



## An Improving Method of Linearity of Magnetic Flux Distribution in Air Gap with Long Iron Cores

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**Abstract** The electromagnetic sensors are comparatively simple and reliable than other sensors. Because nowadays electromagnetics (EM) are applied in many areas of electrical engineering, such as electric machines, transformers, electromechanical energy conversion, antennas and radars, remote sensing, satellite communications, bio electromagnetics, electrocardiograms, electromagnetic sensors etc. But proper design of any electromagnetic devices or sensors requires knowledge based on magnetic circuits and elaborate magnetic circuits analysis of the sensors. The paper introduces a method of increasing linearity of magnetic flux distribution in the air gap with long magnetic cores.

**Keywords** Electromagnetic sensor, magnetic circuit, magnetizing winding, linear displacement sensor, differential equation, magnetizing force, grapho-analytical method, method of variation of constants, integration constant

### Introduction

From classical theory of electromagnetic sensors it is known, that basic metrological performance and work stability of sensors depends on the magnetic flux distribution, linear or nonlinear in the air gap with long magnetic cores of the sensors [1].

In electromagnetic sensors non - uniformity of a magnetic flux density in a air gap or the nonlinearity of distribution of a magnetic flux on coordinate is explained to that the magnetic reluctance of iron core unequal for the various parts of the iron core. This is the main reason for performance degradation, particularly unsteady sensitivity and low precision of electromagnetic sensors, as well as performance degradation have influence on output signal of electromagnetic sensors [2,3].

In connection with there is a necessity of development of new method that can be improve magnetic flux distribution in the air gap between long iron cores of the electromagnetic sensors and provide linear distribution of the magnetic flux on the coordinate. With this purpose in mind, we'll perform analysis long line magnetic circuits as shown in figure 1.

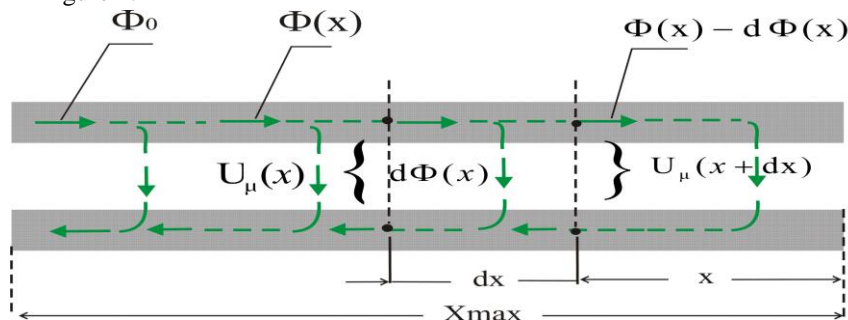


Figure 1: Magnetic circuit with long line iron cores



From the theory of the long line magnetic circuits, it is known that magnetic flux in the gap between long iron cores and magnetic voltage (magnetic flux density) along the line can be written as following system of differential equations:

$$\begin{cases} d\Phi(x) = gU_{\mu}(x)dx \\ dU_{\mu}(x) = 2r_{\mu}\Phi(x)dx \end{cases} \quad (1)$$

where  $g = \mu_0 \frac{b}{\delta}$  - magnetic conductance per unit length of line (specific) in the air gap between long iron cores (H/m),

$r_{\mu} = \rho_{\mu} \frac{1}{A}$  - magnetic reluctance per unit length (specific) of line of the long iron cores, (1/H·m),

$d\Phi(x)$ ,  $U_{\mu}(x)$  - field ramp and magnetic voltage at the elementary part ( $dx$ ) of the magnetic circuits.

Commonly solving system equations (1) we obtain:

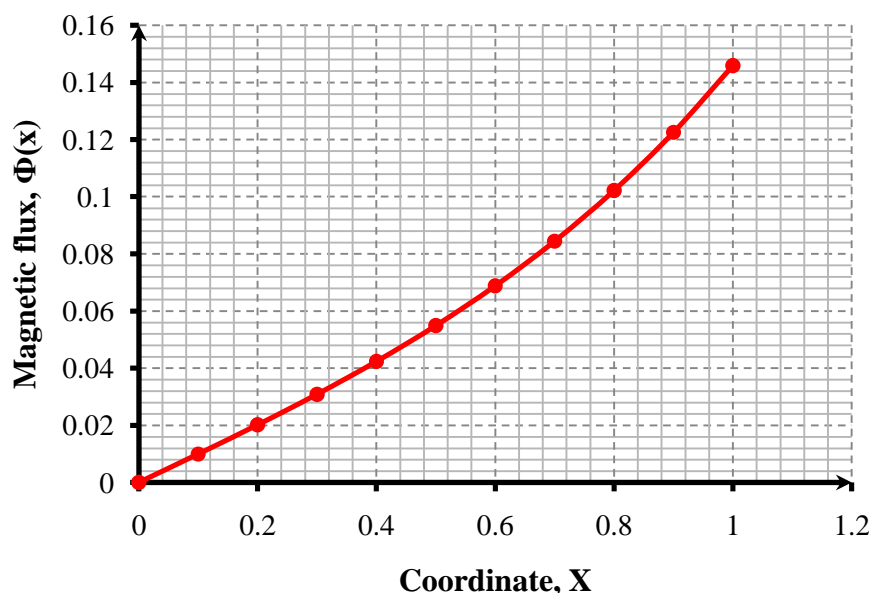
$$\Phi(x) = \frac{gU_{\mu,mmf}}{\gamma \cosh \gamma x_m} \cdot \sinh \gamma x \quad (2)$$

$$U_{\mu}(x) = \frac{d\Phi(x)}{dx} \cdot \frac{1}{g} = \frac{U_{\mu,mmf}}{\cosh \gamma x_m} \cdot \cos \gamma x \quad (3)$$

where  $U_{\mu,mmf} = U_{\mu}(x)$  - magnetomotive force (MMF) of the magnetizing coil,

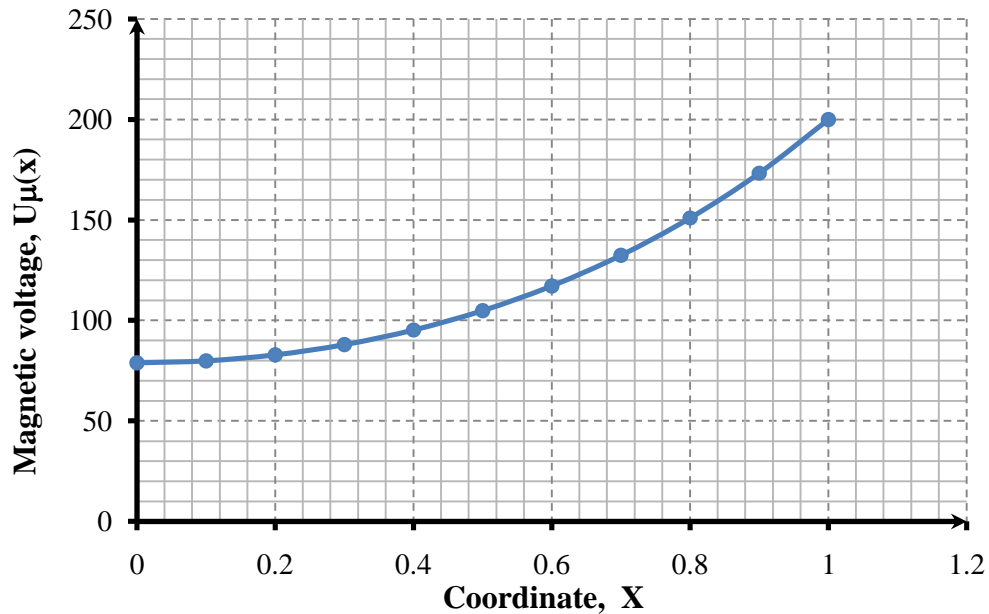
$\gamma = \sqrt{2gr_{\mu}}$  - propagation factor of magnetic flux wave in the magnetic core on the coordinate X.

To examine the obtained results we plot magnetic flux distribution  $\Phi(x)$  and magnetic voltage along the long line  $U_{\mu}(x)$  versus coordinate X. These curves are shown in figures 2 and 3 below.



Figures 2: Magnetic flux distribution  $\Phi(x)$  versus coordinate X





Figures 3: Magnetic voltage  $U_{\mu}(x)$  versus coordinate  $X$

Analyzing the received curves, (see figure 2 and figure 3) it is possible to make a conclusion that with increasing propagation factor of magnetic flux wave  $\gamma$  degree of nonlinearity grows.

Thus magnetic system of existing electromagnetic sensors with simple distribution of magnetic flux do not allow to get linear distribution of a magnetic flux along coordinate  $X$  and therefore metrological performance of the electromagnetic sensors not satisfy to the requirements of automatic control systems.

**Materials and Methods**

Let's consider an offered method of increasing linearity of magnetic flux distribution in the air gap between iron cores by compensation magnetic voltage drops across the magnetic conductor with the magnetomotive force (MMF) of the magnetizing coil [5,6]. The magnetic circuit of the electromagnetic displacement sensor with magnetizing coil is presented in figure 4.

The differential equation for this magnetic system can be written as

$$\frac{d^2\Phi(x)}{dx^2} - g r_{\mu} \Phi(x) = -f_r(x)g \tag{4}$$

where  $f_r(x)$  -magnetizing coil winding law along the coordinate  $X$ .

Pre-requisite and sufficient condition of enforcement of linear distribution of the magnetic flux  $\Phi(x)$  in the air gap is the correct choice of the magnetizing coil winding law  $f_r(x)$  along the coordinate  $X$ .

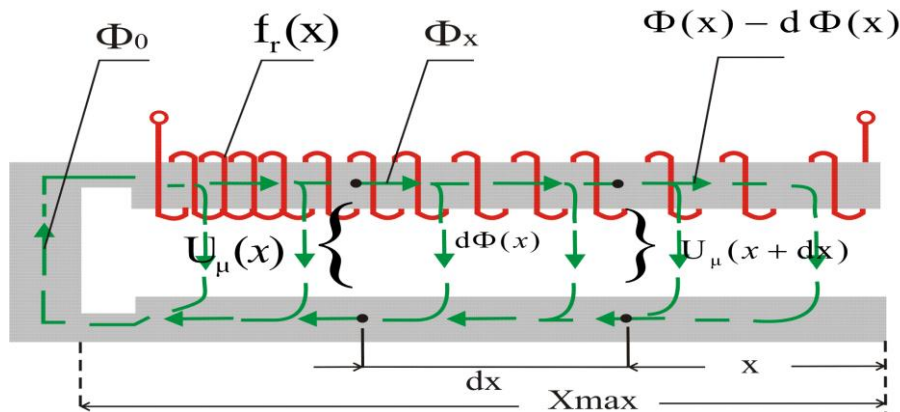


Figure 4: Magnetic circuit of displacement sensor with magnetizing coil



The differential equation (4) is inverse problem. As above mentioned the solution of the equation (4) requires knowing magnetizing coil winding law  $f_r(x)$ . So the choice of function  $f_r(x)$  was performed as follows. We have investigated magnetic circuits and plotted  $U_{\mu,mmf}, U_{\mu}(x)$  which are presented in figure 5.

Using grapho-analytical method we received expected function  $F_r(x)$  and dividing by  $X$  we found per unit length (specific) value of the function  $f_r(x)$  that is:

$$F_r(x) = kx^2 \text{ and } f_r(x) = kx \tag{5}$$

where  $k$  – coefficient, which is determined from the designing parameters of the magnetic system or magnetic circuits and magnitude of magnetomotive force (MMF) of the magnetizing coil  $U_{\mu,mmf}$ .

So the function  $f_r(x)$  can be chosen analytically as follows: according to linearity of magnetic flux distribution along the coordinate  $X$  is:

$$\Phi(x) = kx \tag{6}$$

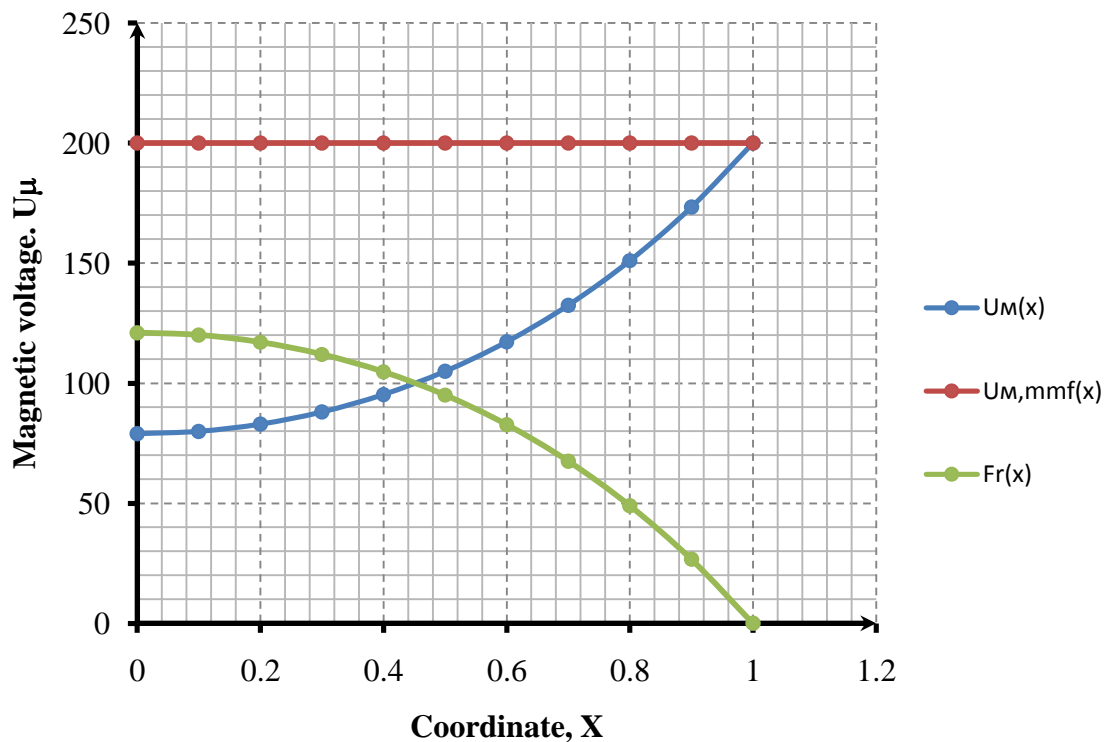


Figure 5: Expected curve of the function  $F_r(x)$

Substituting (6) into (4) we obtain following function of  $f_r(x)$  as:

$$\frac{d^2(kx)}{dx^2} - gr_{\mu}kx = -f_r(x)g$$

hereof  $f_r(x) = r_{\mu}kx$

Substituting (5) into (4) and using method of variation of constants, we will solve differential equation (4) [ 4,7].

$$\Phi(x) = \frac{kx}{r_\mu} + e^{-\gamma x} \int \frac{(e^{\gamma x})'}{e^{\gamma x} (e^{-\gamma x})' - (e^{\gamma x})' e^{-\gamma x}} \left[ \frac{kx}{r_\mu} \right] dx - e^{\gamma x} \int \frac{(e^{-\gamma x})'}{e^{\gamma x} (e^{-\gamma x})' - (e^{\gamma x})' e^{-\gamma x}} \left[ \frac{kx}{r_\mu} \right] dx + C_1 e^{\gamma x} + C_2 e^{-\gamma x}$$

$$\Phi(x) = \frac{kx}{r_\mu} + e^{-\gamma x} \int \frac{\gamma e^{\gamma x}}{-2\gamma} \left[ \frac{kx}{r_\mu} \right] dx - e^{\gamma x} \int \frac{-\gamma e^{-\gamma x}}{-2\gamma} \left[ \frac{kx}{r_\mu} \right] dx + C_1 e^{\gamma x} + C_2 e^{-\gamma x} = \frac{kx}{r_\mu} - \frac{kx}{2r_\mu} \cdot e^{-\gamma x} \cdot \frac{1}{\gamma} \cdot e^{\gamma x} + \frac{kx}{2r_\mu} \cdot e^{\gamma x} \cdot \frac{1}{\gamma} \cdot e^{-\gamma x} + C_1 e^{\gamma x} + C_2 e^{-\gamma x} = \frac{kx}{r_\mu} + C_1 e^{\gamma x} + C_2 e^{-\gamma x} \quad (7)$$

Differentiating the equation (7) on dx, we find magnetic voltage (magnetic intensity) along the line:

$$U_\mu(x) = \frac{d\Phi(x)}{dx} \cdot \frac{1}{g} = \frac{1}{g} \left[ \frac{k}{r_\mu} + \gamma C_1 e^{\gamma x} - \gamma C_2 e^{-\gamma x} \right] = \frac{k}{gr_\mu} + \frac{\gamma}{g} [C_1 e^{\gamma x} - C_2 e^{-\gamma x}] \quad (8)$$

Integration constants C1 and C2 will be determined from the boundary conditions as follows: on condition that

$$x=0 \quad \Phi(x)=0, \quad x=X_m \quad U_\mu(x) = U_{\mu,mmf} \quad (9)$$

Solving in common equations (7) and (8) subject to the boundary conditions (9) we obtain magnitude of the C1 and C2.

$$\begin{cases} C_1 + C_2 = 0 \\ U_{\mu,mmf} = \frac{k}{gr_\mu} + \frac{\gamma}{g} (C_1 e^{\gamma X_m} - C_2 e^{-\gamma X_m}) \end{cases} \quad \begin{cases} C_2 = -C_1 \\ U_{\mu,mmf} - \frac{k}{gr_\mu} = \frac{\gamma}{g} C_1 2 \cosh \gamma X_m \end{cases}$$

$$C_1 = \frac{gr_\mu U_{\mu,mmf} - k}{2r_\mu \gamma \cosh \gamma X_m}; \quad C_2 = \frac{k - gr_\mu U_{\mu,mmf}}{2r_\mu \gamma \cosh \gamma X_m} \quad (10)$$

Substituting equation (10) into equation (7) yields:

$$\Phi(x) = \frac{kx}{r_\mu} + \frac{(gr_\mu U_{\mu,mmf} - k) 2 \sinh \gamma x}{2r_\mu \gamma \cosh \gamma X_m} = \frac{kx}{r_\mu} + \frac{(gr_\mu U_{\mu,mmf} - k) \sinh \gamma x}{r_\mu \gamma \cosh \gamma X_m} \quad (11)$$

Magnetic voltage  $U_\mu(x)$  is determined as follows:

$$U_\mu(x) = \frac{d\Phi(x)}{dx} \cdot \frac{1}{g} = \frac{k}{gr_\mu} + \frac{(gr_\mu U_{\mu,mmf} - k) \cosh \gamma x}{gr_\mu \cosh \gamma X_m}$$

or

$$U_\mu(x) = \frac{k}{gr_\mu} + U_{\mu,mmf} \frac{\cosh \gamma x}{\cosh \gamma X_m} - \frac{k \cdot \cosh \gamma x}{gr_\mu \cosh \gamma X_m} \quad (12)$$



On the basis of the received equations (11) and (12) we have plotted magnetic flux distribution in the air gap and magnetic voltage along long line versus coordinate X which are presented below in figures 6 and 7.

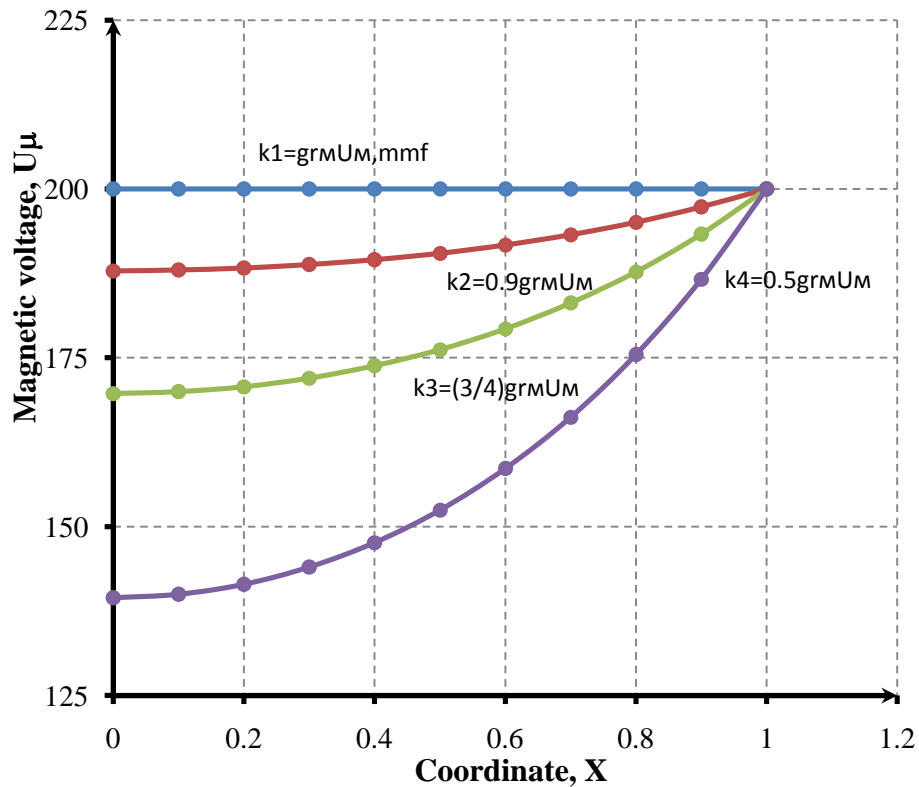


Figure 6: Magnetic voltage versus coordinate X at different K

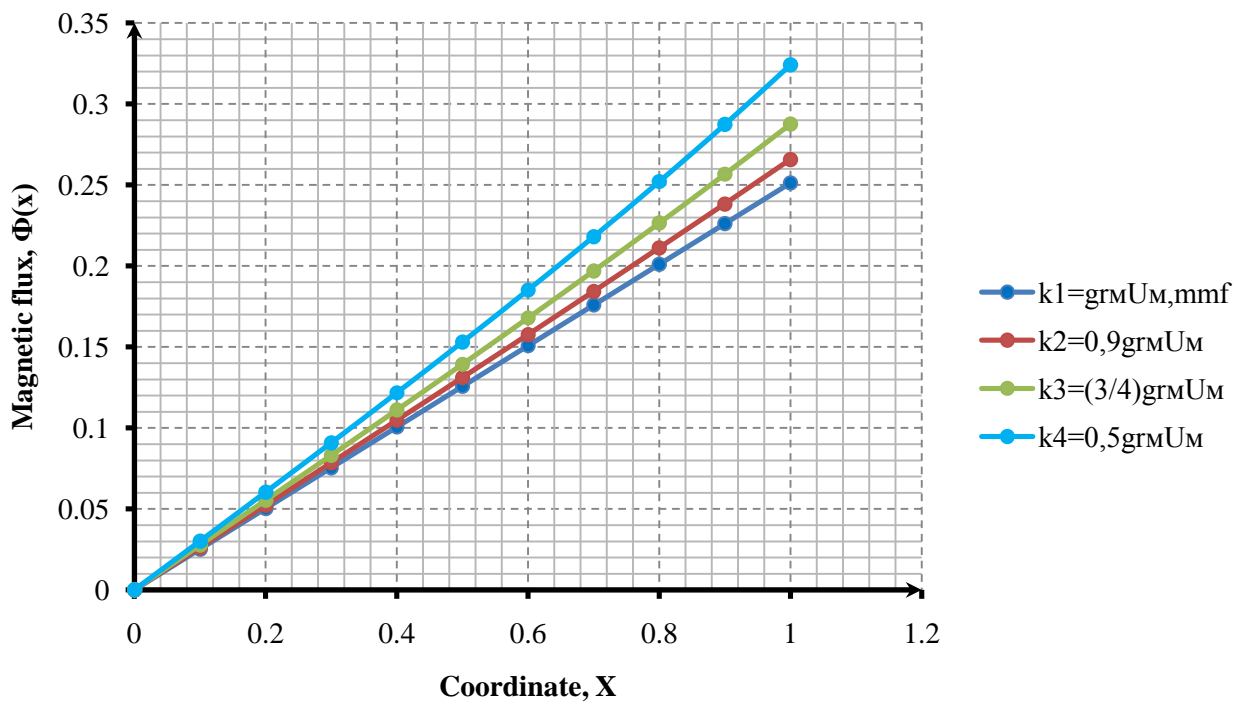


Figure 7: Magnetic flux versus coordinate X at different K

**Conclusion**

From the obtained equations and curves of the magnetic flux distribution and magnetic voltage along the coordinate it is possible to do conclusion that the defined value of the K it is possible to get completely linear magnetic flux distribution and magnetic voltage across the long line. So discovered method allows more linear distribution of magnetic flux in the air gap and magnetic voltage across the long iron cores. It improves the precision of electromagnetic sensors throughout the measurement range, provides more accuracy and constant sensitivity and remove measurement range limitation due to nonlinearity.

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