_0\gamma\left(\vec{\nabla}.\vec{U}_1\right)-\left(\vec{U}_1.\vec{\nabla}\right)p_0 \tag{6}$$

Now, we consider the perturbation in all physical quantities are taken in the form

$$\Psi_1(x,y,z,t)=\Psi_1(z)\exp\left\{\ ik_x\ x+ik_y\ y-i\omega t\right\}. \tag{7}$$

where k_x,k_y ($k^2=k_x^2+k_y^2$) are the horizontal wave numbers and ω denotes the rate at which the system departs from equilibrium. Then, the system of Eqs. (4)-(6) becomes

$$-i\omega\rho_0 u_{x1}=-ik_x p_1+\rho_0\omega_y u_{z1}, \tag{8}$$

$$-i\omega\rho_0 u_{y1}=-ik_y p_1 \tag{9}$$

$$-i\omega\rho_0 u_{z1}=-\frac{dp_1}{dz}-\rho_1 g-\rho_0\omega_y u_{x1}, \tag{10}$$

$$-i\omega\rho_1+\rho_0\left\{ik_x u_{x1}+ik_y u_{y1}+\frac{du_{z1}}{dz}\right\}+u_{z1}\frac{d\rho_0}{dz}=0, \tag{11}$$

$$-ip_1\omega=-\gamma\,p_0\left\{ik_x u_{x1}+ik_y u_{y1}+\frac{du_{z1}}{dz}\right\}+u_{z1}\,g\,\rho_0. \tag{12}$$

From the system of Eqs. (8)-(12), and by deleting some of the variables, we get

$$u_{z1}\frac{d}{dz}\left\{\frac{\left[\frac{g}{c^2}+\frac{\left(\vec{k}\times\vec{\omega}_y\right)_{\vec{e}_z}}{\omega}\right]}{k^2-\frac{\omega^2}{c^2}}\rho_0\right\}-\frac{d}{dz}\left\{\frac{\frac{du_{z1}}{dz}}{k^2-\frac{\omega^2}{c^2}}\rho_0\right\}+$$

$$\rho_0\left\{\begin{aligned}&1+\frac{\frac{g^2k^2}{c^2\omega^2}}{k^2-\frac{\omega^2}{c^2}}+\frac{g\left(\frac{d\rho_0}{dz}\right)}{\rho_0\,\omega^2}+\\&\frac{\frac{1}{\omega^2}}{k^2-\frac{\omega^2}{c^2}}\left[\frac{2\omega g}{c^2}\left(\vec{k}\times\vec{\omega}_y\right)_{\vec{e}_z}-\left(\vec{k}\cdot\vec{\omega}_y\right)^2+\frac{\omega^2\left(\vec{k}\cdot\vec{\omega}_y\right)^2}{c^2k_y^2}\right]\end{aligned}\right\}u_{z1}=0, \qquad (13)$$

where $c=\sqrt{\gamma\frac{p_0}{\rho_0}}$ is the speed of sound.

3. Transition Layer

In this section we consider two compressible plasma layers of densities ρ_1,ρ_2. The two layers be confined between rigid horizontal planes: $z=-h_1$ (the lower boundary) and $z=h_2$ (the upper one) (see Fig.1). On the other hand, on each side of the interface c^2 and $\frac{D\rho_0}{\rho_0}=-\frac{g}{RT_0}=-\frac{g\gamma}{c^2}$ are constant, where R is the gas constant and T_0 is constant. Then under these conditions Eq. (13) becomes

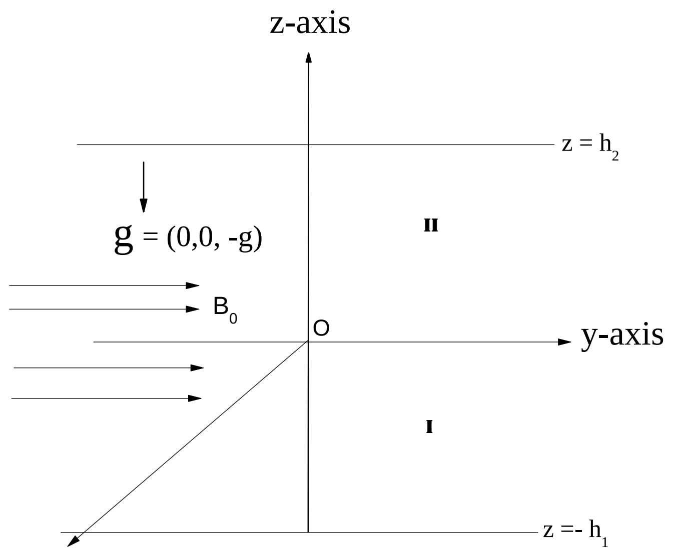

Figure 1

$$\frac{d^2u_{z1}}{dz^2}-\left(\frac{g\gamma}{c^2}\right)\frac{du_{z1}}{dz}-\left\{\begin{array}{l} k^2+\frac{k^2}{c^2}\left[\frac{g^2}{\omega^2}(1-\gamma)-\frac{\omega^2}{k^2}\right]-\frac{1}{\omega^2}\left(\vec{k}\cdot\vec{\omega}_y\right)^2+ \\ \frac{1}{c^2}\left[\frac{\left(\vec{k}\cdot\vec{\omega}_y\right)^2}{k_y^2}+\frac{g}{\omega}(2-\gamma)\left(\vec{k}\times\vec{\omega}_y\right)_{\vec{e}_z}\right] \end{array}\right\}u_{z1}=0\cdot \tag{14}$$

The solution of Eq. (14) in the above two mentioned layers may be written as

$$(u_{z1})_I=A_1\exp(\lambda_1 z)+\bar{A}_1\exp(\bar{\lambda}_1 z), \qquad -h_1\le z\le 0, \tag{15}$$

$$(u_{z1})_{II}=A_2\exp(\lambda_2 z)+\bar{A}_2\exp(\bar{\lambda}_2 z), \qquad 0\le z\le h_2, \tag{16}$$

$$\lambda_1,\bar{\lambda}_1=\frac{g\gamma_1}{2c_1^2}\pm\sqrt{\left(\frac{g\gamma_1}{2c_1^2}\right)^2+\left\{\begin{array}{l} k^2+\frac{k^2}{c_1^2}\left[\frac{g^2}{\omega^2}(1-\gamma_1)-\frac{\omega^2}{k^2}\right]-\frac{1}{\omega^2}\left(\vec{k}\cdot\vec{\omega}_y\right)^2+ \\ \frac{1}{c_1^2}\left[\frac{\left(\vec{k}\cdot\vec{\omega}_y\right)^2}{k_y^2}+\frac{g}{\omega}(2-\gamma_1)\left(\vec{k}\times\vec{\omega}_y\right)_{\vec{e}_z}\right] \end{array}\right\}}, \tag{17}$$

$$\lambda_2,\bar{\lambda}_2=\frac{g\gamma_2}{2c_2^2}\pm\sqrt{\left(\frac{g\gamma_2}{2c_2^2}\right)^2+\left\{\begin{array}{l} k^2+\frac{k^2}{c_2^2}\left[\frac{g^2}{\omega^2}(1-\gamma_2)-\frac{\omega^2}{k^2}\right]-\frac{1}{\omega^2}\left(\vec{k}\cdot\vec{\omega}_y\right)^2+ \\ \frac{1}{c_2^2}\left[\frac{\left(\vec{k}\cdot\vec{\omega}_y\right)^2}{k_y^2}+\frac{g}{\omega}(2-\gamma_2)\left(\vec{k}\times\vec{\omega}_y\right)_{\vec{e}_z}\right] \end{array}\right\}}. \tag{18}$$

Here $A_1, \bar{A}_1, A_2$ and $\bar{A}_2$ are constants.

Now, we need the boundary conditions to match the solutions at the two boundaries and interface, that are, respectively,

(i) On the rigid boundaries at $z=-h_1,\ z=h_2$, the normal velocities vanish on both the bottom and top boundaries, i.e. $(u_{z1})_I=0,\ (u_{z1})_{II}=0$, respectively.

(ii) A cross the interface $z=0$, the normal velocity is continuous, i.e. $(u_{z1})_I=(u_{z1})_{II}$.

(iii) The jump condition is obtained by integrating Eq. (13) across the surface $z=0$ and doing the limit of infinitesimal integration volume. Then, we obtain the condition

$$(u_{z1})_s\Delta_s\left\{\frac{\left[\frac{g}{c^2}+\frac{\left(\vec{k}\times\vec{\omega}_y\right)_{\vec{e}_z}}{\omega}\right]}{k^2-\frac{\omega^2}{c^2}}\rho_0\right\}-\Delta_s\left\{\frac{\frac{du_{z1}}{dz}}{k^2-\frac{\omega^2}{c^2}}\rho_0\right\}+\frac{g}{\omega^2}\Delta_s(\rho_0)(u_{z1})_s=0, \tag{19}$$

Where $\Delta_s(f) = f(z_s+0) - f(z_s-0)$.

Using the above boundary conditions, the dispersion relation may be written as:

$$\begin{aligned}
&\left\{1-\exp\left[-\left(\lambda_1-\bar{\lambda}_1\right)h_1\right]\right\}\times\left\{1-\exp\left[-\left(\lambda_2-\bar{\lambda}_2\right)h_2\right]\right\}\times \\
&\left\{\frac{k\,g}{\omega^2}+\frac{1}{\omega k}\left(\vec{k}\times\vec{\omega}_y\right)_{\vec{e}_z}\right\}\times\left\{\rho_2\left\{1-\frac{\omega^2}{c_2^2k^2}\right\}-\rho_1\left\{1-\frac{\omega^2}{c_1^2k^2}\right\}\right\}+ \\
&\rho_1\left\{1-\frac{\omega^2}{c_1^2k^2}\right\}\times\left\{\lambda_1-\bar{\lambda}_1\exp\left[-\left(\lambda_1-\bar{\lambda}_1\right)h_1\right]\right\}\times\left\{1-\exp\left[-\left(\lambda_2-\bar{\lambda}_2\right)h_2\right]\right\}- \\
&\rho_2\left\{1-\frac{\omega^2}{c_2^2k^2}\right\}\times\left\{\bar{\lambda}_2-\lambda_2\exp\left[-\left(\lambda_2-\bar{\lambda}_2\right)h_2\right]\right\}\times\left\{1-\exp\left[-\left(\lambda_1-\bar{\lambda}_1\right)h_1\right]\right\}=0
\end{aligned} \tag{20}$$

Now, we will define the next dimensionless quantities

$\rho_1=\dfrac{p_\infty}{R_1T_0},\ \rho_2=\dfrac{p_\infty}{R_2T_0},\ M=\dfrac{g\left(\rho_1+\rho_2\right)}{k\,p_\infty},\ \varpi^2=\dfrac{\omega^2}{k\,g},\varpi_y^2=\dfrac{\omega_y^2}{k\,g},\ L=h_1\,k,\ \bar{L}=h_2\,k,$ where p_∞ is the equilibrium pressure at the interface, $\alpha_1=\dfrac{\rho_1}{\rho_2+\rho_1},\ \alpha_2=\dfrac{\rho_2}{\rho_2+\rho_1}$. Also we using $k_y=k\cos\theta,$ where θ is the inclination of the wave vector k to the direction of magnetic field ω_y. Then we will analyze the dispersion relation (20) in the next two cases (the wave number is parallel or perpendicular to the magnetic field, respectively)

(1)When the wave number is parallel to the magnetic field ($\vec{k}\parallel\vec{\omega}_y$)

In this case as we know $\left(\vec{k}\times\vec{\omega}_y\right)_{\vec{e}_z}=0$ and $\left(\vec{k}\cdot\vec{\omega}_y\right)^2=k^2\,\omega_y^2$, then Eq. (20) becomes

$$\begin{aligned}
&\left\{1-\exp\left[-\left(\lambda_3-\bar{\lambda}_3\right)L\right]\right\}\times\left\{1-\exp\left[-\left(\lambda_4-\bar{\lambda}_4\right)\bar{L}\right]\right\}\times \\
&\left\{\alpha_2\left\{1-\frac{\alpha_1M\,\varpi^2}{\gamma_1}\right\}-\alpha_1\left\{1-\frac{\alpha_2M\,\varpi^2}{\gamma_2}\right\}\right\}+ \\
&\alpha_1\varpi^2\left\{1-\frac{\alpha_2M\,\varpi^2}{\gamma_2}\right\}\times\left\{\lambda_3-\bar{\lambda}_3\exp\left[-\left(\lambda_3-\bar{\lambda}_3\right)L\right]\right\}\times\left\{1-\exp\left[-\left(\lambda_4-\bar{\lambda}_4\right)\bar{L}\right]\right\}- \\
&\alpha_2\varpi^2\left\{1-\frac{\alpha_1M\,\varpi^2}{\gamma_1}\right\}\times\left\{\bar{\lambda}_4-\lambda_4\exp\left[-\left(\lambda_4-\bar{\lambda}_4\right)\bar{L}\right]\right\}\times\left\{1-\exp\left[-\left(\lambda_3-\bar{\lambda}_3\right)L\right]\right\}=0.
\end{aligned} \tag{21}$$

where $$\lambda_3,\ \bar{\lambda}_3 = \frac{\alpha_1 M}{2} \pm \sqrt{\left[\frac{\alpha_1 M}{2}\right]^2 + \left[1 - \frac{\alpha_1 M\,\varpi^2}{\gamma_1}\right] - \frac{\alpha_1 M}{\varpi^2}\left[1 - \frac{1}{\gamma_1}\right] + \varpi_y^2\left[\frac{\alpha_1 M}{\gamma_1 \cos\theta} - \frac{1}{\varpi^2}\right]}, \quad (22)$$

$$\lambda_4,\ \bar{\lambda}_4 = \frac{\alpha_2 M}{2} \pm \sqrt{\left[\frac{\alpha_2 M}{2}\right]^2 + \left[1 - \frac{\alpha_2 M\,\varpi^2}{\gamma_2}\right] - \frac{\alpha_2 M}{\varpi^2}\left[1 - \frac{1}{\gamma_2}\right] + \varpi_y^2\left[\frac{\alpha_2 M}{\gamma_2 \cos\theta} - \frac{1}{\varpi^2}\right]}. \quad (23)$$

Note that, if we put $\varpi_y = 0$, the dispersion relation (21) is still as above (the same Eq. (21)) , but $\lambda_3,\ \bar{\lambda}_3, \lambda_4$ and $\bar{\lambda}_4$ takes the forms

$$\lambda_3(\varpi_y = 0),\ \bar{\lambda}_3(\varpi_y = 0) = \frac{\alpha_1 M}{2} \pm \sqrt{\left[\frac{\alpha_1 M}{2}\right]^2 + \left[1 - \frac{\alpha_1 M\,\varpi^2}{\gamma_1}\right] - \frac{\alpha_1 M}{\varpi^2}\left[1 - \frac{1}{\gamma_1}\right]}, \quad (24)$$

$$\lambda_4(\varpi_y = 0),\ \bar{\lambda}_4(\varpi_y = 0) = \frac{\alpha_2 M}{2} \pm \sqrt{\left[\frac{\alpha_2 M}{2}\right]^2 + \left[1 - \frac{\alpha_2 M\,\varpi^2}{\gamma_2}\right] - \frac{\alpha_2 M}{\varpi^2}\left[1 - \frac{1}{\gamma_2}\right]}. \quad (25)$$

If we put $\varpi = i\,n$ in Eqs. (21), (22) and (23) we will discover the formula (Eq. 27) that is derived by Livescu [18] for two immiscible fluids (in the absence of surface tension).

Now, in Eq. (21) and from the relation $M = \frac{g(\rho_1 + \rho_2)}{k\,p_\infty}\left(M^{-1} = \frac{k\,p_\infty}{g(\rho_1 + \rho_2)}\right)$ the decrease in p_∞ is equivalent to an increase in M at the ratio of specific heats (γ) is constant and vice versa. Thus, M represents a measure of the compressibility effects on the rate of growth (the first parameter). The other parameter is the ratio of specific heats (γ).

The incompressible plasma case can be obtained from Eq. (21) by letting $\gamma_1, \gamma_2 \to \infty$ or $p_\infty \to \infty$.
(i) At $\gamma_1, \gamma_2 \to \infty$ the dispersion relation (21) takes the form

$$\begin{aligned}
&\left\{1 - \exp\left[-\left(\lambda_5 - \bar{\lambda}_5\right)L\right]\right\} \times \left\{1 - \exp\left[-\left(\lambda_6 - \bar{\lambda}_6\right)\bar{L}\right]\right\} \times \left\{\alpha_2 - \alpha_1\right\} + \\
&\alpha_1\,\varpi^2 \times \left\{\lambda_5 - \bar{\lambda}_5 \exp\left[-\left(\lambda_5 - \bar{\lambda}_5\right)L\right]\right\} \times \left\{1 - \exp\left[-\left(\lambda_6 - \bar{\lambda}_6\right)\bar{L}\right]\right\} - \\
&\alpha_2\,\varpi^2 \times \left\{\bar{\lambda}_6 - \lambda_6 \exp\left[-\left(\lambda_6 - \bar{\lambda}_6\right)\bar{L}\right]\right\} \times \left\{1 - \exp\left[-\left(\lambda_5 - \bar{\lambda}_5\right)L\right]\right\} = 0.
\end{aligned} \quad (26)$$

Where $$\lambda_5,\ \bar{\lambda}_5 = \frac{\alpha_1 M}{2} \pm \sqrt{\left[\frac{\alpha_1 M}{2}\right]^2 + 1 - \frac{\alpha_1 M}{\varpi^2} - \frac{\varpi_y^2}{\varpi^2}}, \quad (27)$$

$$\lambda_6,\ \bar{\lambda}_6 = \frac{\alpha_2 M}{2} \pm \sqrt{\left[\frac{\alpha_2 M}{2}\right]^2 + 1 - \frac{\alpha_2 M}{\varpi^2} - \frac{\varpi_y^2}{\varpi^2}}. \quad (28)$$

This case equivalent to the incompressible plasmas with exponentially varying density.

(ii) At $p_\infty \to \infty (M = 0)$ (the case of constant density) the dispersion relation (21) becomes

$$\left\{1-\exp\left[-2\sqrt{1-\frac{\varpi_y^2}{\varpi^2}}\,L\right]\right\}\times\left\{1-\exp\left[-2\sqrt{1-\frac{\varpi_y^2}{\varpi^2}}\,\bar{L}\right]\right\}\times\left\{\alpha_2-\alpha_1\right\}+$$
$$\alpha_1\varpi^2\sqrt{1-\frac{\varpi_y^2}{\varpi^2}}\times\left\{1+\exp\left[-2\sqrt{1-\frac{\varpi_y^2}{\varpi^2}}\,L\right]\right\}\times\left\{1-\exp\left[-2\sqrt{1-\frac{\varpi_y^2}{\varpi^2}}\,\bar{L}\right]\right\}+ \quad (29)$$
$$\alpha_2\varpi^2\times\sqrt{1-\frac{\varpi_y^2}{\varpi^2}}\left\{1+\exp\left[-2\sqrt{1-\frac{\varpi_y^2}{\varpi^2}}\,\bar{L}\right]\right\}\times\left\{1-\exp\left[-2\sqrt{1-\frac{\varpi_y^2}{\varpi^2}}\,L\right]\right\}=0.$$

At $L = \bar{L} \to \infty$ the dispersion relation (29) becomes $\varpi^4 - \varpi_y^2\varpi^2 - A^2 = 0,$ where $A = \alpha_2 - \alpha_1 = \dfrac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$ is the Atwood number $(0 < A < 1)$. For the imaginary part of $\varpi = i\omega_r = i\sigma,$ the square of normalized growth rate is $\sigma^2 = \dfrac{1}{2}\left\{-\varpi_y^2 + \sqrt{\varpi_y^4 + 4A^2}\right\}$, and at $\varpi_y = 0,$ we have $\sigma^2 = A$ (the classical case).

From Eqs. (26) and (29) as well as from Eq. (21) it clear that, the magnetic field have a critical strength to suppress the instability completely, just, at $A = 0$. In this case we have no system to study. In other words, the magnetic field has no critical strength to suppress the instability completely at $(\vec{k} \parallel \vec{\omega}_y)$.

In fact the previous formulas (21, 26 and 29) are too complicated such that, analytically, we are not able to determine the role of various parameters of problem. Thus, let us suppose that $A > 0$, the system is stable and $\varpi = i\omega_r = i\sigma$ is purely imaginary. Here, σ is the imaginary part of ϖ. In this case, we will try to solve above equations (21, 26 and 29) numerically. The numerical calculations are plotted in Figs. (2) and (3). In this Figures the thicknesses of the two layers is $L = \bar{L} = 10$ and $A = \dfrac{\rho_2 - \rho_1}{\rho_2 + \rho_1} = 0.5$.

In Fig. 2(a) the square of normalized growth rate of RTI of two incompressible plasma layers at $\gamma_1, \gamma_2 \to \infty$ (Eq. 26) have been plotted against $M^{-1} = \dfrac{k\,p_\infty}{g\left(\rho_1 + \rho_2\right)}$ $(0 \le M^{-1} \le 10)$, while at $M = 0 (p_\infty \to \infty)$ (Eq. 29) the square of normalized growth rate is constant. This Figure shows that, the values of σ^2 at $\gamma_1, \gamma_2 \to \infty$ (equivalent to the incompressible plasma with exponentially varying density) are less than its counterpart at $M = 0$ (equivalent to the incompressible plasma with constant density). Which implies that, the exponentially varying density has a stabilizing effect on the RTI growth rate. Fig. 2(a) also shows that as the equilibrium pressure at the interface (p_∞) increases (less compressible flow), σ^2 increases towards the uniform density result. Moreover, the stability role of magnetic field (ϖ_y) clearly rises where the values of σ^2 at $\varpi_y = 0.5$ are less than their counterpart at $\varpi_y = 0$ and this role increases with the magnetic field increasing (see the values of σ^2 at $\varpi_y = 1$). The same phenomenon holds in Fig. 2(b), when the square of normalized growth rate have been plotted against (ϖ_y) at $M = 0$ and $\gamma_1, \gamma_2 \to \infty$. In Fig. 2(b) also one can see that, the values of σ^2 at $\gamma_1, \gamma_2 \to \infty$ approach to zero before than their counterpart at $M = 0$.

Fig. 2(a)

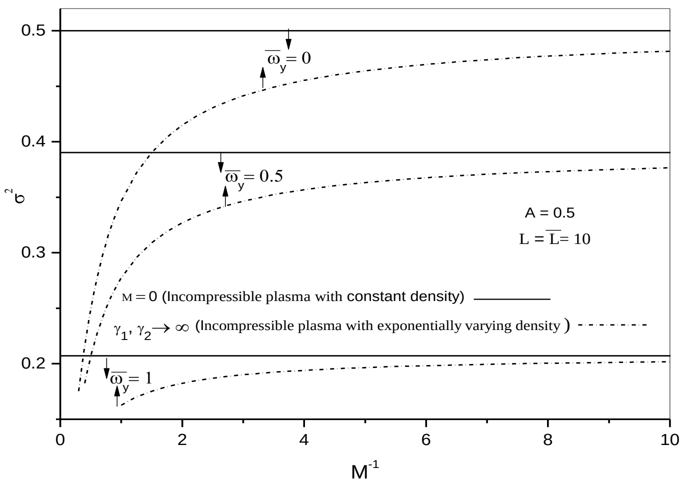

Fig. 2(b)

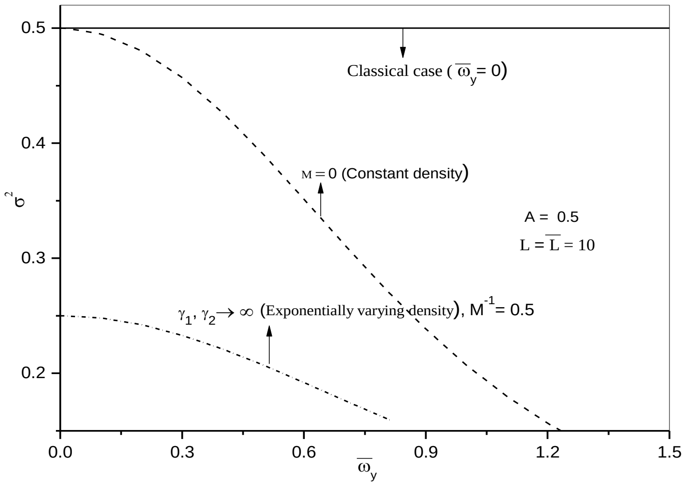

Figure 2: The square of normalized growth rate of RTI of two incompressible plasma layers (i. e. $\gamma_1, \gamma_2 \to \infty$) at $\vec{k} \parallel \vec{\varpi}_y$ (a) has been plotted against $M^{-1} = \dfrac{k\, p_\infty}{g(\rho_1 + \rho_2)}$ $(0 \le M^{-1} \le 10)$, (b) has been plotted against the magnetic field ϖ_y.

The role of compressibility's parameters (the ratio of specific heats (γ) and the static Mach number (M) which is defined by the gravity wave speed and the isothermal speed of sound) have been presented in Figs. 3 (a-d) in the absence (or presence) of external horizontal magnetic (ϖ_y). In Fig. 3(a) the square of normalized growth rate (σ^2) have been plotted against $M^{-1}=\dfrac{k\,p_\infty}{g(\rho_1+\rho_2)}$ $(0\le M^{-1}\le 2)$ at different values of the ratio of specific heats γ_1, γ_2 in the absence of external magnetic field $(\varpi_y=0)$. It can see that, the values of σ^2 decrease as the values of γ_1, γ_2 increasing,

where $\sigma^2(\gamma_1=\gamma_2=10)<\sigma^2(\gamma_1=10, \gamma_2=1)<\sigma^2(\gamma_1=1,\ \gamma_2=10)<\sigma^2(\gamma_1=\gamma_2=1)$. Also one can see that, the values of growth rate in the compressible case $\sigma^2(\gamma_1, \gamma_2)$ are less than their counterpart in the classical case $\sigma^2(M=0)$. Which implies that the compressibility (the ratio of specific heats (γ)) has a stabilizing role on the selected model. This role increases with increasing of γ_1 or γ_2 or the increasing of both of them. Moreover the values of σ^2 at $\gamma_1=10, \gamma_2=1$ are less than their counterpart at $\gamma_1=1, \gamma_2=10$. This implies that the growth rate is more sensitive to the change in compressibility of the lower layer. These results agree with the conclusions given by Livescu [18] for two fluids of different density.

The stability role of external magnetic field $(\vec{\omega}_y \parallel \vec{k})$ of compressible plasmas comes in Figs. 3(b)- 3(d). In Fig. 3(b) the square of normalized growth rate (σ^2) have been plotted against $M^{-1}(0\le M^{-1}\le 2)$ at different values of the ratio of specific heats γ_1, γ_2 and $(\varpi_y=0.5)$. If we compare between the values of σ^2 in Fig. 3(a) and Fig. 3(b), we note that the values of σ^2 in Fig. 3(b) at $\varpi_y=0.5$ are less than their counterpart in Fig. 3(a) at $\varpi_y=0$. Moreover, in Fig. 3(b) we note that the values of σ^2 at $\gamma_1=1, \gamma_2=10$ are less than their counterpart at $\gamma_1=10, \gamma_2=1$, which implies that the growth rate is more sensitive to the change in compressibility of the upper layer. These results disagree with the above result. Absolutely, this exchange ascribes to the presence of magnetic field $(\vec{\omega}_y \parallel \vec{k})$. The same phenomenon holds in Fig. 3(c) when we plot the square of normalized growth rate (σ^2) against the magnetic field $(\vec{\omega}_y)$ at different values of the ratio of specific heats γ_1, γ_2 and $M^{-1}=0.5$, where the values of σ^2 at $\gamma_1=1, \gamma_2=10$ are less than their counterpart at $\gamma_1=10, \gamma_2=1$ for the most values of magnetic field except of the small values of $\vec{\omega}_y$

Fig. 3(d) shows the role of compressibility at $\gamma_1=10, \gamma_2=1$ and $\gamma_1=1, \gamma_2=10$ for different values of the magnetic field $(\varpi_y=0.07, 0.5, 1)$. One can see that the values of σ^2 at $\gamma_1=1, \gamma_2=10$ are less than their counterpart at $\gamma_1=10, \gamma_2=1$, except some values at small values of the magnetic field $(\varpi_y=0.07)$. This implies that the growth rate is more sensitive to the change in compressibility of the upper layer in the presence of magnetic field at $(\vec{\omega}_y \parallel \vec{k})$.

In Figs. 3(a)–3(c), one can see that the stability that happens in the presence of ratio of specific heats (γ) (compressible plasmas) is confined between the instability of incompressible plasma (with constant density, that happens at $M=0$ (Eq. (29))) from top and incompressible plasma (with exponentially varying density, that happens at both $\gamma_1, \gamma_2 \to \infty$ (Eq. (26))) from bottom.

In Figs. 3(a), 3(b) and 3(d), one can see that the increase of $M^{-1}\left(=\dfrac{k\,p_\infty}{g(\rho_1+\rho_2)}\right)$ (is equivalent to an increase in (p_∞) at the ratio of specific heats (γ) is constant) leads to a increase in the square of the growth rate, but the values of growth rate as a functions of (p_∞) are less than their counterpart in the classical case. Which implies that the compressibility (the static Mach number (M) or the pressure (p_∞) at the interface) has a stabilizing role on the selected model. This role declines with increasing of (p_∞) .

Fig. 3(a)

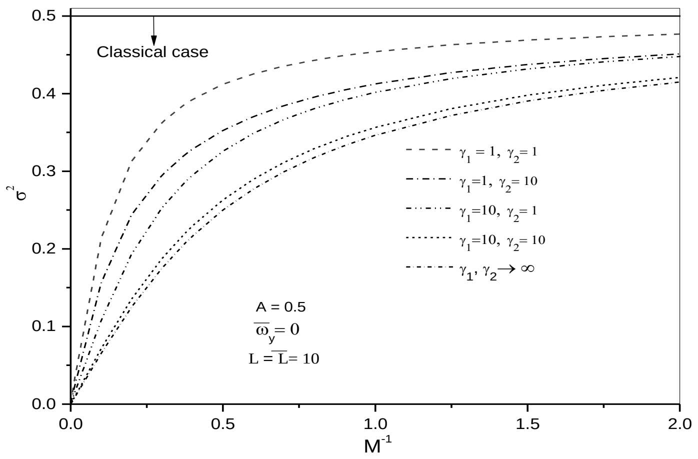

Fig. 3(b)

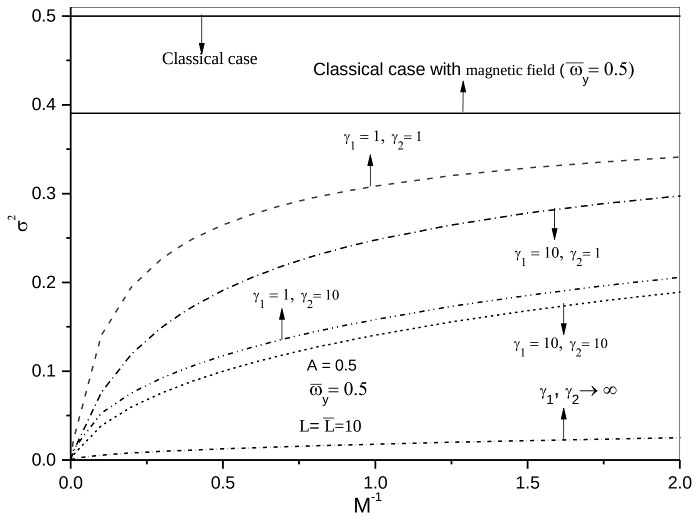

Fig. 3(c)

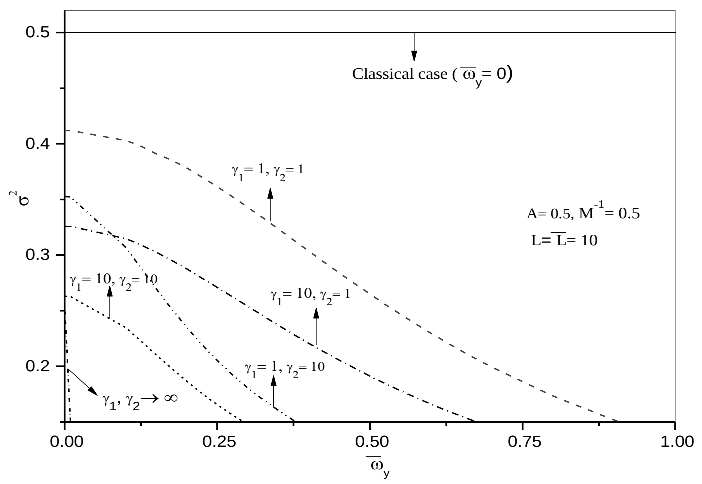

Fig. 3(d)

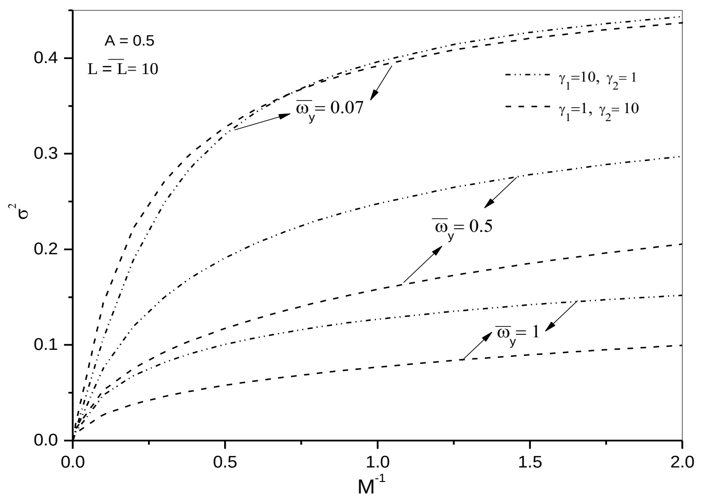

Figure 3: The role of compressibility's parameters (the ratio of specific heats (γ) and the static Mach number (M)) in the presence and absence of magnetic field (at $\vec{k} \parallel \vec{\omega}_y$).

(2) When the wave number perpendicular to the magnetic field (i. e. $\vec{k} \perp \vec{\omega}_y$)

In this case we have $\left(\vec{k} \times \vec{\omega}_y\right)_{\vec{e}_z} = k\,\omega_y$ while $\left(\vec{k} \cdot \vec{\omega}_y\right) = 0$, then Eq. (20) becomes

$$\begin{aligned}&\left\{1-\exp\left[-\left(\lambda_7-\bar{\lambda}_7\right)L\right]\right\}\times\left\{1-\exp\left[-\left(\lambda_8-\bar{\lambda}_8\right)\bar{L}\right]\right\}\times\\&\left\{\frac{1}{\varpi^2}+\frac{\varpi_y}{\varpi}\right\}\left\{\alpha_2\left\{1-\frac{\alpha_1 M\,\varpi^2}{\gamma_1}\right\}-\alpha_1\left\{1-\frac{\alpha_2 M\,\varpi^2}{\gamma_2}\right\}\right\}+\\&\alpha_1\left\{1-\frac{\alpha_2 M\,\varpi^2}{\gamma_2}\right\}\times\left\{\lambda_7-\bar{\lambda}_7\exp\left[-\left(\lambda_7-\bar{\lambda}_7\right)L\right]\right\}\times\left\{1-\exp\left[-\left(\lambda_8-\bar{\lambda}_8\right)\bar{L}\right]\right\}-\\&\alpha_2\left\{1-\frac{\alpha_1 M\,\varpi^2}{\gamma_1}\right\}\times\left\{\bar{\lambda}_8-\lambda_8\exp\left[-\left(\lambda_8-\bar{\lambda}_8\right)\bar{L}\right]\right\}\times\left\{1-\exp\left[-\left(\lambda_7-\bar{\lambda}_7\right)L\right]\right\}=0,\end{aligned} \tag{30}$$

$$\lambda_7,\,\bar{\lambda}_7=\frac{\alpha_1 M}{2}\pm\sqrt{\left[\frac{\alpha_1 M}{2}\right]^2+\left[1-\frac{\alpha_1 M\varpi^2}{\gamma_1}\right]-\frac{\alpha_1 M}{\varpi^2}\left[1-\frac{1}{\gamma_1}\right]+\frac{\varpi_y\alpha_1 M}{\varpi}\left(\frac{2}{\gamma_1}-1\right)}, \tag{31}$$

$$\lambda_8,\,\bar{\lambda}_8=\frac{\alpha_2 M}{2}\pm\sqrt{\left[\frac{\alpha_2 M}{2}\right]^2+\left[1-\frac{\alpha_2 M\,\varpi^2}{\gamma_2}\right]-\frac{\alpha_2 M}{\varpi^2}\left[1-\frac{1}{\gamma_2}\right]+\frac{\varpi_y\alpha_2 M}{\varpi}\left(\frac{2}{\gamma_2}-1\right)}. \tag{32}$$

In the absence of magnetic field ($\varpi_y=0$), then Eq. (30) tends to Eq. (21) and $\lambda_7(\varpi_y=0)$, $\bar{\lambda}_7(\varpi_y=0)$ are equivalent to $\lambda_3(\varpi_y=0)$, $\bar{\lambda}_3(\varpi_y=0)$ as in Eq. (24), respectively, while $\lambda_8(\varpi_y=0), \bar{\lambda}_8(\varpi_y=0)$ are equivalent to $\lambda_4(\varpi_y=0)$, $\bar{\lambda}_4(\varpi_y=0)$ as in Eq. (25), respectively. Appearance of the term $\frac{\varpi_y}{\varpi}$ in the system (30)-(32) makes physical interpretations harder without taking a special cases and numerical calculations. It is useful to consider the case of small-compressibility $(M<<1)$ and $L=\bar{L}\to\infty$. In this case the dispersion relation (30) can be written in the formula

$$\begin{aligned}&\left\{\frac{1}{\varpi^2}+\frac{\varpi_y}{\varpi}\right\}\left\{\alpha_2\left(1-\frac{\alpha_1 M\,\varpi^2}{\gamma_1}\right)-\alpha_1\left(1-\frac{\alpha_2 M\,\varpi^2}{\gamma_2}\right)\right\}+\\&\qquad\alpha_1\,\lambda_7\left\{1-\frac{\alpha_2 M\,\varpi^2}{\gamma_2}\right\}-\alpha_2\,\bar{\lambda}_8\left\{1-\frac{\alpha_1 M\,\varpi^2}{\gamma_1}\right\}=0,\end{aligned} \tag{33}$$

$$\lambda_7=1+\frac{\alpha_1 M}{2}-\frac{\alpha_1 M\,\varpi^2}{2\gamma_1}-\frac{\alpha_1 M}{2\,\varpi^2}\left[1-\frac{1}{\gamma_1}\right]+\frac{\varpi_y\alpha_1 M}{2\,\varpi}\left[\frac{2}{\gamma_1}-1\right], \tag{34}$$

$$\bar{\lambda}_8=-1+\frac{\alpha_2 M}{2}+\frac{\alpha_2 M\,\varpi^2}{2\gamma_2}+\frac{\alpha_2 M}{2\,\varpi^2}\left[1-\frac{1}{\gamma_2}\right]-\frac{\varpi_y\alpha_2 M}{2\,\varpi}\left[\frac{2}{\gamma_2}-1\right]. \tag{35}$$

Substituting from Eqs. (34) and (35) into Eq. (33) under the condition $(M<<1)$, the dispersion relation may be written in the form

$$\xi_1 \overline{\varpi}^4 + \xi_2 \overline{\varpi}^2 + \xi_3 \overline{\varpi} + \xi_4 = 0, \tag{36}$$

$$\xi_1 = \left\{\left(\frac{1}{\gamma_2} - \frac{1}{\gamma_1}\right)\alpha_1 \alpha_2 \overline{\varpi}_y - \left(\frac{\alpha_1}{2\gamma_1} + \frac{\alpha_2}{\gamma_2}\right)\alpha_1 - \left(\frac{\alpha_1}{\gamma_1} + \frac{\alpha_2}{2\gamma_2}\right)\alpha_2\right\} M,$$

$$\xi_2 = \left[\left(\frac{1}{\gamma_2} - \frac{1}{\gamma_1}\right)\alpha_1 \alpha_2 + \frac{1}{2}\left(\alpha_1^2 - \alpha_2^2\right)\right] M + A\overline{\varpi}_y + 1,$$

$$\xi_3 = \frac{1}{2}\left\{\left(\frac{2}{\gamma_1} - 1\right)\alpha_1^2 + \left(\frac{2}{\gamma_2} - 1\right)\alpha_2^2\right\} M \overline{\varpi}_y, \tag{37}$$

$$\xi_4 = A + \frac{1}{2}\left[\left(\frac{1}{\gamma_1} - 1\right)\alpha_1^2 + \left(\frac{1}{\gamma_2} - 1\right)\alpha_2^2\right] M.$$

Under the above appreciation we study the incompressible case of Eq. (36)

(i) If we put $\gamma_1, \gamma_2 \to \infty$ (The density changes with the vertical coordinate), then Eq. (36) is

$$\overline{\varpi}^2 - \frac{\frac{M}{2}\left\{\alpha_1^2 + \alpha_2^2\right\}\overline{\varpi}_y}{\frac{1}{2}\left[\alpha_1^2 - \alpha_2^2\right] M + A\overline{\varpi}_y + 1}\overline{\varpi} + \frac{A - \frac{M}{2}\left\{\alpha_1^2 + \alpha_2^2\right\}}{\frac{1}{2}\left[\alpha_1^2 - \alpha_2^2\right] M + A\overline{\varpi}_y + 1}, \tag{38}$$

At $\overline{\varpi} = \omega_r^* + i\omega_i^*$ ($\omega_i^* = \sigma$), the solution of this equation is

$$\overline{\varpi} = \omega_r^* + i\sigma = \frac{\frac{M}{2}\left\{\alpha_1^2 + \alpha_2^2\right\}\overline{\varpi}_y}{2\left\{\frac{1}{2}\left[\alpha_1^2 - \alpha_2^2\right] M + A\overline{\varpi}_y + 1\right\}} \pm i\sqrt{\left\{\frac{\alpha_2 - \alpha_1 - \frac{M}{2}\left\{\alpha_1^2 + \alpha_2^2\right\}}{\frac{1}{2}\left[\alpha_1^2 - \alpha_2^2\right] M + A\overline{\varpi}_y + 1} - \left[\frac{\frac{M}{2}\left\{\alpha_1^2 + \alpha_2^2\right\}\overline{\varpi}_y}{2\left\{\frac{1}{2}\left[\alpha_1^2 - \alpha_2^2\right] M + A\overline{\varpi}_y + 1\right\}}\right]^2\right\}}. \tag{39}$$

Then the square of the normalized growth rate given by

$$\sigma^2_{\gamma_1,\gamma_2\to\infty} = \frac{A - \frac{1}{2}\left\{\alpha_1^2 + \alpha_2^2\right\} M}{\frac{1}{2}\left\{\alpha_1^2 - \alpha_2^2\right\} M + A\overline{\varpi}_y + 1} - \left[\frac{\left\{\alpha_1^2 + \alpha_2^2\right\} M \overline{\varpi}_y}{2\left\{\alpha_1^2 - \alpha_2^2\right\} M + 4A\overline{\varpi}_y + 4A}\right]^2. \tag{40}$$

(ii) If we put $M = 0$ (constant density case) in Eq. (36) the dispersion relation becomes $\left\{A\overline{\varpi}_y + 1\right\}\overline{\varpi}^2 + A = 0$ and if we put $\omega^* = \omega_r^* + i\omega_i^*$, Then

$$\omega_i^{*^2} = \sigma_{M=0}^2 = \frac{A}{1+A\varpi_y} . \tag{41}$$

From Eqs. (40) and (41), one can see that $\sigma^2_{\gamma_1,\gamma_2\to\infty} < \sigma^2_{M=0}$. Which implies that, the exponentially varying density has a stabilizing effect on the RTI growth rate as that is found in the first case (as $\vec{k} \parallel \vec{\varpi}_y$). Also, the incensing of magnetic field (at $\vec{k} \perp \vec{\varpi}_y$) tends to more stability in the selected model. In Eq. (41), we note the magnetic field has no critical strength to suppress the instability completely.

(iii) For the general case (at $\vec{k} \perp \vec{\varpi}_y$)

The condition of Rayleigh–Taylor instability of compressible plasmas can be obtained easily from the constant term of (36) that given as:

$$A < \frac{1}{2}\left[\left(1-\frac{1}{\gamma_1}\right)\alpha_1^2 + \left(1-\frac{1}{\gamma_2}\right)\alpha_2^2\right]M . \tag{42}$$

Thus the system remains unstable for all the values of Atwood number smaller than the value given by condition (42).

Now, at $\omega^* = \omega_r^* + i\omega_i^*$, Eq. (36) can be put as:

$$\xi_1\left\{\omega_r^4 + \omega_i^4 - 6\omega_r^2\,\omega_i^2 + 4i\omega_r\,\omega_i(\omega_r^2-\omega_i^2)\right\} + \xi_2\left\{\omega_r^2 - \omega_i^2 + 2i\omega_r\,\omega_i\right\} + \xi_3\left\{\omega_r + i\omega_i\right\} + \xi_4 = 0, \tag{43}$$

Thus, both the real and imaginary parts are, respectively, in the form

$$\xi_1\left\{\omega_r^4 + \sigma^4 - 6\omega_r^2\,\sigma^2\right\} + \xi_2\left\{\omega_r^2 - \sigma^2\right\} + \xi_3\,\omega_r + \xi_4 = 0 , \tag{44}$$

$$4\xi_1\,\omega_r\left\{\omega_r^2 - \sigma^2\right\} + 2\xi_2\,\omega_r + \xi_3 = 0 . \tag{45}$$

In fact we cannot combined the above two equations to generate a single equation that gives the normalized growth rate $\sigma(=\omega_i^*)$ as a function of the parameters of problem (ϖ_y, M and γ_1,γ_2). But, we can put them in the form

$$\omega_r^6 - \frac{1}{4}\left\{\frac{\xi_2^2}{4\xi_1^2} - 2\frac{\xi_2}{\xi_1}\right\}\omega_r^4 - \frac{1}{4}\left\{\frac{\xi_4}{\xi_1} - \frac{\xi_2^2}{2\xi_1^2}\right\}\omega_r^2 - \frac{\xi_3^2}{64\,\xi_1^2} = 0 , \tag{46}$$

$$\sigma^2 = \omega_i^2 = \omega_r^2 + \frac{\xi_2}{2\xi_1} + \frac{\xi_3}{4\omega_r\,\xi_1} . \tag{47}$$

Now, we will numerically solve these two equations (46) and (47) (or (44) and (45)) to determine the behavior of normalized growth rate (σ) against of (ϖ_y, M and γ_1, γ_2). The numerically calculations for our current case have been presented in Fig. 4.

Fig. 4(a) shows the role of compressibility (the role of ratio of specific heats (γ)) in the absence of magnetic field. The square of normalized growth rate (σ^2) has plotted against M^{-1}. One can see that the values of σ^2 decreases as the values of ratio of specific heats increasing (as the first case). Also, In contrast to the first case, one can see that the values of σ^2 at $\gamma_1 = 1, \gamma_2 = 10$ are less than their counterpart at $\gamma_1 = 10, \gamma_2 = 1$. Which implies that the growth rate is more sensitive to the change in compressibility of the upper layer in the absence of magnetic field at $(\vec{\omega}_y \perp \vec{k})$.

Figs. 4(b), shows the role of ratio of specific heats in the presence of magnetic field $(\varpi_y = 0.5)$. One can see that, the values of σ^2 in Fig. 4(b) at $(\varpi_y = 0.5)$ are less than their counterpart in Fig. 4(a) at $(\varpi_y = 0)$. Also, the values of σ^2 at $\gamma_1 = 1, \gamma_2 = 10$ are less than their counterpart at $\gamma_1 = 10, \gamma_2 = 1$ (as Fig. 3(a)).

Fig. 4(a)

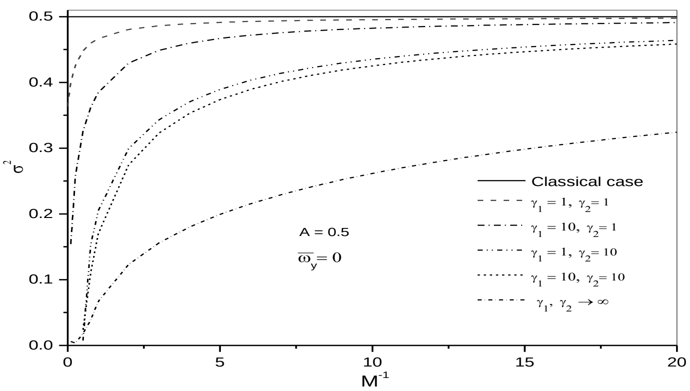

Fig. 4(b)

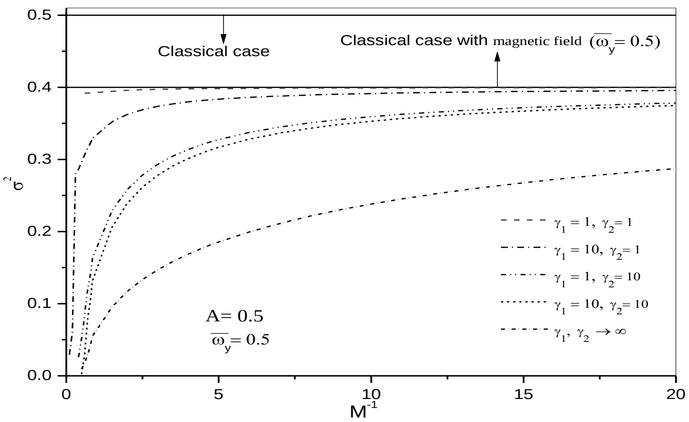

Figure 4: The role of compressibility's parameters (the ratio of specific heats (γ) and the static Mach number (M)) in the presence and absence of magnetic field (at $\vec{k} \perp \vec{\omega}_y$).

This phenomenon have been emphasized in Fig. 5, where the square of normalized growth rate (σ^2) have plotted against the magnetic field $(\overline{\varpi}_y)$ at $M = 0.1$. This Figure shows that, the values of (σ^2) decreases as the values of $(\overline{\varpi}_y)$ increasing as $\sigma^2(\gamma_1 = 1, \gamma_2 = 10) < \sigma^2(\gamma_1 = 10, \gamma_2 = 1)$.

Fig. 5

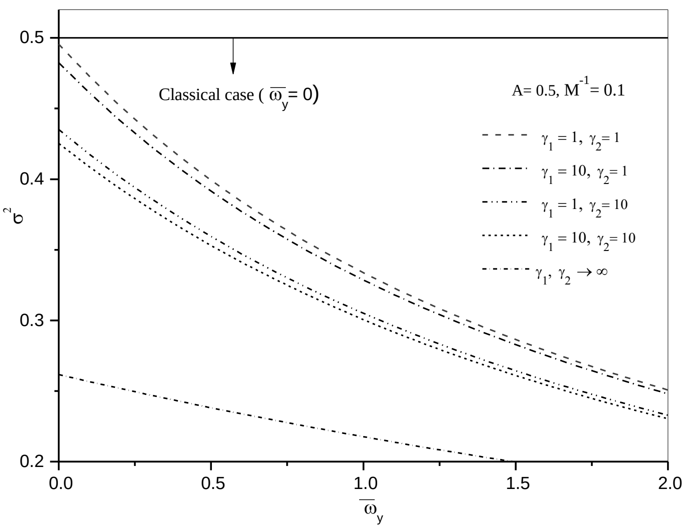

Figure 5: The square of normalized growth rate (σ^2) *against the magnetic field* $(\overline{\varpi}_y)$ *at* $M = 0.1$.

4. Conclusion

In summary, the compressibility effect on the Rayleigh–Taylor instability of two plasmas layers have been investigated in inhomogeneous plasma rotating uniformly in an external transverse magnetic field. The general dispersion relation is obtained. The dispersion relation is analyzed at the external transverse magnetic field parallels and is perpendicular to the wave number and is numerically solved. We can summarize the results as follows:

(i) In the absence of (or presence) external transverse magnetic field, the compressibility have been characterized by two parameters (γ and p_∞). For the parameter γ (the ratio of specific heats values), the compressibility has stability role on the growth rate. This role increases with increase of the ratio of specific heats values. While the influences of equilibrium pressure at the interface p_∞ on the RTI growth rates are destabilized. (ii) The incompressible exponentially varying density plasma ($\gamma \to \infty$) has a lower growth rate than the incompressible constant density plasma. This implies that, the exponentially varying density has a stabilizing effect on the RTI growth rates.

(iii) In the absence of magnetic field (at $\vec{k} \parallel \vec{\omega}_y$) the growth rate is more sensitive to the change in compressibility of the lower layer (the change in γ parameter), while in the presence of magnetic field the growth rate becomes more sensitive to the change in compressibility of the upper layer.

(iv) In the case of magnetic field is perpendicular to the wave number ($\vec{k} \perp \vec{\omega}_y$) and for our appreciation selected $(M << 1)$, the growth rate is more sensitive to the change in compressibility of the upper layer (the change in γ parameter). This sensitivity satisfies in the absence of (or presence) external transverse magnetic field

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