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Research Article

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Study on the Large- Scale RC Beams Shear Strengthened With FRP Sheets

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Abstract This paper presents a nonlinear Finite Element Analysis (FEA) that has been carried out to simulate the behavior of modes failure of Reinforced Concrete (RC) beams strengthened in shear by Fiber Reinforced Polymer (FRP) laminates. Sixty two beams were adoptable in FEM model software using ANSYS. Six beams of them were adoptable in ANSYS and validation of experimental results, three of this sex beams were control beam without strengthened and other beams were strengthened with Basalt Fiber Reinforced Polymers (BFRP). From the analyses of relationships the load with deflection, crack pattern, first crack load and ultimate load was obtained and compared with the experimental results available in laboratory. The load deflection plots obtained from numerical studies show good agreement with the experimental plots. There was a difference at the behavior between the RC beams strengthened with and without BFRP layers. The prediction models that were proposed by considering all common parameters that influence the ultimate shear capacity of strengthened beam including beam width, concrete strength, effective height of the beam, FRP thickness, height of FRP sheet, strengthening configuration (completely wrapped, U-jacketing and side bonding). Therefore, the models of the experimental beams can be adoptable in ANSYS and validation of experimental results can be done using ANSYS.

Keywords ANSYS Program, finite element analysis (FEA), FRP Sheets

Introduction

The application of fiber reinforced polymers has received as an external reinforcement a lot of attention from structural engineering. The ferries reinforcement outer shell to enhance or rehabilitate reinforced concrete structures has gained popularity in the current century. Strands and furs can be used externally bonded to increase shear strength of reinforced concrete beams.

Experimental testing has been used extensively as a mean of analyzing individual elements and concrete strength effects under loading. While this is the way that produces a real life response, it is very time consuming and the use of materials can be very expensive. Analytical modeling of reinforced concrete beams has been carried out externally with FRP slices using FEM that adopted by ANSIS.

FEM accuracy is evaluated compared to experimental results, which are in good agreement.

The load-deflection curves from the finite element analysis agree well with the experimental results.

Some of the research work carried out on comparative study between experimental and analytical work in FRP strengthening describe below as Amer Ibrahim [1], Ahmed. M. S (2014) [2], Saifullah [3], Patil [4], performed numerical analysis on RC beams by ANSYS finite element program and the results show that the general behavior of the finite element models represented by the load-deflection curves at mid span show good agreement with the test data.

Finite Element Modeling by Ansys

Concrete Modeling

Concrete is a quasi-brittle material and has different behavior in compression and tension. Development of a model for the behavior of concrete is a challenging task for researchers. The Solid 65 element was used to model the concrete. This element has eight nodes with three degrees of freedom at each node – translations in the nodal x-, y- and z-directions. This element is capable of plastic deformation, cracking in three orthogonal directions, and crushing. The geometry and node locations for this element type are shown in Figure1.

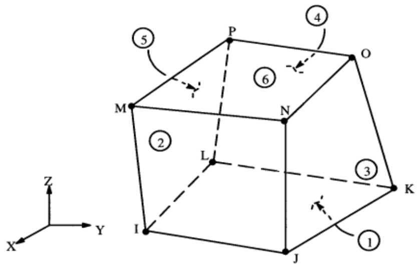

Figure 1: Solid 65 – 3-D reinforced concrete solid

Steel Reinforcement

A Link8 element was used to model the steel reinforcement. Two nodes are required for this element. Each node has three degrees of freedom, translations in the nodal x, y, and z directions. The element is also capable of plastic deformation. The geometry and node locations for this element type are shown in Figure 2.

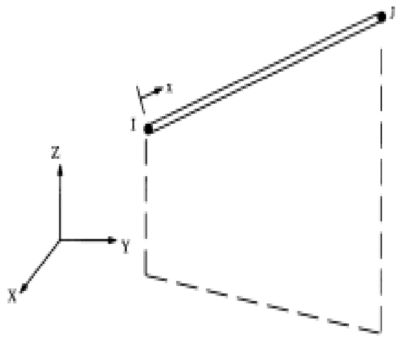

Figure 2: Link 8 – 3-D spar

FRP Composite

A layered solid element, solid 185, was used to model the FRP composites. The element allows for up 100 different material layers with different orientations and orthotropic material properties in each layer. The element has three degrees of freedom at each node and translations in the nodal x, y, and z directions. The geometry, node locations, and the coordinate system are shown in Figure 3.

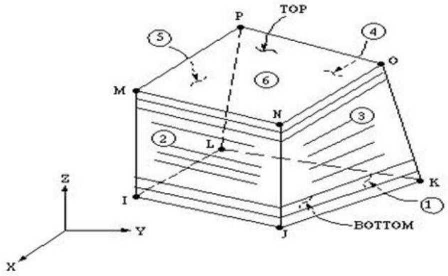

Figure 3: Solid 185 – 3-D layered structural solid

Steel Plates

An eight-node solid element, Solid 45, was used for the steel plates at the supports in the beam models. The element is defined with eight nodes having three degrees of freedom at each node translations in the nodal x, y, and z directions. The geometry and node locations for this element type are shown in Figure 4.

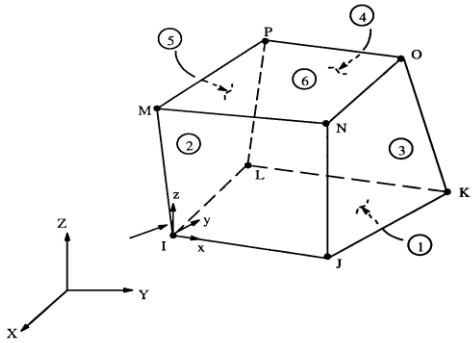

Figure 4: Solid 45 – 3-D solid

The Correlation between Theoretical and Experimental

Six rectangular reinforced concrete (RC) beams were verified in the present study [2] details of this beams specimens are summarized in the Figure 5. To obtain the same main reinforcement ratio ρs equal 3.0% area concrete for all beams, three bar sizes were used 10, 25 and 32 mm diameters and the web bars consisted of 10.0 mm diameter stirrups with yield strength of 430 MPa at spacing equal 200 mm at shear span and 150mm at flexural span, as shown in Figure 6.

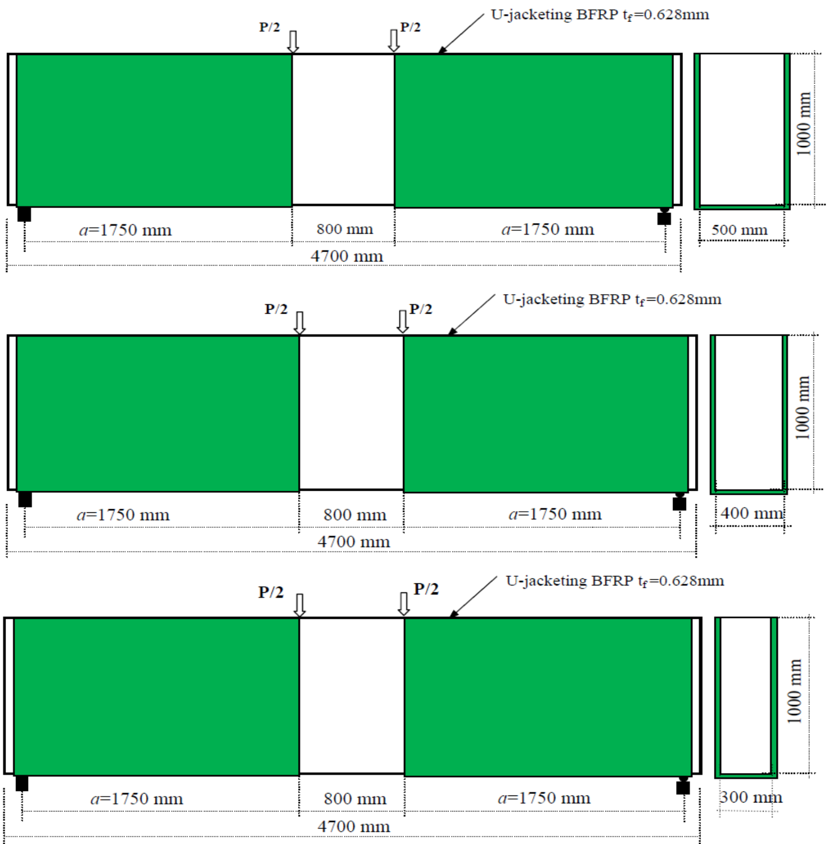

Figure 5: Geometrical details of the RC beams strengthened with U-jacketing

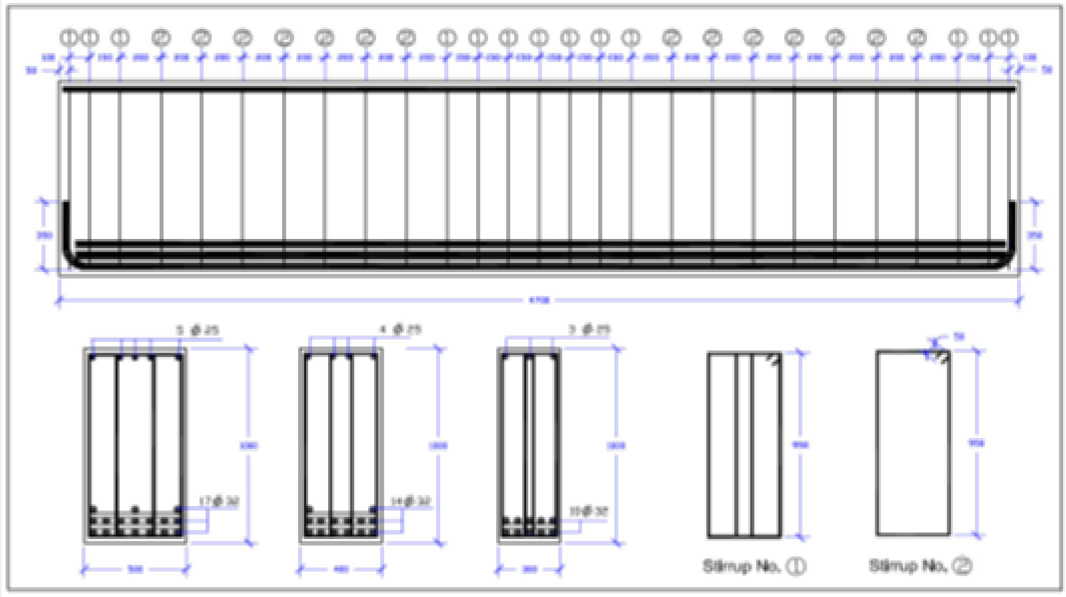

Figure 6: Geometrical details of the large- scale RC beams

Analysis and Discussion Obtained test result:

The pattern of cracks, shear capacity and modes of failure were observed for all beams for the different beam series as follows.

Beams of Series Large-Scale Dimension (C-1000/300)

For the large-scale dimension, the first crack for control beam C-1000/300(without strengthened) was observed at bottom concrete surface at mid-span of the beam at cracking load equal to 300 kN, and by increasing applied load, it increases the crack at shear zone and the first crack was observed at shear zone at cracking load equal to 360 KN. The final mode of failure was of shear type one in shear-span zone. The experimental test specimen failed at a corresponding applied ultimate load-carrying capacity equals to 1260kN and obtained theoretical applied ultimate load-carrying capacity equals to 1240 KN as show at Figure7 and the percentage of the results of the theoretical test and its similarities in the practical is 98.4%.

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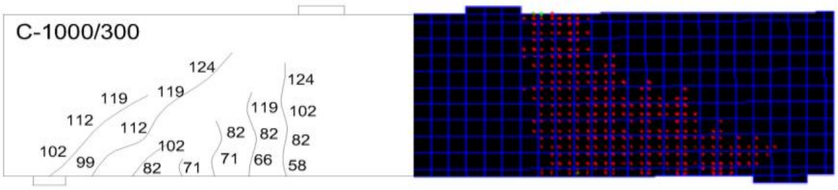
a) Experimental



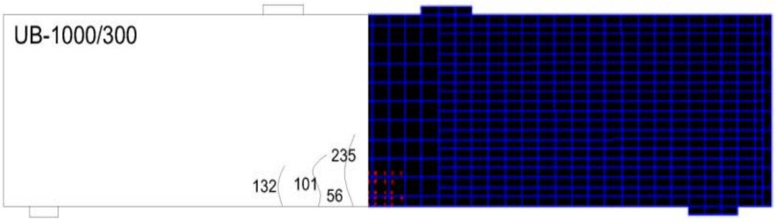
b) Theoretical

Figure 7: Crack pattern of beam (C-1000/300)

Beams of Series Large-Scale Dimension (UB-1000/300)

The first crack for large-scale dimension strengthened beam (U-B-1000/300) strengthened with four-layers BFRP sheets in the form of (U-jacketing) started vertically at bottom concrete surface in flexural zone under point of load application at cracking load equal to 360 kN, and propagated vertically up to level of two-third of the height for beam. The ultimate shear capacity of U-B-1000/300 is 2440 kN at experimental test. The mode of failure was observed BFRP tensile rupture failure type, at obtained theoretical the ultimate shear capacity is 2351.76 KN and the mode of failure was observed BFRP tensile rupture failure type as show at Figure 8 and the percentage of the results of the theoretical test and its similarities in the practical is 96.38 %.

a) Experimental

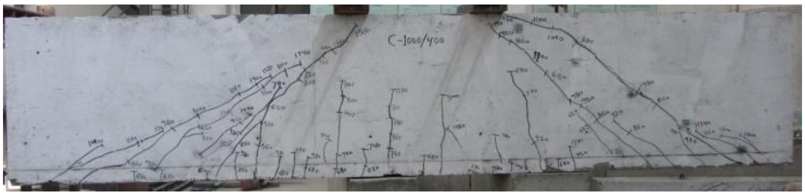


b) Theoretical)

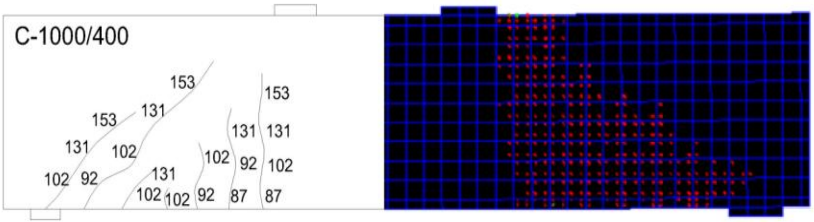
Figure 8: Crack pattern of beam (UB-1000/300)

Beams of Series Large-Scale Dimension (C-1000/400)

For the large-scale dimension, the first crack for control beam C-1000/400 (without strengthened) was observed at bottom concrete surface at mid-span of the beam at cracking load equal to 440 KN, and by increasing applied load, it increases the crack at shear zone and the first crack was observed at shear zone at cracking load equal to620 KN. The final mode of failure was of shear type one in shear-span zone. The experimental test specimen failed at a corresponding applied ultimate load-carrying capacity equals to 1600 KN and obtained theoretical applied ultimate load-carrying capacity equals to 1538.9 KNas show at Figure 9 and the percentage of the results of the theoretical test and its similarities in the practical is 96.18%.



a) Experimental



b) Theoretical)

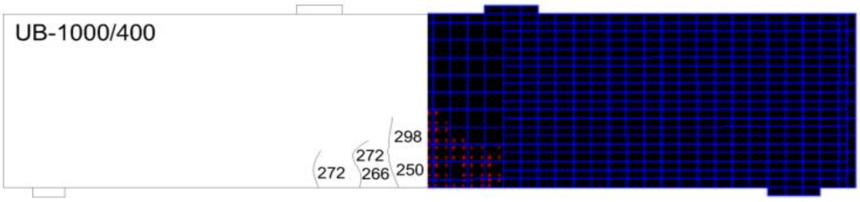
Figure 9: Crack pattern of beam (C-1000/400)

Beams of Series Large-Scale Dimension (UB-1000/400)

The first crack for large-scale dimension strengthened beam UB-1000/400 (strengthened with four-layers BFRP sheets in the form of U-jacketing) started vertically at bottom concrete surface in flexural zone under point of load application at cracking load equal to 360 kN, and propagated vertically up to level of two-third of the height for beam. as The ultimate shear capacity of U-B-1000/400 is 3050 kN at experimental test. The mode of failure was observed BFRP debonding failure type, at obtained theoretical the ultimate shear capacity is

2981.52 KN and the mode of failure was observed BFRP debonding failure type as show at Figure 10 and the percentage of the results of the theoretical test and its similarities in the practical is 97.77%.

a) Experimental

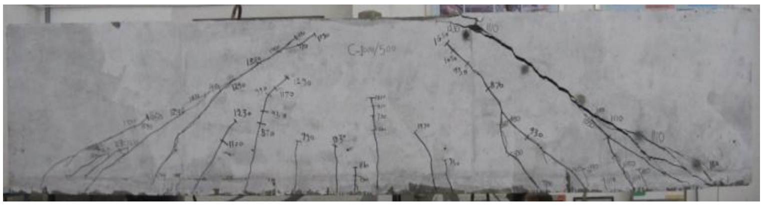


b) Theoretical

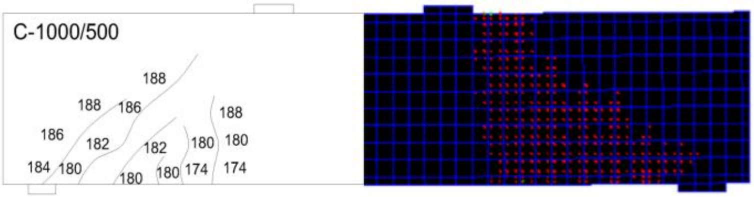
Figure 10: Crack pattern of beam (UB-1000/400)

Beams of Series Large-Scale Dimension (C-1000/500)

For the large-scale dimension, the first crack for control beam C-1000/500 (without strengthened) was observed at bottom concrete surface at mid-span of the beam at cracking load equal to 630 KN, and by increasing applied load, it increases the crack at shear zone and the first crack was observed at shear zone at cracking load equal to 810 KN. The final mode of failure was of shear type one in shear-span zone. The experimental test specimen failed at a corresponding applied ultimate load-carrying capacity equals to 1950 KN and obtained theoretical applied ultimate load-carrying capacity equals to 1886 KN as show at Figure 11 and the percentage of the results of the theoretical test and its similarities in the practical is 96.72 %.



a) Experimental



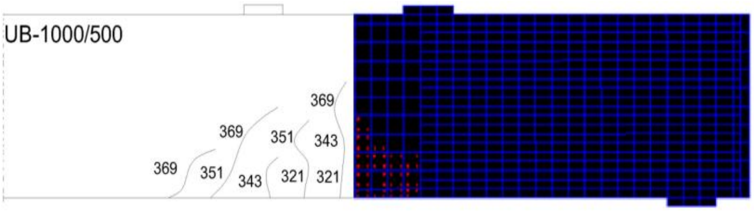
b) Theoretical

Figure 11: Crack pattern of beam (C-1000/500)

Beams of Series Large-Scale Dimension (UB-1000/500)

The first crack for large-scale dimension strengthened beam UB-1000/500 (strengthened with four-layers BFRP sheets in the form of U-jacketing) started vertically at bottom concrete surface in flexural zone under point of load application at cracking load equal to 360 kN, and propagated vertically up to level of two-third of the height for beam. as The ultimate shear capacity of U-B-1000/400 is 3860 kN at experimental test. The mode of failure was observed BFRP debonding failure type, at obtained theoretical the ultimate shear capacity is 3690 KN and the mode of failure was observed BFRP debonding failure type as show at Figure 12 and the percentage of the results of the theoretical test and its similarities in the practical is 95.6 %.

a) Experimental



b) Theoretical

Figure 12: Crack pattern of beam (UB-1000/500)

The Summarized Theoretical Study for Beams in the Present

The study of the effect of the new variables on the prediction of the final shear strength of reinforced concrete (RC) beams with fiber reinforced polymer (FRP) sheets is what has been discussed. Fifty six specimens were analyzed by considering the effect of beam width, concrete strength, shear span-to-depth ratio, FRP thickness, and strengthening configuration (completely wrapped, U-jacketing, and side bonding. Experimental results of beams collected from previous published work were analyzed to verify the accuracy of the proposed model. The results show that lateral strain along the top and the bottom of beams are affected by all these variables. This was not considered in previous beams strengthened with FRP sheets with higher accuracy than existing models , coefficients of variation reaching 19.53% for side bonding ,17.07% for U-jacketing , and 17.46%for completely wrapped , respectively. The test program was mainly intended to cover the testing of theoretical study on the Large-Scale (RC) beams shear strengthened with FRP sheets taking into account the following parameters:

1. The change in characteristic strength for concrete.
2. The change in distance between sheets.
3. The change in the number of layers of strengthens.
4. The change in the shape of strengthen
5. The change in the area section of stirrups

Effect of the Distance Between FRB Sheets on the Strength of Strengthen Beams

Twenty high strength beams having 4.7m total length with rectangular cross-section of 0.3m breadth and constant height equals to 1m. These beams were tested over a simple span of 4.5m under two equal loads far from support 1.75m. All beams were reinforced with ten tension bars from high tensile steel of 32.0 mm diameter and three compression bars of 16.0 mm diameter plus stirrups of 10.0 mm diameter at variable spacing, as shown in Figure (13).

These beams were divided into four series (A-B-C-D) according to the degree of high strength concrete and the distance between sheets.

Series (A) consists of five reinforced concrete beams with (Cube strength = 250kg/cm2) designated as A.0, A.1, A.2, A.3 and A.4.

Series (B) consists of five reinforced concrete beams with (Cube strength = 400kg/cm2) designated as B.0, B.1, B.2, B.3 and B.4.

Series (C) consists of five reinforced concrete beams with (Cube strength = 500kg/cm2) designated as C.0, C.1, C.2, C.3 and C.4.

Series (D) consists of five reinforced concrete beams with (Cube strength = 700kg/cm2) designated as D.0, D.1, D.2, D.3 and D.4.

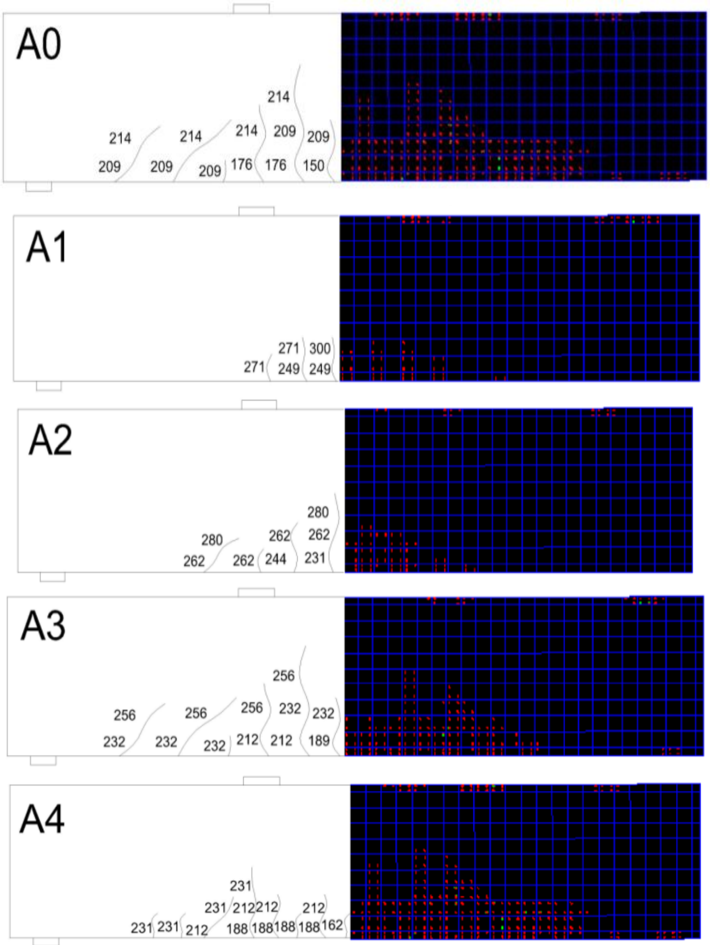


Figure13. Crack pattern for effect of the distance between FRB sheets

Load-Deflection Curves for series "A"

For series "A" (strength = 250 kg/cm2), the rate of deflection decreasing is considered small for beam A.0 but it is considerably big for beams A.1, A.2, A.3 and A.4. The difference between these rates for beams A.1, A.2, A.3 and A.4 are considerably small. Also the measured maximum load and deflection at failure point for strengthened beams are considered very big for that of the original beam as shown in figure (14).

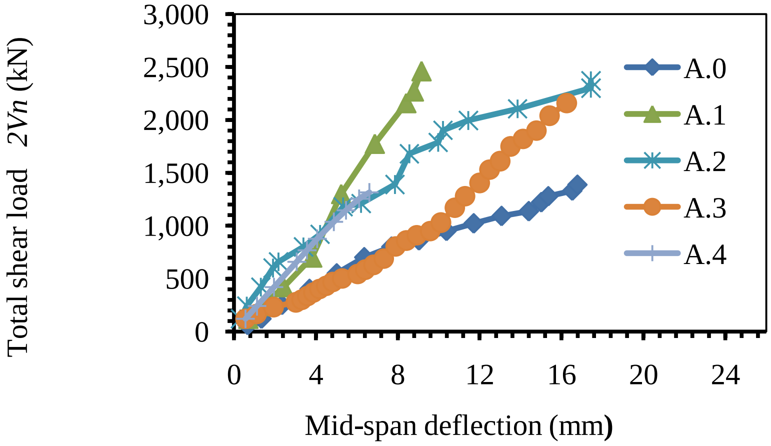


Figure 14: Load–Deflection relationship for effect of the distance between FRB sheets for series "A"

Load-Deflection Curves for Series "B"

For series "B" (strength = 400 kg/cm2), the rate of deflection decreasing is considered small for beam B.0 but it is considerably big for beams B.1, B.2, B.3 and B.4. The difference between these rates for beams B.1, B.2, B.3 and B.4 are considerably small. Also the measured maximum load and deflection at failure point for strengthened beams are considered very big for that of the original beam as shown in figure (15).

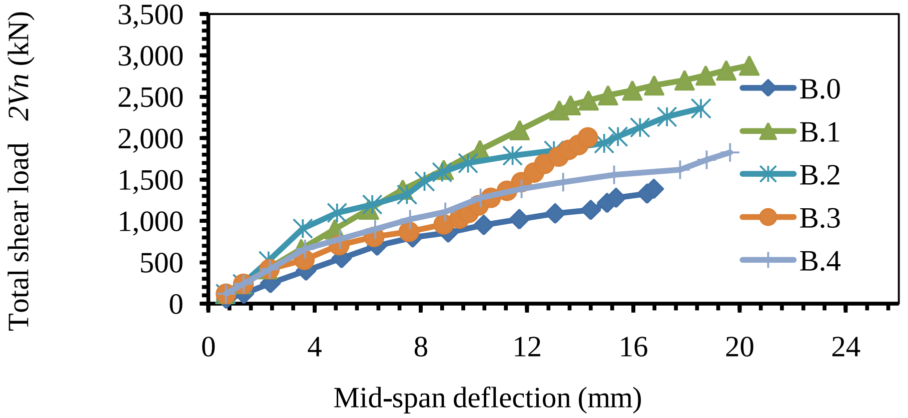


Figure15: Load–Deflection relationship for effect of the distance between FRB sheets for series "B"

Load-Deflection Curves for Series "C"

For series "C" (strength = 500 kg/cm2), the rate of deflection decreasing is considered small for beam AC.0 but it is considerably big for beams C.1, C.2, C.3 and C.4.

The difference between these rates for beams C.1, C.2, C.3 and C.4 are considerably small. Also the measured maximum load and deflection at failure point for strengthened beams are considered very big for that of the original beam as shown in figure (16).

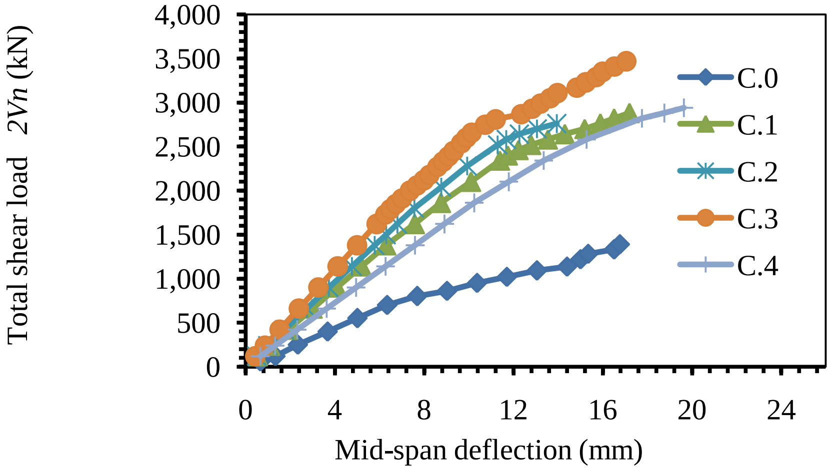


Figure 16: Load–Deflection relationship for effect of the distance between FRB sheets for series "C"

Load-Deflection Curves for Series "D"

For series "D" (strength = 700 kg/cm2), the rate of deflection decreasing is considered small for beam AD.0 but it is considerably big for beams D.1, D.2, D.3 and D.4.

The difference between these rates for beams D.1, D.2, D.3 and D.4 are considerably small. Also the measured maximum load and deflection at failure point for strengthened beams are considered very big for that of the original beam as shown in figure (17).

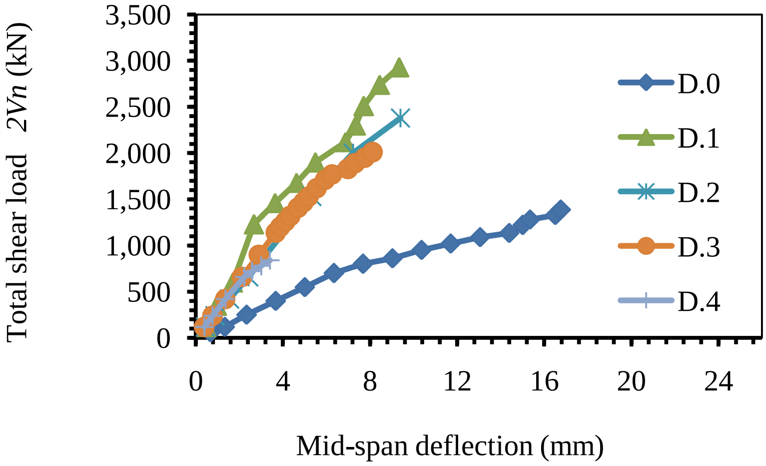


Figure17: Load–Deflection relationship for effect of the distance between FRB sheets for series "D"

The Effect of Configuration FRB Sheets on the Shape of Strengthening

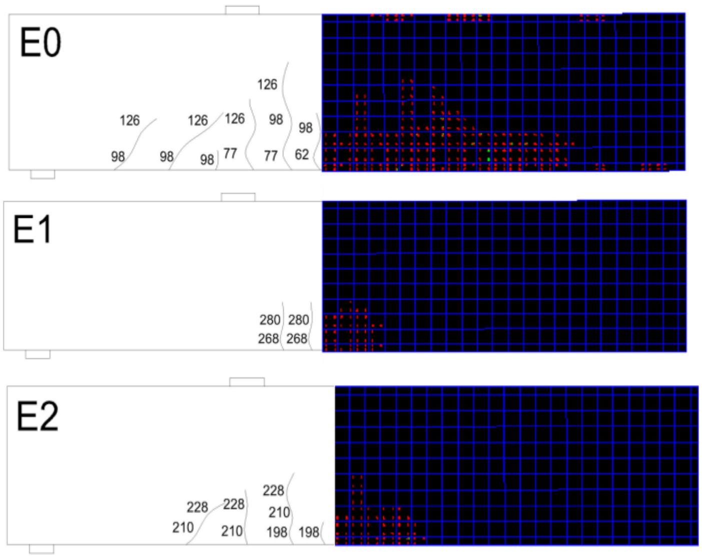
Sixteen high strength beams having 4.7m total length with rectangular cross-section of 0.3m breadth and constant height equals to 1m. These beams were tested over a simple span of 4.5m under two equal loads far from support 1.75m. All beams were reinforced with ten tension bars from high tensile steel of 32.0mm diameter and three compression bars of 16.0mm diameter plus stirrups of 10.0mm diameter at variable spacing, as shown in figure (18). These beams were divided into four series (E-F-G-H) according to the degree of high strength concrete and the shape of strengthen.

Series (E) consists of four reinforced concrete beams with (cube strength=250 kg/cm2) designated as E.0, E.1, E.2 and E.3.

Series (F) consists of four reinforced concrete beams with (cube strength= 400 kg/cm2) designated as F.0, F.1, F.2 and F.3.

Series (G) consists of four reinforced concrete beams with (cube strength= 500 kg/cm2) designated as G.0, G.1, G.2 and G.3.

Series (H) consists of four reinforced concrete beams with (cube strength= 700 kg/cm2) designated as H.0, H.1, H.2 and H.3.



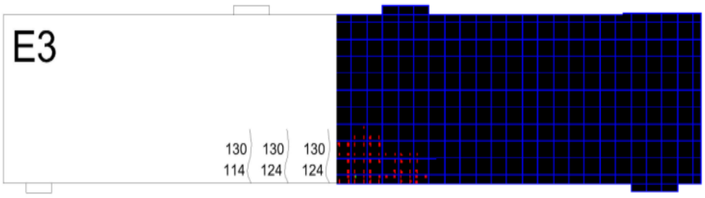


Figure18: Crack pattern for effect of configuration FRB Sheets on the Shape of Strengthening

Load-Deflection Curves for Series "E"

For series "E" (strength = 250 kg/cm2), the rate of deflection decreasing is considered small for beam E.0 but it is considerably big for beams E.1, E.2 and E.3. The difference between these rates for beams E.1, E.2 and E.3 are considerably small. Also the measured maximum load and deflection at failure point for strengthened beams are considered very big for that of the original beam as shown in figure (19).

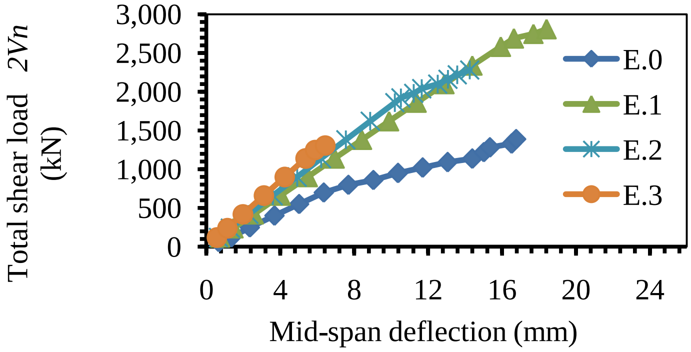


Figure19: Load–Deflection Relationship for Effect of Configuration FRB Sheets on the Shape of Strengthening for series "E"

Load-Deflection Curves for series "F"

For series "F" (strength = 400 kg/cm^2), the rate of deflection decreasing is considered small for beam BB.0 but it is considerably big for beams F.1, F.2 and F.3. The difference between these rates for beams F.1, F.2 and F.3 is considerably small. Also the measured maximum load and deflection at failure point for strengthened beams are considered very big for that of the original beam, as shown in figure (20).

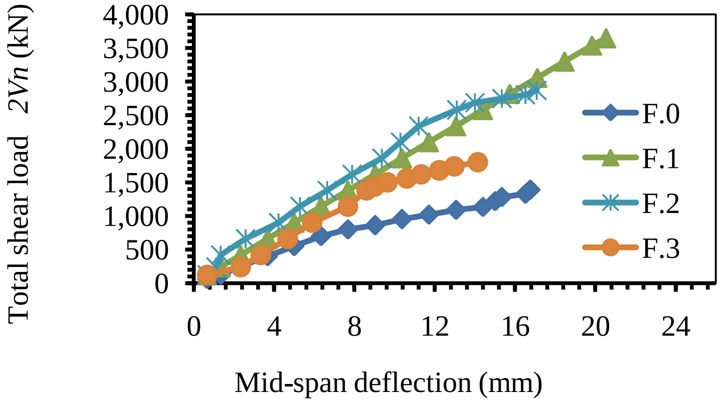


Figure 20: Load–Deflection Relationship for Effect of Configuration FRB Sheets on the Shape of Strengthening for series "F"

Load-Deflection Curves for Series "G"

For series "G" (strength = 500 kg/cm2), the rate of deflection decreasing is considered small for beam G.0 but it is considerably big for beams G.1, G.2 and G.3. The difference between these rates for beams G.1, G.2 and G.3

are considerably small. Also the measured maximum load and deflection at failure point for strengthened beams are considered very big for that of the original beam, as shown in figure (21).

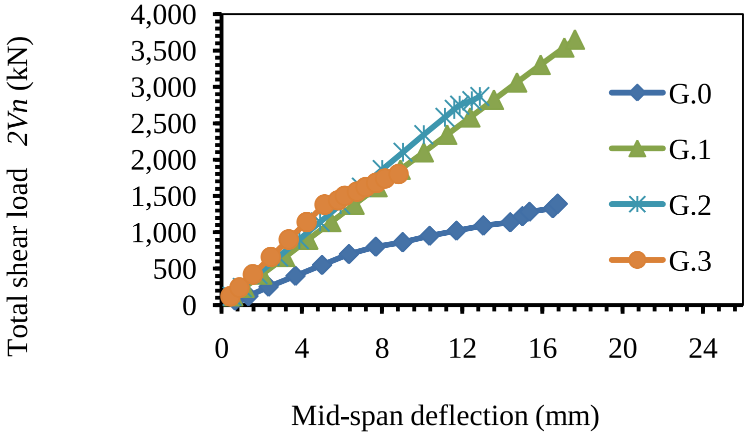


Figure 21: Load–Deflection Relationship for Effect of Configuration FRB Sheets on the Shape of Strengthening for series "G"

Load-Deflection Curves for Series "H"

For series "H" (strength = 700 kg/cm2), the rate of deflection decreasing is considered small for beam BD.0 but it is considerably big for beams H.1, H.2 and H.3. The difference between these rates for beams H.1, H.2 and H.3 are considerably small. Also the measured maximum load and deflection at failure point for strengthened beams are considered very big for that of the original beam, as shown in figure (22).

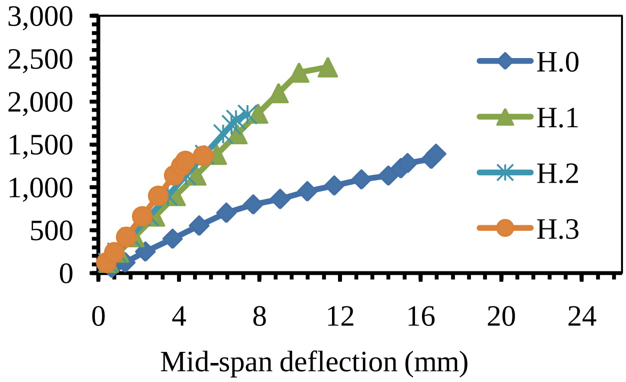


Figure 22: Load–Deflection Relationship for Effect of Configuration FRB Sheets on the Shape of Strengthening for series "H"

The Effect Change of the Number of Layers of Strengthening

Twenty high strength beams having 4.7m total length with rectangular cross-section of 0.3m breadth and constant height equals to 1m. These beams were tested over a simple span of 4.5m under two equal loads far from support 1.75m. All beams were reinforced with ten tension bars from high tensile steel of 32.0 mm diameter and three compression bars of 16.0mm diameter plus stirrups of 10.0mm diameter at variable spacing, as shown in figure(23). These beams were divided into four series (I-J-K-L) according to the degree of high strength concrete and the shape of strengthen.

Series (I) consists of five reinforced concrete beams with (cube strength= 250 kg/cm2) designated as I.0, I.1, I.2, I.3 and I.4.

Series (J) consists of five reinforced concrete beams with (cube strength= 400 kg/cm2) designated as J.0, J.1, J.2, J.3 and J.4.

Series (K) consists of five reinforced concrete beams with (cube strength= 500 kg/cm2) designated as K.0, K.1, K.2, K.3 and K.4.

Series (L) consists of five reinforced concrete beams with (cube strength= 700 kg/cm2) designated as L.0, L.1, L.2, L.3 and L.4.

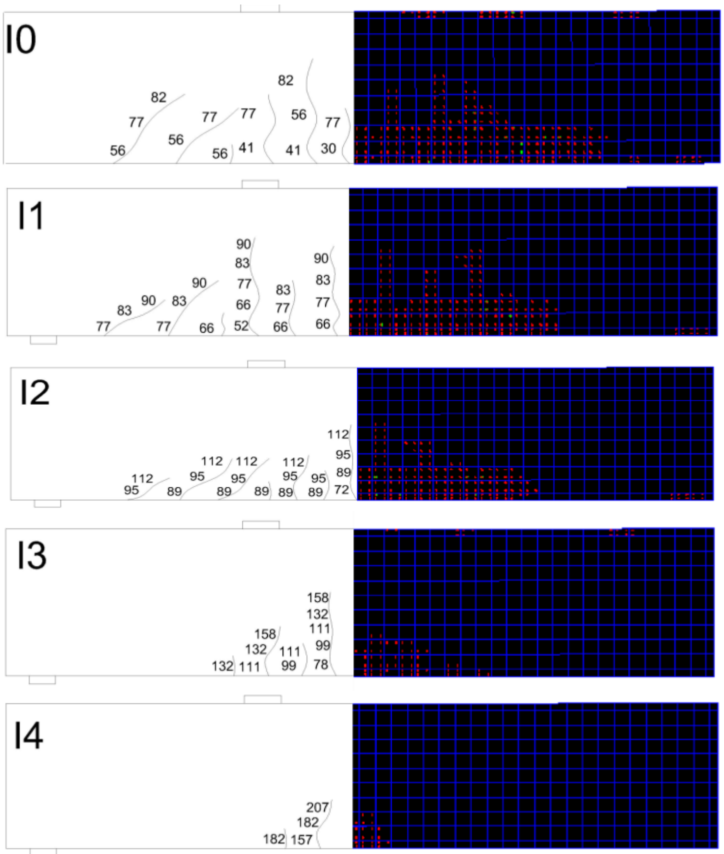


Figure 23: Crack pattern for effect change of the number of layers of strengthening

Load-Deflection Curves for Series "I"

For series "I" (strength = 250 kg/cm2), the rate of deflection decreasing is considered small for beam CA.0 but it is considerably big for beams I.1, I.2, I.3 and I.4. The difference between these rates for beams I.1, I.2, I.3 and I.4 are considerably small. Also the measured maximum load and deflection at failure point for strengthened beams are considered very big for that of the original beam as shown in figure (24).

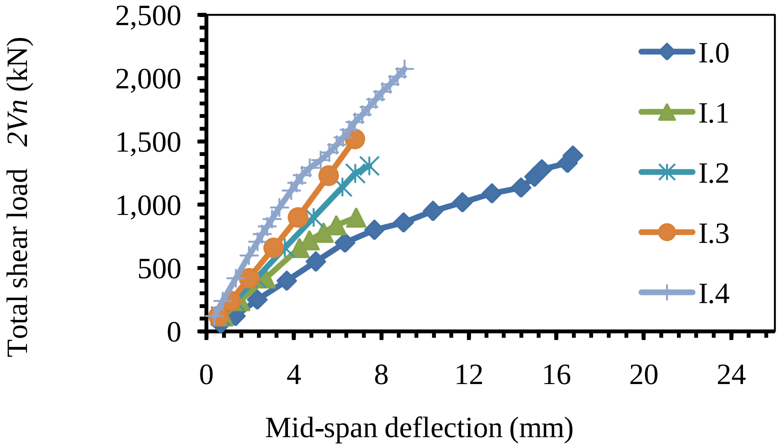


Figure 24: Load–Deflection Relationship for effect change of the number of layers of strengthening for series "I"

Load-Deflection Curves for series "J"

For series "J" (strength = 400 kg/cm2), the rate of deflection decreasing is considered small for beam CB.0 but it is considerably big for beams J.1 , J.2 , J.3 and J 4. The difference between these rates for beams J.1, J.2, J.3 and J.4 are considerably small. Also the measured maximum load and deflection at failure point for strengthened beams are considered very big for that of the original beam as shown in figure (25).

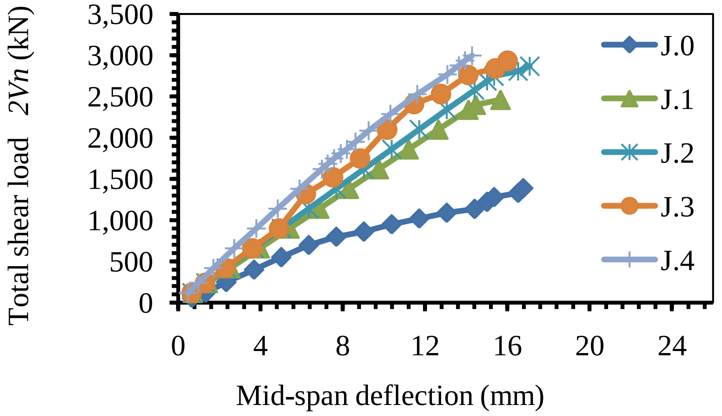


Figure 25: Load–Deflection Relationship for effect change of the number of layers of strengthening for series "J"

Load-Deflection Curves for series "K"

For series "K" (strength = 500 kg/cm2), the rate deflection of decreasing is considered small for beam CC.0 but it is considerably big for beams K.1 , K.2 , K.3 and K.4.The difference between these rates for beams K.1 , K.2 , K.3 and K.4 is considerably small. Also the measured maximum load and deflection at failure point for strengthened beams are considered very big for that of the original beam as shown in figure (26).

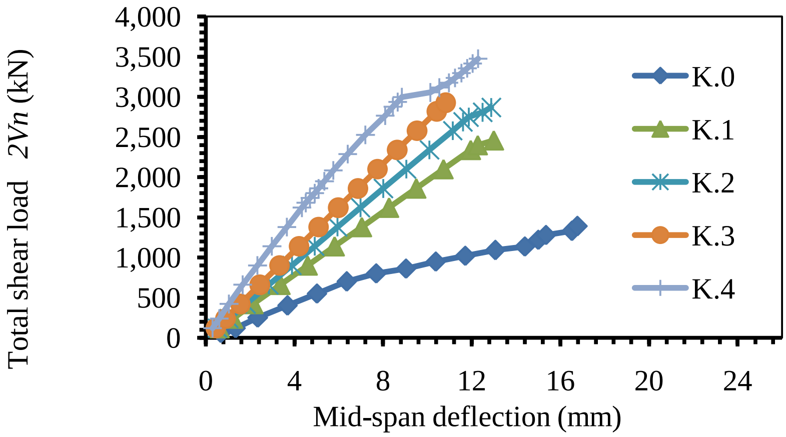


Figure 26: Load–Deflection Relationship for effect change of the number of layers of strengthening for series "K"

Load-Deflection Curves for series "L"

For series "L" (strength = 700 kg/cm2), the rate of decreasing is considered small for beam L.0 but it is considerably big for beams L.1, L.2, L.3 and L.4. The difference between these rates for beams L.1, L.2, L.3 and L.4 is considerably small. Also the measured maximum load and deflection at failure point for strengthened beams are considered very big for that of the original beam as shown in figure (27).

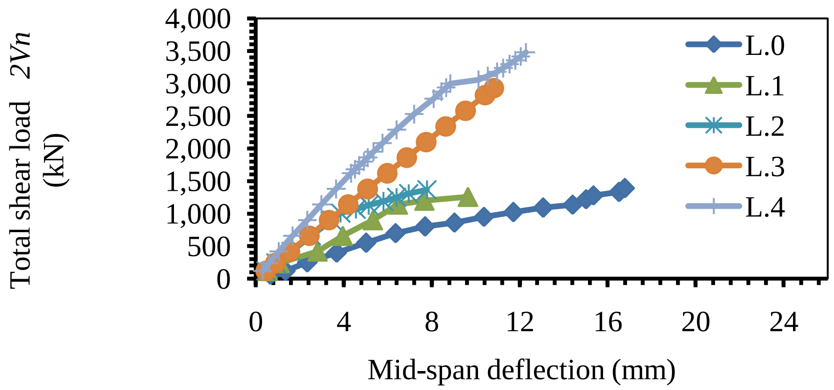


Figure 27: Load–Deflection Relationship for effect change of the number of layers of strengthening for series "L"

Conclusions

Based on the experimental and analytical results and observations, it could be concluded that the ultimate load carrying capacity of the strengthened beam is higher when compared to the control beam. The results obtained with Ansys analysis were the following:

Concrete change in resistance is directly proportional to the load and inversely with the deflection. In this research shows the change in resistance and maximum load beam and the change in deflection. The change in the distance between the sheets will change the Surface area to strengthen will be changed by increasing or decreasing Consequent Carrying beam loads of capacity and deflection will change .The change in the number of layers of strengthen increases the durability of the beam to the loads and will Decrease deflection.

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