



Development of Updated Rating Curves/Equations for Niger and Benue River Systems

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Abstract The stage – discharge relation is an hydraulic equation that transforms continuous measurements of water levels to discharges using the developed relationship. It is simple, accurate and economical alternative to direct computation of discharge in open channels. This paper presents the development of updated rating curves/equations for Aboh, Baro, Idah, Lokoja and Onitsha on the Niger river while Ibi, Makurdi, Umaisha and Yola on the Benue in Nigeria. The power curve and polynomial equations were derived using Microsoft Excel Solver Tools. The exponents on the flow depth parameter (β) fall between 1.30 and 2.75, the range usually reported in literature for wide natural channels. The predictive ability of the stage – discharge equations were assessed using coefficient of determination (R^2), Index of agreement (d), Model efficiency (EF), and all the equations produced perfect fits. The errors associated with the transformation of stage data to discharge were evaluated using the mathematical measures of mean absolute error (MAE), mean bias error (MBE), and root mean square error (RMSE). The MAE for the stations on the Niger River ranged between 11 – 157.6m³/sec, while MBE; 0.07 – 76.80m³/sec and RMSE; 31.45 – 21.8m³/sec. The stations on the Benue river produced R^2 , d and EF values of about 0.9, indicating very good fit. The ranges for MAE, MBE and RMSE are 9.60 – 260m³/sec, 0.04 – 332m³/sec, 18.1 – 289.0m³/sec, indicating poor performances in some stations. In view of the poor performances of MAE, MBE and RMSE measures, this paper recommends the need for improved hydrological data acquisition and management in the Niger and Benue river systems.

Keywords stage-discharge curve; streamflow measurement, statistical methods, error analysis

1. Introduction

The availability of quality streamflow data is a prerequisite for planning, water resources management, and design of water resources system. The utility of stream flow data includes hydraulic modeling, development of resilience strategies for flood risk and floodplain management, flood frequency analysis etc. Conversely, a continuous decline in hydrometeorological data collection and management has been reported [1-3]. The adverse impact of this decline in dearth of real-time data for the design of development projects. Furthermore, a review of pertinent literature reveal that river discharges are almost never directly measured, but obtained from surrogate variables such as stage or water-depth, which can be measured easily and more accurately [4-5]. This is because direct measurements of stream discharge are labour intensive, expensive and sometimes impractical during high floods. A relation is obtained through the stage and discharge data using least-squares regression, that is, fitting a non linear power law model through the data. This stage-discharge relationship is known as flow rating curve, or a flood rating curve based on Annual Maximum flood series (AM). The main advantage of the stage-discharge rating curve is that continuous measurement of water levels, either by chart recorder or by sophisticated solid-state equipment may be converted to continuous record of the river discharge. The use of stage-discharge for stream discharge predictions is a standard practice for agencies like the US Geological Survey (USGS), and the stage discharge relation is recommended by BS 3680: Part 3C:1983 “Methods for determination of the Stage-Discharge Relation” as a code of practice for measurement of liquid flow in open channels and flow measurement structures.



It must be emphasized that rating curve established at a gauging station has to be updated periodically, due to factors such as; scour and sedimentation of the river bed, morphological changes due to floods, and changes due to backwater effects. The existence of backwater effect in open channel flow is governed by the Froude number (Fr) of the flow given by Equation 1

$$Fr = \frac{U}{\sqrt{gh}} \quad (1)$$

Where U is the flow velocity, h is the water depth and g is the acceleration due to gravity. When $Fr < 1$, the flow is subcritical, the backwater effects propagate upstream and downstream. In the case of supercritical flow, $Fr > 1$, the effects of disturbance propagates only downstream. The sources of backwater are found downstream reservoirs, tributaries, tides, ice, dams and other obstructions that influence the flow at the upstream gauging station. Other very important source of complexity, peculiar of some streams during unsteady flow, is hysteresis (also known as loop rating) which results when the water surface slope changes due to either rapidly rising or rapidly falling water levels in a channel control reach [6]. The rating curve can be represented adequately by Equation 2.

$$Q = C(H-H_0)^\beta \quad (2)$$

Where Q is the stream discharge in m^3/sec , H is the water level or stage in the river in m, H_0 is a correction factor for water level at zero flow, and C and β are constant. The value of H_0 was determined by trial and error. The values of C and β are found by least squares regression analysis. Equation 2 is compatible with the Manning Formula where the cross-sectioned area of flow A and hydraulic radius R are functions of (H- H_0).

$$Q = \frac{A}{n} R^{2/3} S_o^{1/2} = \frac{n A^{5/3} S_o^{1/2}}{p^{2/3}} = \left(\frac{n B^{5/3} S_o^{1/2}}{p^{2/3}} \right) * H^{1.67} = CH^{1.67} \quad (3)$$

Comparing Equation 2 and 3, it can be shown that the coefficient β in Equation 2 should have value around 1.67. In fact the value of β depends on the channel section type. Consequently, there are cases, where the rating curve will be a compound curve consisting of several segments for different flow ranges. Statistical methods are available in statistical softwares (e.g Excel, Minitab, etc) to fit curves in the form of Equation 2 or Polynomial curves to measured stages and discharges [7].

This paper focuses on the development of improved rating curves for the Niger and Benue River system using data from hydrological stations. The hydrological stations are situated at Aboh, Baro, Idah, Lokoja and Onitsha on the Niger and Ibi, Makurdi, Umaisha and Yola on the Benue. The Stage-Discharge data were derived from updated data series from 2000 to 2016; Section 2 of the paper contains data and methods. The predicted model's (i.e stage-discharge regression equation accuracy) were evaluated after calibration through statistical error analysis. Section 3, contains the calibrated stage-discharge equations together with numerical values of statistical error analyses with comments on the tables and figures. In Section 4, comments on importance, validity and generality of conclusions will be made. In the lower Niger River Basin, no previous study has developed updated stage-discharge equations, therefore the stage-discharge models will be useful to hydrologic practice and design of water resource systems, in view of the decline of hydro meteorological data collection for development of integrated water resources management tools.

2. Data and Methods

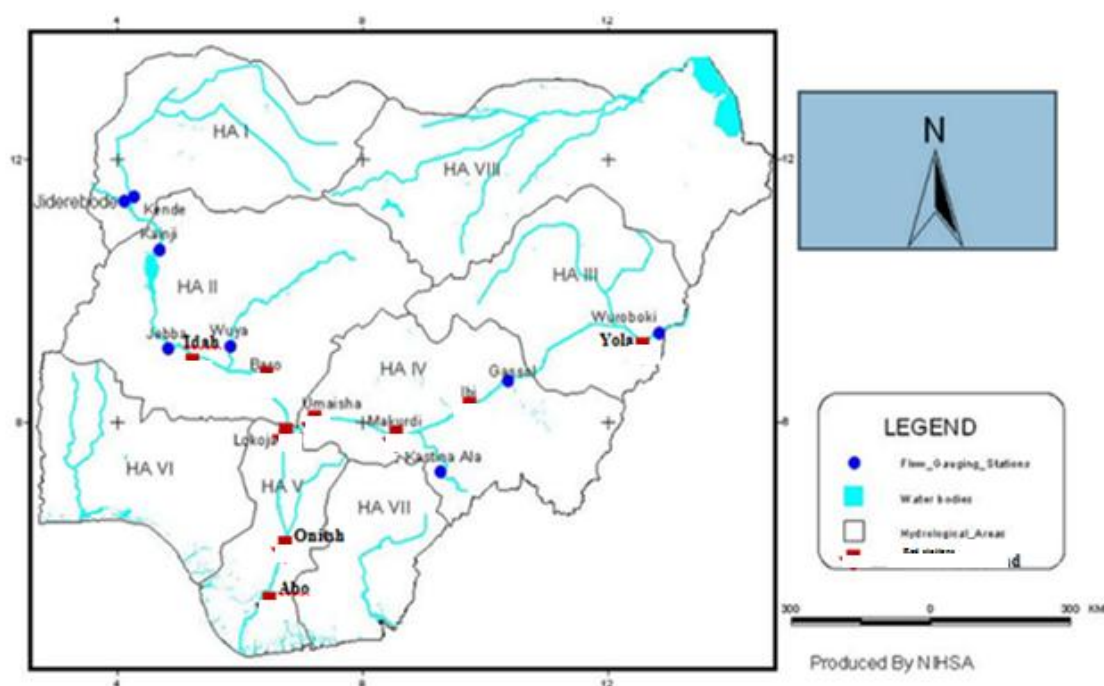
2.1. Data and Study Stations

The stage-discharge data of the selected hydrological stations were obtained from the Nigerian Inland Waterways Authority (NIWA), Lokoja Nigeria. The gauging stations are situated in the Niger and Benue River systems. The characteristics of the study stations are given in Table 1 while Figure 1 shows the locations of the study stations.



Table 1: Characteristics of some Selected Gauge Stations

(1) S/N	(2) Station	(3) Latitude (N)	(4) Longitude (E)	(5) River	(6) Catchment Km ²	(7) Annual Streamflow Max. m ³ /s Min. m ³ /s	
1	Aboh	05°32'	06°31'	Niger	1,112,830	18671.41	1228.42
2	Baro	08°35'	06°23'	Niger	729,510	8852.21	103.45
3	Idah	07°06'	06°43'	Niger	1,105,780	26,760.24	826.32
4	Lokoja	07°49'	06°44'	Niger	750,790	28,360	248.75
5	Onitsha	06°10'	06°45'	Niger	1,125,170	26,607.53	426.84
6	Ibi	08°11'	09°45'	Benue	275,370	12,454.94	12.68
7	Makurdi	07°45'	08°32'	Benue	317,430	16,034.93	30.48
8	Umaisha	08°00'	07°14'	Benue	343,210	18,408.97	7.71
9	Yola	09°14'	12°28'	Benue	112,680	6641.30	8.93

*Figure 1: Hydrological stations (in red rectangles)*

2.2. Methods

2.2.1. Development of Stage-Discharge Equation

The stage-discharge Equation were calibrated using the Microsoft Excel Solver Tools to obtain the parameters, C and β in Equation 2 while the correction factor, H_0 for water level at zero flow was derived by trial and error. The best value of H_0 was adjudged together with the parameter C and β through the coefficient of determination (R^2) and statistical error value of the indices used.

2.2.2. Statistical Error Analysis

The accuracy of the predicted stage-discharge model Equations were evaluated through statistical error analysis [8-9] using the indices; coefficient of determination (R^2), mean absolute error (MAE), root mean square error (RMSE), Mean Bias Error (MBE), Nash-Sutcliffe Efficiency (NSE), Index of agreement (d) and Model of Efficiency (EF). The above mathematical measures are expressed in Equations 4 – 10.



$$\text{Coefficient of determination } (R^2) = \frac{\sum_{i=1}^N (Q_o - \bar{Q}_o) (Q_p - \bar{Q}_p)}{\sqrt{\sum_{i=1}^N (Q_o - \bar{Q}_o)^2} * \sqrt{\sum_{i=1}^N (Q_p - \bar{Q}_p)^2}} \quad (4)$$

$$\text{MAE} = N^{-1} \sum_{i=1}^N |Q_p - Q_o| \quad (5)$$

$$\text{RMSE} = N^{-1} \left[N^{-1} \sum_{i=1}^N (Q_p - Q_o)^2 \right]^{1/2} \quad (6)$$

$$\text{MBE} = N^{-1} \sum_{i=1}^N (Q_p - Q_o) \quad (7)$$

$$\text{Nash - Sutcliffe Efficiency} = 1 - \frac{\sum_{i=1}^N (Q_o - Q_p)^2}{\sum_{i=1}^N (Q_o - \bar{Q}_o)^2} \quad (8)$$

$$\text{Index of Agreement } d = 1 - \frac{\sum_{i=1}^N (Q_o - Q_p)^2}{\sum_{i=1}^N (|Q_p - \bar{Q}| + |Q_o - \bar{Q}|)^2} \quad 0 \leq d \leq 1 \quad (9)$$

$$\text{Model Efficiency (EF)} = 1 - \frac{\sum_{i=1}^N (Q_p - Q_o)^2}{\sum_{i=1}^N (\bar{Q} - Q_o)^2} \quad 0 \leq \text{EF} \leq 1 \quad (10)$$

Where N is the sample size, Q_o is the observed (or measured) discharge, Q_p is the predicted discharge, \bar{Q} is the average observed (or measured) discharge. R^2 statistics gives an indication of the explanatory power of the equation, in terms of how the stage-discharge equations approximate or fit the data points. The higher the value of R^2 , the more successful the fit or the explanatory power, if R^2 is small, it indicates a poor fit, possibly the need to search for an alternative model. Both MAE and RMSE can range from 0 to infinity and the lower their values the better. The index of agreement (d) is a description measure for making cross-comparison between models. The range of d is similar to that of R^2 and lies between 0 (no correlation) and 1 (perfect fit). The Nash-Sutcliffe Efficiency (NSE) lies between 1.0 (perfect fit) and $-\infty$. Bias can be described by MBE.



3. Result and Discussion

Table 2 presents the derived stage-discharge power law and polynomial equation.

Table 2: Updated State – Discharge Curve Equation

Station	EQ	Rating Curve Equation	R ²	d	EF	MAE	MBE	NSE	RMSE	Remarks
Aboh	11	$Q = 160.12H^2 - 343.76H + 1419.2$	0.999	0.984	0.984	25.3	8.67	0.9842	35.34	Full range
	12	$Q = 597.25 (H - 0.3)^{1.303}$	0.9998	0.887	0.8954	22.78	3.33	0.868	42.5	<7.0m
	13	$Q = 99.73 (H + 0.4)^{2.141}$	0.999	0.889	1.00	29.45	10.27	1.00	31.45	>7.0m
Baro	14	$Q = 254.13 (H + 0.1)^{1.717}$	0.9995	1.0	0.989	11.93	4.36	0.999	14.48	Full range
Idah	15	$Q = 235.59 (H + 0.6)^{1.817}$	1.00	1.0	0.986	79.0	76.80	0.986	210.83	<6.0m
	16	$Q = 61.14 (H + 1.2)^{2.419}$	0.997	0.999	0.988	170.48	61.28	0.997	210.0	>6.0m
	17	$Q = 235.53 H^2 - 409.91H + 1402.3$	0.9999	1.0	1.0	139.70	3.34	1.0	166.11	Full range
Lokoja	18	$Q = 171.56 H^2 + 500.95H - 35.04$	0.9997	1.0	1.0	71.60	0.0311	1.0	107.76	Full range
	19	$Q = 268.49 (H + 0.6)^{1.8712}$	0.9995	1.0	1.0	84.02	17.79	0.999	125.14	Full range
Onitsha	20	$Q = 134.47 H^2 + 145.33H - 51.53$	0.998	1.0	1.0	157.6	0.007	0.998	244	Full range
Ibi	21	$Q = 183.64H^2 - 487.6H + 353.64$	0.9998	0.979	0.958	235.9	200	0.950	289.02	<6.0m
	22	$Q = 57.89 H^2 + 1330.67H - 5133.1$	0.9992	0.865	0.932	234	332	0.930	270.45	>6.0m
Makurdi	23	$Q = 256.77 (H+0.6)^2 - 1170.3(H-0.6) + 1360$	0.9973	0.994	0.903	260	316.3	0.972	249.72	<7.0m
	24	$Q = 73.15 (H+0.2)^2 - 1049.5H - 7110.3$	0.9999	0.991	0.913	115.3	50.2	0.9978	136.7	>7.0m
Umaisha	25	$Q = 238.7 (H+0.2)^2 - 1129.3(H+0.2) - 13544$	0.9996	1.0	0.998	70.7	0.04	0.999	94.6	Full range
Yola	26	$Q = 21.88 (H+0.4)^{2.747}$	0.9997	1.0	1.0	9.60	4.31	0.9998	18.1	Full range

Table 2 shows that two kinds of equations have been developed; power law and second degree polynomial equations. Sanyu and Sumiko, 1994 study reported that potential Evapotranspiration (ET) increased Northward, whereas rainfall increases Southward, and that surplus rainfall over ET was found around Latitude 7° down to the Niger Delta. Consequently, most rivers above Latitude 7° are ephemeral and also under the influence of the recent Sahelian drought. The updated stage-discharge curve equations agree with these findings.

The equations for gauging stations situated above Latitude 7° all have zero flow adjustment factors, reflecting the ephemeral or intermittent nature of those rivers. Also the values of the exponent, β on Equation 2 fall within the range reported by previous researchers for relatively wide rivers [4, 10].

Furthermore, for irregular non-uniform flow, Equation 2 cannot be expected to apply throughout the range of stage; this stage-discharge curve may change from a parabola to an odd curve or vice versa. Accordingly, in Table 2, second degree polynomial equations were obtained along with the power law equation as alternative. Figure 2 presents the improved stage-discharge curve equations for some of the gauge stations.

Also in Table 2, seven different efficiency measures for the evaluation of the stage-discharge equations were investigated.



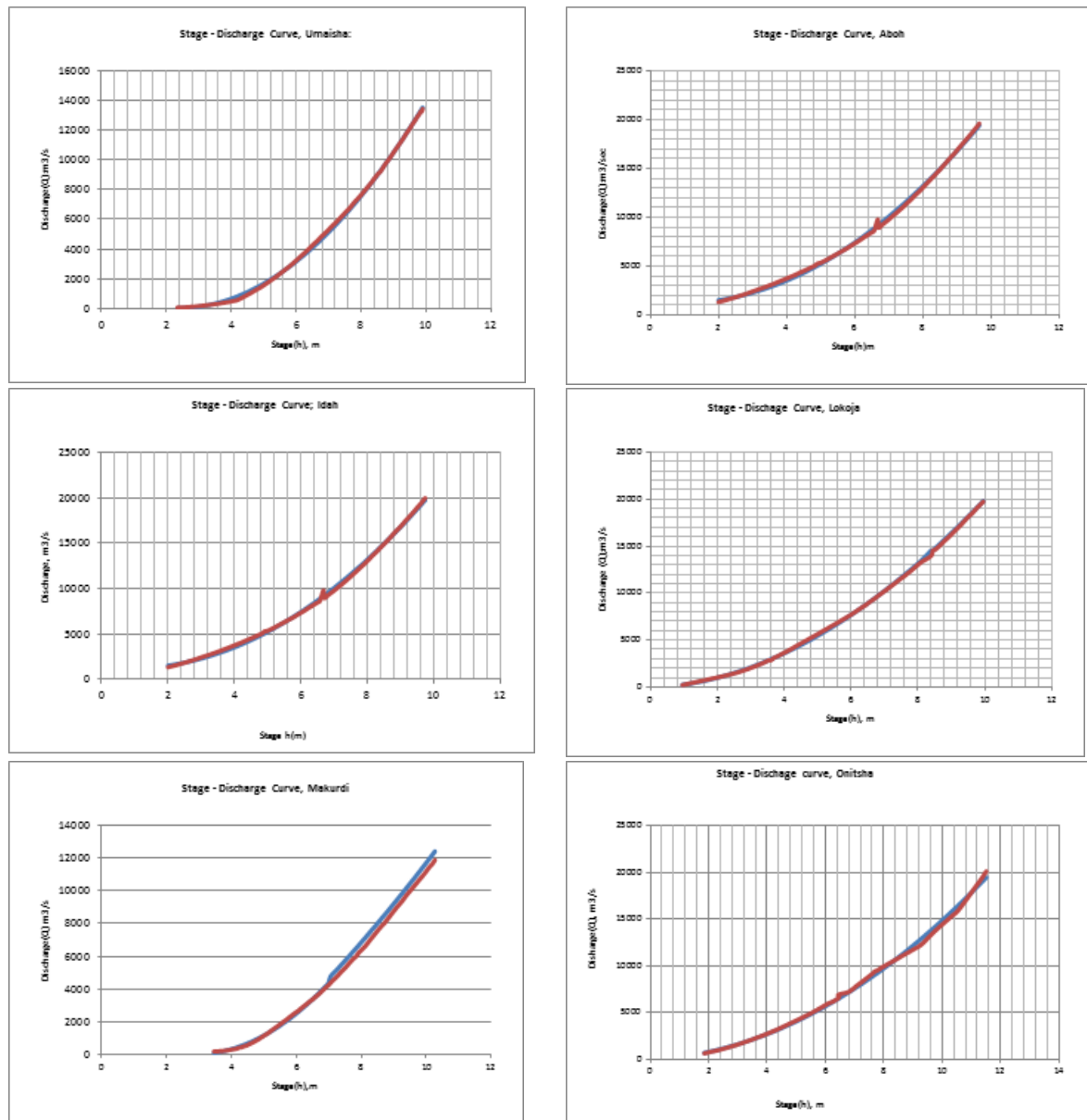


Figure 2: Plots of stage-discharge curves

The coefficient of determination (R^2), The “Index of Agreement” (d), Model Efficiency (EF) and Nash-Sutcliffe Efficiency (NSE), all having their rating between 0 and 1. A value of 1.0 indicates the equation perfectly fits the data, whereas a value of 0.0 indicates poor performance of the statistical measure. Judging on the performance of R^2 , d and EF, the stage-discharge equations performed perfectly. Since the stage-discharge equation/curve is meant to transform continuous stage data to a continuous record of stream discharge, some error is likely to be associated with the reported discharge data [11]. The statistical indices of MAE, MBE and RMSE were used as mathematical measures to evaluate how well the stage-discharge curve/equations fit the observed data of the gauging stations. All the indices have the same dimensions as the observed discharge, so the computed errors are reported in the units of the predicted discharge. The MAE, MBE and RMSE indices ranged between 0 and ∞ , and lower values show better agreement between observed and predicted discharges. Table 2 shows performance rating evaluated by MAE was generally low. The performed moderately based on MAE index except at Idah on the Niger River, Ibi and Makurdi on the Benue River. The RMSE measure performed moderately well except at Ibi, Makurdi and Umaisha on the Benue River. In view of the poor performances of some of the mathematical measures, there is a need to improve hydrological data gathering in the Niger and Benue river systems.



4. Conclusion and Recommendation

The rating curve is a very important tool in surface hydrology. It is extremely used to transform continuous measurement of stage data to a continuous record of stream discharge in natural and/or artificial open channel. Streamflow data is used in Flood Frequency Analysis, Rainfall-Runoff modeling, Floodplain management etc. This paper presents the development of updated stage-discharge curve equations for 9 hydrological stations in the Niger and Benue river systems in Nigeria. Since the stream data are transformed from stage data, errors and uncertainties are introduced and they have been evaluated using mathematical measures of MAE, MBE and RMSE while the degree of agreement between observed and predicted streamflow were assessed using R^2 , d and EF indices and the updated or improved stage – discharge equations performed perfectly well. The poor performance of the RMSE index suggests the need for quality hydro meteorological data collection and management. In view the critical role water resources management play in sustained economic development, poverty alleviation, and attainment of millennium Development Goals (MDGs) in the Niger River Basin. This paper recommends the need for institutional reforms to acknowledge the importance of hydrometry, data management and manpower development.

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