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Research Article

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A new local energy-preserving scheme for the RLW equation

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Abstract In this paper, a local energy-preserving scheme for solving the regularized long wave (RLW) equation are constructed. By using the implicit midpoint method and the AVF method for the spatial and temporal discretization respectively, we present an numerical algorithm for the RLW equation. The new method is proved to be local energy preserving. With suitable boundary conditions, such as periodic boundary conditions, the algorithms admit global energy conservation law. Numerical experiments are conducted to show the performance of the proposed conservative scheme.

Keywords RLW equation; Local energy-preserving; Average Vector Field (AVF) method; energy

1. Introduction

The regularized long wave (RLW) equation

$$u_t + u_x - \sigma u_{xxt} + \frac{\nu}{2}(u^2)_x = 0, \quad a < x < b, 0 < t < T, \tag{1.1}$$

subject to the initial and boundary condition

$$u(a,t) = u(b,t), \quad 0 < t < T, \tag{1.2}$$
$$u(x,0) = u_0(x), \quad a < x < b.$$

was first put forward by Peregrine [1] to describe a model for long waves on the surface of water in a channel, and later by Benjamin et al. [2]. P. J. Olver [3] presented that the RLW equation posses only three independent conservation laws:

Mass conservation law

$$\mathrm{S} = \int_a^b u dx = C_1. \tag{1.3}$$

Global momentum conservation law (GMCL)

$$\mathrm{M} = \frac{1}{3}\int_a^b u^2 + \sigma(u_x)^2 dx = C_2. \tag{1.4}$$

Global energy conservation law (GECL)

$$\mathrm{E} = \frac{1}{2}\int_a^b (\nu u^3 + 3u^2) dx = C_3. \tag{1.5}$$

This equation has drawn much attention for decades and has motivated a series of studies in physics and mathematics. Various numerical methods have been used to investigate the solutions for the RLW equation [6, 4, 5, 7]. In particularly the finite difference method in [8], finite element methods such as the Galerkin method in [9], collocation methods in [10] and so on.

In this paper, we will focus on the local energy preserving algorithm for the RLW equation. First we apply the second order AVF method in time discritization and then apply the implicit midpoint method in space discretization to construct a local energy-preserving scheme.

The rest of this paper is organized as follows: In Section 2, a local energy-preserving algorithm is developed for the RLW equation. We prove that the proposed scheme preserves the total energy and mass in the discrete forms. Numerical experiments are presented to verify the performance of the scheme in Section 3.

2. Local energy-preserving algorithm

2.1. Multisymplectic structure of the RLW equation

First we introduce the conjugate variables $u=\varphi_x, v=u_x, \omega=u_t$ and $p=\frac{1}{2}\varphi_t+\frac{1}{2}\varphi_x+\frac{\nu}{2}\varphi_x^2-\sigma\varphi_{xxt}$, let $Z=(\varphi,u,v,\omega,p)^T$, equation (1.1) can be reformulated into the following PDEs

$$\begin{aligned}
&-\frac{1}{2}u_t-\frac{1}{2}u_x-p_x=0,\\
&\frac{1}{2}\varphi_t-\frac{\sigma}{2}v_t+\frac{1}{2}\varphi_x-\frac{\sigma}{2}\omega_x=p-\frac{\nu}{2}u^2,\\
&\frac{\sigma}{2}u_t=\frac{\sigma}{2}\omega,\\
&\frac{\sigma}{2}u_x=\frac{\sigma}{2}v,\\
&\varphi_x=u.
\end{aligned} \tag{2.1}$$

Thus it can be written as a multi-symplectic system

$$MZ_t+KZ_x=\nabla_Z S(Z)$$

where

$$S(Z)=up-\frac{\nu}{6}u^3+\frac{\sigma}{2}v\omega,$$

$$M=\begin{pmatrix} 0 & -\frac{1}{2} & 0 & 0 & 0\\ \frac{1}{2} & 0 & -\frac{\sigma}{2} & 0 & 0\\ 0 & \frac{\sigma}{2} & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, K=\begin{pmatrix} 0 & -\frac{1}{2} & 0 & 0 & -1\\ \frac{1}{2} & 0 & 0 & -\frac{\sigma}{2} & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & \frac{\sigma}{2} & 0 & 0 & 0\\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Proposition 2.1 *The system (2.1) possesses a local energy conservation law (LECL)*

$$\partial_t(-\frac{1}{2}u_x\varphi-\frac{\sigma}{2}uv_x)+\partial_x(up+\frac{\sigma}{2}\omega v-\frac{\nu}{6}u^3+\frac{1}{2}u_t\varphi+\frac{\sigma}{2}uv_t)=0. \tag{2.2}$$

Proof. Multiplying (2.1) by φ_x, u_x, v_x, ω_x and p_x respectively gives

$$
\begin{aligned}
&-\frac{1}{2}u_t\varphi_x-\frac{1}{2}u_x\varphi_x-p_x\varphi_x=0,\\
&\frac{1}{2}\varphi_t u_x-\frac{\sigma}{2}v_t u_x+\frac{1}{2}\varphi_x u_x-\frac{\sigma}{2}\omega_x u_x=pu_x-\frac{\nu}{2}u^2u_x,\\
&\frac{\sigma}{2}u_t v_x=\frac{\sigma}{2}\omega v_x,\\
&\frac{\sigma}{2}u_x\omega_x=\frac{\sigma}{2}v\omega_x,\\
&\varphi_x p_x=up_x.
\end{aligned}
\tag{2.3}
$$

Adding the above equations (2.3) together, we have

$$
\begin{aligned}
-p_x\varphi_x-\frac{\sigma}{2}\omega_x u_x+\frac{\sigma}{2}u_x\omega_x+\varphi_x p_x &= pu_x-\frac{1}{2}u^2u_x+\frac{\sigma}{2}\omega v_x+\frac{\sigma}{2}v\omega_x+up_x\\
&=\partial_x(up+\frac{\sigma}{2}\omega v-\frac{\nu}{6}u^3)
\end{aligned}
$$

According the commutative law and discrete Leibnitz rules, we can obtain local energy conservation law (2.2). With the initial and periodic boundary conditions (1.2), we obtain the global energy conservation law (GECL) (1.5).

2.2. Local energy-preserving scheme

In this subsection, we propose the new local energy-preserving schemes for the RLW equation. We first introduce some notations: the spatial domain $I=[a,b]$ and $L=b-a$, $x_j=a+hj$, $j=0,1,2,\cdots N-1$, where $h=(b-a)/N$ is the spatial length. $t_k=k\tau$, $k=0,1,2,\cdots$, where τ is temporal step span.

Some operators are also defined. Let u_j^k be the approximation of $u(x,t)$ at the node (x_j,t_k). We define the finite difference operators

$$\delta_t f_j^k=\frac{1}{\tau}(f_j^{k+1}-f_j^k),\ \ \delta_x f_j^k=\frac{1}{h}(f_{j+1}^k-f_j^k)$$

and averaging operators

$$A_t f_j^k=\frac{1}{2}(f_j^{k+1}+f_j^k),\ \ A_x f_j^k=\frac{1}{2}(f_{j+1}^k+f_j^k).$$

Now we apply the implicit midpoint scheme in time and the AVF method in space to construct the local energy-preserving algorithm for RLW equation.

LEP scheme: First we use the implicit midpoint scheme in time and obtain semi-discrete system

$$
\begin{aligned}
&-\frac{1}{2}\delta_t u^k-\frac{1}{2}\partial_x A_t u^k-\partial_x A_t p^k=0,\\
&\frac{1}{2}\delta_t\varphi^k-\frac{\sigma}{2}\delta_t v^k+\frac{1}{2}\partial_x A_t\varphi^k-\frac{\sigma}{2}\partial_x A_t\omega^k=A_t p^k-\frac{\nu}{2}(A_t u^k)^2,\\
&\frac{\sigma}{2}\delta_t u^k=\frac{\sigma}{2}A_t\omega^k,\\
&\frac{\sigma}{2}\partial_x A_t u^k=\frac{\sigma}{2}A_t v^k,\\
&\partial_x A_t\varphi^k=A_t u^k.
\end{aligned}
$$

Then we use the AVF method in space

$$-\frac{1}{2}\delta_t A_x u_j^k - \frac{1}{2}\delta_x A_t u_j^k - \delta_x A_t p_j^k = 0,$$

$$\frac{1}{2}\delta_t A_x \varphi_j^k - \frac{\sigma}{2}\delta_t A_x v_j^k + \frac{1}{2}\delta_x A_t \varphi_j^k - \frac{\sigma}{2}\delta_x A_t \omega_j^k = \int_0^1((1-\xi)A_t p_j^k + \xi A_t p_{j+1}^k) - \frac{\nu}{2}((1-\xi)A_t u_j^k + \xi A_t u_{j+1}^k)^2 d\xi,$$

$$\frac{\sigma}{2}\delta_t A_x u_j^k = \frac{\sigma}{2}\int_0^1((1-\xi)A_t \omega_j^k + \xi A_t \omega_{j+1}^k)d\xi,$$

$$\frac{\sigma}{2}\delta_x A_t u_j^k = \frac{\sigma}{2}\int_0^1((1-\xi)A_t v_j^k + \xi A_t v_{j+1}^k)d\xi,$$

$$\delta_x A_t \varphi_j^k = \int_0^1((1-\xi)A_t u_j^k + \xi A_t u_{j+1}^k)d\xi.$$

The full-discrete system can be written as

$$\begin{aligned}
&-\frac{1}{2}\delta_t A_x u_j^k - \frac{1}{2}\delta_x A_t u_j^k - \delta_x A_t p_j^k = 0,\\
&\frac{1}{2}\delta_t A_x \varphi_j^k - \frac{\sigma}{2}\delta_t A_x v_j^k + \frac{1}{2}\delta_x A_t \varphi_j^k - \frac{\sigma}{2}\delta_x A_t \omega_j^k = A_x A_t p_j^k - \frac{\nu}{6}[(A_t u_j^k)^2 + (A_t u_{j+1}^k)^2 + (A_t u_j^k)(A_t u_{j+1}^k)],\\
&\frac{\sigma}{2}\delta_t A_x u_j^k = \frac{\sigma}{2} A_t A_x \omega_j^k,\\
&\frac{\sigma}{2}\delta_x A_t u_j^k = \frac{\sigma}{2} A_t A_x v_j^k,\\
&\delta_x A_t \varphi_j^k = A_t A_x u_j^k.
\end{aligned} \tag{2.4}$$

Eliminating the auxiliary variables φ, v, ω and p yields the system

$$\delta_t A_x^3 A_t u_{j-2}^k + \delta_x A_x^2 A_t^2 u_{j-2}^k - \sigma \delta_x^2 \delta_t A_x A_x u_{j-2}^k + \frac{\nu}{6}\delta_x A_x A_t[(A_t u_{j-2}^k)^2 + (A_t u_{j-1}^k)^2 + (A_t u_{j-1}^k)(A_t u_{j-2}^k)] = 0.$$

Omitting the average operator A_x and A_t we can get a two time level scheme

$$A_x^2 \delta_t u_{j-1}^k + \delta_x A_x A_t u_{j-1}^k - \sigma \delta_x^2 \delta_t u_{j-1}^k + \frac{\nu}{6}\delta_x[(A_t u_{j-1}^k)^2 + (A_t u_j^k)^2 + (A_t u_{j-1}^k)(A_t u_j^k)] = 0. \tag{2.5}$$

Theorem 2.2 *The scheme (2.4) or (2.5) preserve the discrete local energy conservation law*

$$\begin{aligned}
\mathsf{E}(x_j,t_k) = \;& \delta_t(-\frac{1}{2}\delta_x u_j^k \cdot A_x \varphi_j^k - \frac{\sigma}{2} A_x u_j^k \cdot \delta_x v_j^k)\\
&+ \delta_x[A_t p_j^k \cdot A_t u_j^k + \frac{\sigma}{2} A_t \omega_j^k \cdot A_t v_j^k - \frac{\nu}{6}(A_t u_j^k)^3 + \frac{1}{2}\delta_t u_j^k \cdot A_t \varphi_j^k + \frac{\sigma}{2}\delta_t v_j^k \cdot A_t u_j^k] = 0
\end{aligned} \tag{2.6}$$

Proof. Multiplying (2.4) by $\delta_x A_t \varphi_j^k$, $\delta_x A_t u_j^k$, $\delta_x A_t v_j^k$, $\delta_x A_t \omega_j^k$ and $\delta_x A_t p_j^k$ respectively, we have

$$-\frac{1}{2}\delta_t A_x u_j^k \cdot \delta_x A_t \varphi_j^k - \frac{1}{2}\delta_x A_t u_j^k \cdot \delta_x A_t \varphi_j^k - \delta_x A_t p_j^k \cdot \delta_x A_t \varphi_j^k = 0,$$

$$\begin{aligned}
&\frac{1}{2}\delta_t A_x \varphi_j^k \cdot \delta_x A_t u_j^k - \frac{\sigma}{2}\delta_t A_x v_j^k \cdot \delta_x A_t u_j^k + \frac{1}{2}\delta_x A_t \varphi_j^k \cdot \delta_x A_t u_j^k - \frac{\sigma}{2}\delta_x A_t \omega_j^k \cdot \delta_x A_t u_j^k\\
&= A_t A_x p_j^k \cdot \delta_x A_t u_j^k - \frac{\nu}{6}[(A_t u_j^k)^2 + (A_t u_{j+1}^k)^2 + (A_t u_j^k)(A_t u_{j+1}^k)] \cdot \delta_x A_t u_j^k,
\end{aligned}$$

$$\frac{\sigma}{2}\delta_t A_x u_j^k \cdot \delta_x A_t v_j^k = \frac{\sigma}{2} A_t A_x \omega_j^k \cdot \delta_x A_t v_j^k, \tag{2.7}$$

$$\frac{\sigma}{2}\delta_x A_t u_j^k \cdot \delta_x A_t \omega_j^k = \frac{\sigma}{2} A_t A_x v_j^k \cdot \delta_x A_t \omega_j^k,$$

$$\delta_x A_t \varphi_j^k \cdot \delta_x A_t p_j^k = A_t A_x u_j^k \cdot \delta_x A_t p_j^k.$$

Summing the above equations (2.8)

$$\begin{aligned}&-\frac{1}{2}\delta_t A_x u_j^k \cdot \delta_x A_t \varphi_j^k + \frac{1}{2}\delta_t A_x \varphi_j^k \cdot \delta_x A_t u_j^k - \frac{\sigma}{2}\delta_t A_x v_j^k \cdot \delta_x A_t u_j^k + \frac{\sigma}{2}\delta_x A_t v_j^k \cdot \delta_t A_x u_j^k \\ &= A_t A_x p_j^k \cdot \delta_x A_t u_j^k + A_t A_x u_j^k \cdot \delta_x A_t p_j^k - \frac{\nu}{6}[(A_t u_j^k)^2 + (A_t u_{j+1}^k)^2 + (A_t u_j^k)(A_t u_{j+1}^k)] \cdot \delta_x A_t u_j^k \\ &+ \frac{\sigma}{2} A_t A_x \omega_j^k \cdot \delta_x A_t v_j^k + \frac{\sigma}{2} A_t A_x v_j^k \cdot \delta_x A_t \omega_j^k.\end{aligned} \tag{2.8}$$

By using the discrete Leibniz rule and commutative law we can deduce

$$-\frac{1}{2}\delta_t A_x u_j^k \cdot \delta_x A_t \varphi_j^k + \frac{1}{2}\delta_t A_x \varphi_j^k \cdot \delta_x A_t u_j^k = \delta_x(-\frac{1}{2}\delta_t u_j^k \cdot A_t \varphi_j^k) + \delta_t(\frac{1}{2}\delta_x u_j^k \cdot A_x \varphi_j^k),$$

$$-\frac{\sigma}{2}\delta_t A_x v_j^k \cdot \delta_x A_t u_j^k + \frac{\sigma}{2}\delta_x A_t v_j^k \cdot \delta_t A_x u_j^k = \delta_x(-\frac{\sigma}{2}\delta_t v_j^k \cdot A_t u_j^k) + \delta_t(\frac{\sigma}{2} A_x u_j^k \cdot \delta_x v_j^k),$$

$$A_t A_x p_j^k \cdot \delta_x A_t u_j^k + A_t A_x u_j^k \cdot \delta_x A_t p_j^k = \delta_x (A_t p_j^k \cdot A_t u_j^k),$$

$$\frac{\nu}{6}[(A_t u_j^k)^2 + (A_t u_{j+1}^k)^2 + (A_t u_j^k)(A_t u_{j+1}^k)] \cdot \delta_x A_t u_j^k = \frac{\nu}{6}\delta_x (A_t u_j^k)^3,$$

$$\frac{\sigma}{2} A_t A_x \omega_j^k \cdot \delta_x A_t v_j^k + \frac{\sigma}{2} A_t A_x v_j^k \cdot \delta_x A_t \omega_j^k = \delta_x(\frac{\sigma}{2} A_t \omega_j^k \cdot A_t v_j^k),$$

So we can reduce the equation (2.8) to discrete local energy conservation law (2.6).

Remark 2.3 *The discrete LECL (2.6) is independent of boundary conditions and is consist with the continuous LECL (2.2).*

Summing the discrete energy conservation law (2.6) over index j reads

$$\begin{aligned}&\delta_x \sum_{j=1}^{N} [A_t p_j^k \cdot A_t u_j^k + \frac{\sigma}{2} A_t \omega_j^k \cdot A_t v_j^k - \frac{\nu}{6}(A_t u_j^k)^3 + \frac{1}{2}\delta_t u_j^k \cdot A_t \varphi_j^k + \frac{\sigma}{2}\delta_t v_j^k \cdot A_t u_j^k] \\ &+ \delta_t \sum_{j=1}^{N} (-\frac{1}{2}\delta_x u_j^k \cdot A_x \varphi_j^k - \frac{\sigma}{2} A_x u_j^k \cdot \delta_x v_j^k) = 0.\end{aligned}$$

With the initial and periodic boundary conditions (1.2), we can obtain the following discrete global energy conservation law.

Corollary 2.4 *For the initial and periodic boundary conditions (1.2), the LEP scheme preserves the discrete GECL*

$$\mathsf{E}^{k+1} = \mathsf{E}^k = \cdots = \mathsf{E}^1 = \mathsf{E}^0, \tag{2.9}$$

where $\mathsf{E}^k = h\sum_{j=1}^{N}(-\frac{1}{2}\delta_x u_j^k \cdot A_x \varphi_j^k - \frac{\sigma}{2} A_x u_j^k \cdot \delta_x v_j^k)$.

3. Numerical experiments

In this section, we conduct some numerical experiments to verify the theoretical results of the LEP scheme.

The RLW equation have the following soliton solution [1]

$$u(x,t) = 3c\,sech^2(k(x - x_0 - \varepsilon t)),$$

where $k = \frac{1}{2}\sqrt{\frac{\nu c}{\sigma(1+\nu c)}}, \delta = 1 + \nu c$ and $3c$ is amplitude and ε is velocity.

The relative mass error, relative momentum error and relative energy error $t = t_k$ are defined as

$$RI_1^k = \frac{|I_1^k - I_1^0|}{|I_1^0|}, RI_2^k = \frac{|I_2^k - I_2^0|}{|I_2^0|}, RI_3^k = \frac{|I_3^k - I_3^0|}{|I_3^0|}, k = 0,1,2,\ldots,N \tag{3.1}$$

where

$$I_1^k = h\sum_{j=0}^{N-1}(A_x u_j^k) = h\sum_{j=0}^{N-1}\frac{u_{j+1}^k + u_j^k}{2}$$

is the discrete global mass;

$$I_2^k = \frac{h}{2}\sum_{j=0}^{N-1}((A_x u_j^k)^2 + \sigma(\delta_x u_j^k)^2) = \frac{h}{2}\sum_{j=0}^{N-1}(\frac{u_{j+1}^k + u_j^k}{2})^2 + \frac{\sigma}{h}(u_{j+1}^k - u_j^k)$$

is the discrete global momentum and

$$I_3^k = \frac{h}{3}\sum_{j=0}^{N-1}(\varepsilon(A_x u_j^k)^3 + 3(A_x u_j^k)^2) = \frac{h}{3}\sum_{j=0}^{N-1}(\varepsilon(\frac{u_{j+1}^k + u_j^k}{2})^3 + 3(\frac{u_{j+1}^k + u_j^k}{2})^2$$

is the discrete global energy.

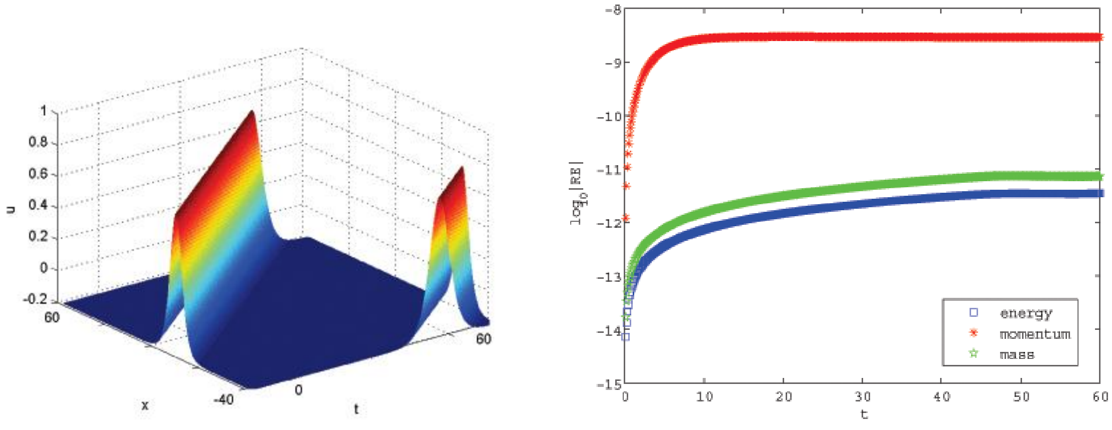
Example 3.1 (Single solitary wave)

We consider the RLW equation with initial boundary conditions

$$u(x,0) = 3c\,sech^2(k(x - x_0)), u(a,t) = u(b,t),$$

where $\varepsilon = \sigma = 1$, $c = 0.3$, $x_0 = 0$.

Computations are done with grid number $N = 800$, temporal step $\tau = 0.1$ and spatial step $h = 0.125$, $-40 \le x \le 60$. The relative error of mass, energy and momentum are computed by using local structure preserving schemes LMP and LEP.



(a) (b)

Figure 1: The numerical solution of the LEP scheme with $\tau = 0.125$ and $h = 0.1$. (a) Propagation of solitary wave from $t = 0$ to $t = 60$. (b) The conservation properties of the LEP scheme.

Fig. 1 displays the results of single soliton obtained by using the LEP scheme. As it can be seen from Fig. 1(a), the propogation of solitary wave over time interval [0,60] is travelling from left to right as required and the shape of the solution is preserved accurately. Fig. 1(b) shows that the relative mass and energy are conserved to the machine accuracy. We can also see that the error of energy growth linearly.

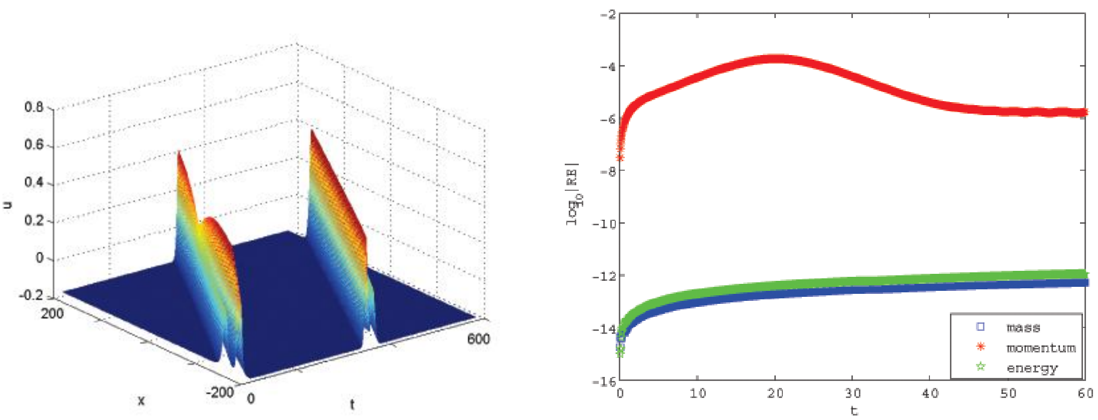
Example 3.2 (Two-solitary wave)

In the following simulations, we will study interaction of two positive solitary waves.

$$u(x,0) = 3c_1 sech^2(k_1(x-x_1)) + 3c_2 sech^2(k_2(x-x_2)), \tag{3.2}$$

where $\varepsilon = \sigma = 1$, $c_1 = 0.2, c_2 = 0.1$, $x_1 = -177, x_2 = -147$.

We apply the LEP scheme to the solitary waves with initial condition (3.2) over time interval [0,600].



(a) (b)

Figure 2: The numerical solution of the LEP scheme with $\tau = 0.5$ and $h = 0.5$. (a) Propagation of solitary wave from $t = 0$ to $t = 600$. (b) The conservation properties of the LEP scheme.

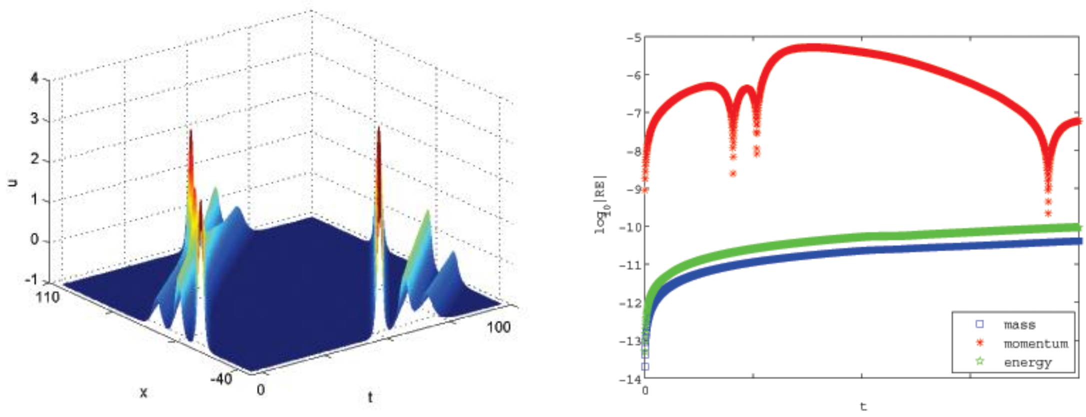
Computations are carried out with grid number $N = 1200$, temporal step $\tau = 0.5$ and spatial step $h = 0.5$, $-200 \le x \le 200$. The interaction process of two solitary obtained by using the LEP scheme can be viewed in Fig. 2(a). The quantities mass, energy and momentum versus time are depicted in Fig. 2(b).

Example 3.3 (Three-solitary wave)

$$u(x,0) = 3c_1 sech^2(k_1(x-x_1)) + 3c_2 sech^2(k_2(x-x_2)) + 3c_3 sech^2(k_3(x-x_3)),$$

where $\varepsilon = \sigma = 1$, $c_1 = 1, c_2 = 0.5, c_3 = 0.25$, $x_1 = 0, x_2 = 18, x_3 = 35$.

Computations are done with grid number $N = 400$, space step $h = 0.1$ temporal step $\tau = 0.05$ and spatial step $h = 0.1$, $-40 \le x \le 110$.



(a) (b)

Figure 3: The numerical solution of the LEP scheme with $\tau = 0.05$ and $h = 0.1$. (a) Propagation of solitary wave from $t = 0$ to $t = 100$. (b) The conservation properties of the LEP scheme.

Fig. 3 provides the results of three-solitary wave with the time step $\tau = 0.05$ and the space step $h = 0.1$ by using the LEP scheme. As it can be seen from Fig. 3(a), the propogation of solitary wave over time interval [0,100] is travelling from left to right as required and the shape of the solution is preserved accurately. Fig. 3(b) shows that the relative mass and energy are conserved to the machine accuracy. The error of energy growth linearly.

We can draw a clear conclusion from the numerical results that the LEP scheme provides highly accurate numerical solutions and preserves the local mass and energy to machine accuracy.

4. Conclusions

The local/global conservation laws, such as symplectic and multisymplectic conservation laws, local energy and momentum conservation laws, usually play an important role in physics and applications for PDEs. In this work, we have proposed a local energy-preserving algorithm to simulate the RLW equation. The new scheme is a conservative scheme, which not only preserve discrete local energy but also preserve the local mass precisely. The merit of the scheme is that with suitable boundary conditions, for example with periodic boundary conditions, this algorithm conserve the global mass and energy precisely. Numerical results indicate that the present scheme can well simulate different solitary wave behaviors of the RLW equation in long term computation and also show excellent performance in preserving geometry structure.

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