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Effects of Thermal Radiation on MHD Free Convection Reactive Flow with Time-Dependent Suction

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Abstract Investigations on the effect of thermal radiation and magnetic field on unsteady free convection flow with time-dependent suction and chemical reaction in a porous medium are carried out. All the fluid properties are assumed constant except the influence of the density variation with temperature and concentration. The Boussinesq approximation is used for the density variation with temperature and concentration. The governing equations are non dimensionalised and the regular perturbation technique is used for solving the nonlinear partial differential equations governing the flow variables (velocity, temperature and concentration). Numerical results for the velocity, temperature and concentration profiles, are obtained by using NDSolve of the software Mathematica. Finally, the analytical solutions for the flow variables are graphically shown and discussed.

Keywords Free Convection, Uniform Magnetic Field, Radiation, Chemical Reaction, Porous plate, Magneto-Hydrodynamic (MHD), Density Variation

Introduction

In industries, MHD systems are very significant in a moving conducting fluid producing electrical energy by extraction, reactors cooling in connection with nuclear engineering, filtration of solids from liquids, gas turbines, propulsion in airplanes, missiles and air vehicles. Over time the theory of MHD free convective flow through a porous medium has been of great importance in many facets of life such as in various transport processes both naturally and artificially in the areas of science and engineering applications. Many researchers have succeeded in carrying out related studies on these as found in [1-10]. For instance, Israel-Cookey et al. [1] examined the impact of viscous dissipation and radiation on the issue of unsteady magnetohydrodynamic free-convection stream past an endless vertical warmed plate in some optically thin surroundings with time dependent suction. Elgazery [6] examined numerically the problem of unsteady free convection flow with heat and mass transfer from an isothermal vertical plate in a porous medium. Kim [11] analysed unsteady MHD convective heat transfer past a semi-infinite vertical porous moving plate with variable suction, the plate moved with steady velocity in the direction of the flow and the stream velocity increasing exponentially with a small perturbation law. Chamkha [12] then extended the problem of [11] to mass transfer and heat absorption effects.

Several researchers have investigated on the effects of uniform magnetic field and thermal radiation [13-19] For example, Kumar et al. [13] extended the work in [9] by including thermal diffusion effect on MHD free convective radiating flow past an impulsively started vertical plate embedded in a porous medium using Laplace transform technique to solve the governing equations.

Other researchers worked on the effects of chemical reaction with or without magnetic field and radiation. For example, using finite difference method Sarada & Shanker [20] studied numerically the impact of chemical reaction on an unsteady MHD flow past an infinite vertical porous plate with variable suction and heat convective mass transfer. Sinha [21] carried out a parametric study on the effect of first order chemical reaction on an unsteady MHD free convective flow of an incompressible, electrically conducting and heat absorbing

fluid past an infinite vertical porous plate in the presence of uniform transverse magnetic field. Rao et al. [22] studied the effects of chemical reaction on an unsteady magnetohydrodynamics free convection fluid flow past a semi-infinite vertical plate implanted in a permeable medium with heat absorption. While Shivaiah & Rao [23] studied the impact of chemical reaction on unsteady MHD free convective fluid flow past a vertical porous plate in the presence of injection or suction. Finite element method was used to solve the dimensionless equations. Using Runge- Kutta fourth order with shooting method, Hemalatha et al. [24] examined the impacts of both thermal radiation and chemical reaction on MHD free convection flow past a moving vertical plate consisting heat source and convective surface boundary condition in the presence of heat generation. Kowsalya & Begam [25] carried out a research on the impacts of thermal diffusion and hall current on MHD unsteady mass transfer flow past a semi-vast vertical permeable plate embedded in a permeable medium in a slip flow regime in the existence of variable suction, thermal radiation as well as chemical reaction. Ahmed & Kalita [26] presented a paper on the impacts of chemical reactions and magnetic field on the mass and heat transfer of Newtonian fluid over a boundless vertical wavering plate with variable mass dissemination. Gurivireddy et al. [27] aimed at examining the effect of thermodiffusion on an unsteady simultaneous convective flow of heat and mass transfer of an incompressible, electrically conducting, heat generating and absorbing fluid along a semi-unbounded moving permeable plate implanted in a permeable medium with the presence of pressure slope, thermal radiation field and chemical reaction. Hence based on these works we have studied the combined effects of thermal radiation, and magnetic field with chemical reaction.

Formulation of the Problem

Consider an unsteady convective flow of an incompressible viscous, chemically reacting and radiating hydromagnetic fluid through an infinite porous plate with time – dependent suction.We assumed u' and v' to be the component of velocity in the directions of x' and y' respectively. The plate is long enough in the x' −direction, so all the physical variables are functions of y and t only.The plate temperature and concentration are varying with time. A uniform magnetic field of B_0 is applied normal to the plate. The plate moves uniformly along the positive x-direction with velocity U_0 . All the fluid properties are assumed constant except the influence of the density variation with temperature and concentration and hence the Boussinessq approximation. The flow is consequently governed by the equations of continuity, momentum, energy and species concentration below:

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = \upsilon\frac{\partial^2 u'}{\partial y'^2} + \frac{\partial U'}{\partial t} - \left(\frac{\mu^2\sigma_c}{\rho}B_0'^2 + \frac{\upsilon}{k}\right)(u' - U') + g\beta_T(T' - T_\infty') \tag{2}$$

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{K}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{\mu}{\rho c_p}\left(\frac{\partial u'}{\partial y'}\right)^2 - \frac{K}{\rho c_p}\frac{\partial q_r'}{\partial y'} \tag{3}$$

$$\frac{\partial C'}{\partial t'} + v'\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} - Kr(c' - c_\infty') + \frac{K_T D_m}{T_m}\frac{\partial^2 T'}{\partial y'^2} \tag{4}$$

With the initial and boundary conditions

$$u' = 0 \quad T' = T_w', \quad C' = C_w' \quad on \quad y' = 0$$

$$u' = U'(t) = v_0'(1 + \varepsilon e^{i\omega' t'}), T' = T'_\infty, C' = C_\infty' \quad as \quad y' \longrightarrow \infty \tag{5}$$

The Rosseland approximation is followed by taking the radiative heat flux in equation (3) as:

$$q_r' = \frac{-4\sigma * \nabla T'^4}{3\delta} \quad , \tag{6}$$

where $\sigma *$ is the Stephen – Boltzmann constant, δ is the mean absorption coefficient. Expanding T'^4 in Taylor's series about T_∞' (the free stream temperature) with the assumption that the temperature difference between the fluid and porous medium is small; also neglecting higher order terms the heat flux is then expressed as follows:

$$q_r' = \frac{-4\sigma *}{3\delta}\nabla(4T_\infty'^3 T' - 3T_\infty'^4) \tag{7}$$

Where

$$\frac{\partial q_r'}{\partial y} = \frac{16\sigma * T_\infty'^3}{3\delta}\frac{\partial^2 T'}{\partial y'^2} \tag{8}$$

Again, from equation (1) it is clearly shown that v' is a function of t only and also a constant. We assume

$$v' = -v_0(1+\varepsilon e^{\omega' t'}) \quad (9)$$

Such that $\varepsilon \ll 1$ and the negative sign depicts that the suction velocity is toward the porous plate.

Using the dimensionless quantities as follows:

$$y = \frac{v_0' y'}{\vartheta}, \quad u = \frac{u}{U_0}, \quad \omega = \frac{\vartheta}{v_0'^2}\omega',$$

$$t = \frac{v_0'^2}{\vartheta} t', \quad U = \frac{U'}{U_0}, \quad \theta = \frac{T'-T_\infty'}{T_w'-T_\infty'},$$

$$C = \frac{C'-C_\infty'}{C_w'-C_\infty'}, \quad \vartheta = \frac{\mu}{\rho}, \quad Kr = \frac{K_\tau'}{V_0'^2},$$

$$Sc = \frac{\vartheta}{D}, \quad Gr = \frac{\vartheta g B_T (T_w'-T_\infty')}{U_0 V_0'^2}, \quad \chi^2 = \frac{\vartheta^2}{k v_0'^2} \quad (10)$$

$$Grm = \frac{\vartheta g B_c (C_w'-C_\infty')}{U_0 V_0'^2}, \quad M^2 = \frac{\vartheta^2 \delta_c B_0'^2}{\rho v_0'^2}, \quad Pr = \frac{\mu Cp}{k},$$

$$Ec = \frac{U_0^2}{C_p (T_w'-T_\infty')}, \quad Sr = \frac{D_m K_T}{\vartheta T_m} \frac{(T_w'-T_\infty')}{C_w'-C_\infty'}$$

Where Kr is the chemical reaction, Sc is the Schmidt number, Gr is the Grashof number, Grm is the Modified Grashof number, M^2 is the Magnetic field, Pr is the Prandtl number, χ^2 is the Porosity, Ec is the Eckert number, Sc is the Soret number, equations (1) – (5) are reduced, thereby generating a system of nonlinear partial differential equations.:

$$\frac{\partial u}{\partial t} - (1+\varepsilon e^{i\omega t})\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \frac{\partial U}{\partial t} - (M^2+\gamma^2)(u-U) + Gr\theta + GrmC \quad (11)$$

$$Pr\frac{\partial \theta}{\partial t} - Pr(1+\varepsilon e^{i\omega t})\frac{\partial \theta}{\partial y} = PrEc\left(\frac{\partial u}{\partial y}\right)^2 + (1+R^2)\frac{\partial^2 \theta}{\partial y^2} \quad (12)$$

$$\frac{\partial C}{\partial t} - (1+\varepsilon e^{i\omega t})\frac{\partial C}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} - KrC + Sr\frac{\partial^2 \theta}{\partial y^2} \quad (13)$$

With the corresponding dimensionless initial and boundary conditions:

$$u = 0, \quad \theta = 1, \; C = 1 \quad at \; y = 0$$

$$u = (1+\varepsilon e^{i\omega t}), \; \theta = 0, \; C = 0 \quad as \; y \to \infty \quad (14)$$

Method of Solution

Using regular perturbation method, the nonlinear partial differential equations are reduced to ordinary differential equations, since they cannot be solved in closed forms. We assume that the solutions of the equations can be expanded using Taylor's expansion in ε.

$$u(y,t) = u_o(y) + \varepsilon e^{i\omega t} u_1(y) + O(\varepsilon^2)$$

$$\theta(y,t) = \theta_o(y) + \varepsilon e^{i\omega t} \theta_1(y) + O(\varepsilon^2) \quad (15)$$

$$C(y,t) = C_o(y) + \varepsilon e^{i\omega t} C_1(y) + O(\varepsilon^2)$$

Substituting equation (15) into the set of equations (11) - (14), equating nonharmonic, harmonic terms and neglecting the higher order terms of $O(\varepsilon^2)$, the following set of approximations are obtained.

$$u_0'' + u_0' - (M^2+\chi^2)u_0 = -(M^2+\chi^2) - Gr\theta_0 - GmC_0 \quad (16)$$

$$u_1'' + u_1' - [(M^2+\chi^2)+i\omega]u_1 = -u_0' - [(M^2+\chi^2)+i\omega] - Gr\theta_1 - GmC_1 \quad (17)$$

$$(1+R^2)\theta_0'' + Pr\theta_0' = -PrEcu_0'^2 \quad (18)$$

$$(1+R^2)\theta_1'' + Pr\theta_1' - Pri\omega\theta_1 = -Pr\theta_0' - 2PrEcu_0'u_1' \quad (19)$$

$$C_0'' + C_0' - KrScC_0 = -ScSr\theta_0'' \quad (20)$$

$$C_1'' + ScC_1' - Sc(i\omega + Kr)C_1 = -ScC_0' - ScSr\theta_1'' \quad (21)$$

The corresponding boundary conditions are:

$$u_0 = 0, u_1 = 0, \; \theta_0 = 1, \theta_1 = 0, C_0 = 1, C_1 = 0 \quad at \; y = 0 \quad (22)$$

$$u_0 \to 1, \; u_1 \to 1, \quad \theta_0 \to 0, \; \theta_1 \to 0, \; C_0 \to 0, \; C_1 \to 0 \qquad as \; y \to \infty$$

For O (ε) equations.

The O (ε) equations (17) – (21) are still coupled non-linear equations, and exact solutions are still impossible to obtain. Therefore by expanding $u_0,\ u_1,\ \theta_0, \theta_1,\ C_0\ and\ C_1$ around the Eckert number (Ec<<1) we have:

$$u_0(y) = u_{01}(y) + Ecu_{02}(y)$$
$$u_1(y) = u_{11}(y) + Ecu_{12}(y)$$
$$\theta_0(y) = \theta_{01}(y) + Ec\theta_{02}(y)$$
$$\theta_1(y) = \theta_{11}(y) + Ec\theta_{12}(y) \quad (23)$$
$$C_0(y) = C_{01}(y) + EcC_{02}(y)$$
$$C_1(y) = C_{11}(y) + EcC_{12}(y)$$

Substituting (23) into the set of equations (18) – (22), the following equations are obtained.

$$u''_{01} + u'_{01} - \delta_1 u_{01} = -\delta_1 - Gr\theta_{01} - GmC_{01} \quad (24)$$
$$(1 + R^2)\theta''_{01} + Pr\theta'_{01} = 0 \quad (25)$$
$$C''_{01} + ScC'_{01} - KrScC_{01} = -ScSr\theta''_{01} \quad (26)$$
$$u''_{02} + u'_{02} - \delta_1 u_{02} = -Gr\theta_{02} - GmC_{02} \quad (27)$$
$$C''_{02} + ScC'_{02} - KrScC_{02} = -ScSr\theta''_{02} \quad (28)$$
$$(1 + R^2)\theta''_{02} + Pr\theta'_{02} = -Pru'^2_{01} \quad (29)$$

where $\delta_1 = M^2 + \chi^2$

subject to the boundary conditions:

$$u_{01} = 0 = u_{02}, \theta_{01} = 1, \theta_{02} = 0, C_{01} = 1, C_{02} = 0 \quad \text{at } y = 0 \quad (30)$$
$$u_{01} = 1, u_{02} = 0, \theta_{01} = 0 = \theta_{02}, C_{01} = 0 = C_{02} \quad \text{as } y \to \infty$$

for O (1) equations and

$$u''_{11} + u'_{11} - \delta_2 u_{11} = -u'_{01} - \delta_2 - Gr\theta_{11} - GmC_{11} \quad (31)$$
$$(1 + R^2)\theta''_{11} + Pr\theta'_{11} - Pri\omega\theta_1 = -Pr\theta'_{01} \quad (32)$$
$$C''_{11} + ScC'_{11} - \delta_3 ScC_{11} = -ScC'_{01} - ScSr\theta''_{11} \quad (33)$$
$$u''_{12} + u'_{12} - \delta_2 u_{12} = -u'_{02} - \delta_2 - Gr\theta_{12} - GmC_{12} \quad (34)$$
$$(1 + R^2)\theta''_{12} + Pr\theta'_{12} - Pri\omega\theta_{12} = -Pr\theta'_{02} - 2Pru'_{01}u'_{11} \quad (35)$$
$$C''_{12} + ScC'_{12} - \delta_3 ScC_{12} = -ScC'_{02} - ScSr\theta''_{12} \quad (36)$$

subject to the boundary conditions:

$$u_{11} = 0 = u_{12}, \theta_{11} = 0 = \theta_{12}, C_{11} = 0 = C_{12} \quad \text{at } y = 0$$
$$u_{11} = 1, u_{12} = 0, \theta_{11} = 0 = \theta_{12}, C_{11} = 0 = C_{12} \quad \text{as } y \to \infty \quad (37)$$

for O (Ec) equations, where; $\delta_2 = (M^2 + \chi^2) + i\omega$, $\delta_3 = (i\omega + Kr)$.

Solving equations (24) – (29) satisfying the boundary conditions equation (30) and also solving equations (31) – (36) with boundary conditions equation (37). Then, substituting the solutions into equation (23) and also using equation (15). The following solutions for velocity, temperature and species concentration profiles are obtained:

$$u(y,t) = A_1 e^{m_3 y} + \alpha_2 - \alpha_3 e^{m_1 y} - \alpha_4 e^{m_2 y} + Ec(A_4 e^{m_3 y} + \alpha_{18} e^{m_1 y} + \alpha_{19} e^{m_2 y} + \alpha_{20} e^{2m_1 y} + \alpha_{21} e^{2m_2 y} + \alpha_{22} e^{2m_3 y} + \alpha_{23} e^{(m_1+m_2)y} + \alpha_{24} e^{(m_1+m_3)y} + \alpha_{25} e^{(m_2+m_3)y}) + \varepsilon e^{i\omega t}(A_7 e^{m_6 y} + \alpha_{30} e^{m_1 y} + \alpha_{31} e^{m_2 y} + \alpha_{32} e^{m_3 y} + \alpha_{33} e^{m_4 y} + \alpha_{34} e^{m_5 y} + \alpha_{35} + Ec\big(A_{10} e^{m_6 y} + \alpha_{70} e^{m_1 y} + \alpha_{71} e^{m_2 y} + \alpha_{72} e^{m_3 y} + \alpha_{73} e^{m_4 y} + \alpha_{74} e^{m_5 y} + \alpha_{75} e^{2m_1 y} + \alpha_{76} e^{2m_2 y} + \alpha_{77} e^{2m_3 y} + \alpha_{78} e^{(m_1+m_2)y} + \alpha_{79} e^{(m_1+m_3)y} + \alpha_{80} e^{(m_1+m_4)y} \alpha_{81} e^{(m_1+m_5)y} + \alpha_{82} e^{(m_1+m_6)y} + \alpha_{83} e^{(m_2+m_3)y} + \alpha_{84} e^{(m_2+m_4)y} + \alpha_{85} e^{(m_2+m_5)y} + \alpha_{86} e^{(m_2+m_6)y} + \alpha_{87} e^{(m_3+m_4)y} + \alpha_{88} e^{(m_3+m_5)y} + \alpha_{89} e^{(m_3+m_6)y}\ ) \quad (38)$$

$$\theta(y,t) = e^{m_1 y} + Ec(A_2 e^{m_1 y} + \alpha_5 e^{2m_1 y} + \alpha_6 e^{2m_2 y} + \alpha_7 e^{2m_3 y} + \alpha_8 e^{(m_1+m_2)y} + \alpha_9 e^{(m_1+m_3)y} + \alpha_{10} e^{(m_2+m_3)y}) + \varepsilon e^{i\omega t}(A_5 e^{m_1 y} + \alpha_{26} e^{m_4 y} + Ec\big(A_8 e^{m_4 y} + \alpha_{36} e^{m_1 y} + \alpha_{37} e^{2m_1 y} + \alpha_{38} e^{2m_2 y} + \alpha_{39} e^{2m_3 y} + \alpha_{40} e^{(m_1+m_2)y} + \alpha_{41} e^{(m_1+m_3)y} + \alpha_{42} e^{(m_1+m_4)y} + \alpha_{43} e^{(m_1+m_5)y} + \alpha_{44} e^{(m_1+m_6)y} + \alpha_{45} e^{(m_2+m_3)y} + \alpha_{46} e^{(m_2+m_4)y} + \alpha_{47} e^{(m_2+m_5)y} + \alpha_{48} e^{(m_2+m_6)y} + \alpha_{49} e^{(m_3+m_4)y} + \alpha_{50} e^{(m_3+m_5)y} + \alpha_{51} e^{(m_3+m_6)y})) \quad (39)$$

$$C(y,t) = (1 - \alpha_1) e^{m_2 y} + \alpha_1 e^{m_1 y} + Ec(A_3 e^{m_2 y} + \alpha_{11} e^{m_1 y} + \alpha_{12} e^{2m_1 y} + \alpha_{13} e^{2m_2 y} + \alpha_{14} e^{2m_3 y} + \alpha_{15} e^{(m_1+m_2)y} + \alpha_{16} e^{(m_1+m_3)y} + \alpha_{17} e^{(m_2+m_3)y}) + \varepsilon e^{i\omega t}(A_6 e^{m_5 y} + \alpha_{27} e^{m_1 y} + \quad (40)$$

$$\alpha_{28}e^{m_2y} + \alpha_{29}e^{m_4y} + Ec\big(A_9e^{m_5y} + \alpha_{52}e^{m_1y} + \alpha_{53}e^{m_2y} + \alpha_{54}e^{m_4y} + \alpha_{55}e^{2m_1y} + \alpha_{56}e^{2m_2y} + \alpha_{57}e^{2m_3y} + \alpha_{58}e^{(m_1+m_2)y} + \alpha_{59}e^{(m_1+m_3)y} + \alpha_{60}e^{(m_1+m_4)y} + \alpha_{61}e^{(m_1+m_5)y} + \alpha_{62}e^{(m_1+m_6)y} + \alpha_{63}e^{(m_2+m_3)y} + \alpha_{64}e^{(m_2+m_4)y} + \alpha_{65}e^{(m_2+m_5)y} + \alpha_{66}e^{(m_2+m_6)y} + \alpha_{67}e^{(m_3+m_4)y} + \alpha_{68}e^{(m_3+m_5)y} + \alpha_{69}e^{(m_3+m_6)y}\big)\big)$$

Result and Discussion

On this study, we have performed perturbation analysis on the effect of thermal radiation and magnetic field on free convection time-dependent flow with chemical reaction. Our computed results have been done with relevant physical parameters and are revealed on graphs to show the influence of the governing parameters of the flow. Most especially the influence of radiation R, magnetic field M, chemical reaction Kr, on temperature $\theta(y,t)$, velocity U(y,t), and concentration C(y,t). The various graphical results are discussed below as follows: In Figure 4.1 it is observed that the velocity increases with increasing radiation parameters. Physically, the velocity increases because increase in radiation leads to increased dominance of conduction over radiation which increases the buoyancy force of the boundary layer. figure 4.2 illustrates the effect of magnetic field on the velocity distribution of the fluid. The profile shows that velocity decreases with increase in magnetic field parameter, M. This observation showcases the action of the Lorentz force which has the tendency of condensing the momentum boundary layer thereby retarding the flow. Far from the plate where the effect of the magnetic field is not felt, the velocity assumes the free stream flow. The effects of the thermal buoyancy force, Gr on the velocity distribution is shown in Figure 4.3; depicting that increase in the Grashof number Gr increases the velocity. This is because of the increase in the convection current. Figure 4.4 shows the effect of chemical reaction on the velocity distribution. The profile reveals that the velocity decreases with increase in chemical reaction parameter, due to the presence of viscous dissipation. We notice that near the plate there is a rise in the velocity profile and far from the plate it assumes the free stream velocity. The effect of the Eckert number on the velocity profile is illustrated in Figure 4.5. The effect is significant close to the plate. It simply shows that the velocity at the boundary layer increases as a result of increase in the Eckert number. The velocity is enhanced because of the heat energy stored in fluid. Figure 4.6 depicts the velocity increases with increase in the values of Soret. There is more pronouncement of that in the vicinity of the plate than in the free stream. Physically, this is true as increase in Soret increases the driving force for mass diffusion. The influence of thermal radiation on the temperature distribution is very significant in Figure 4.7. The profile indicates that the fluid temperature increases as the thermal radiation is increased. In the real sense it is true as the addition of thermal radiation enhances the further diffusion of energy. The effect of increasing magnetic field on the temperature profile is illustrated in Figure 4.8. It is observed that increase in magnetic field increases temperature of the distribution. The effect is more significant at higher values of the magnetic field. The magnetic field applied heats up the fluid, thereby increasing the temperature. Figure 4.9 illustrates the effect of chemical reaction Kr on the concentration. The profile shows that increase in chemical reaction decreases the concentration. This is because increase in the chemical reaction parameter leads to decrease in the concentration of species in the boundary layer and increases mass transfer. Also chemical process reduces concentration profile at the boundary layer. The influence of magnetic field on the concentration distribution is illustrated in Figure 4.10. Increase in the magnetic field parameter M, results in decrease in the concentration profile. It can be seen that the effect is significant at M= 1,3 compared to lower values. This observation is true physically as the magnetic force greatly reduce the concentration at the boundary layer. Figure 4.11 represents the effect of radiation parameter R on the concentration profiles. It is found that the concentration increases with increase in the radiation parameter. The trend in figure 4.12 shows that the concentration increases with increase in Soret. This effect is very significant at higher values of Soret. This physically shows that the ratio of thermal diffusion is increased.

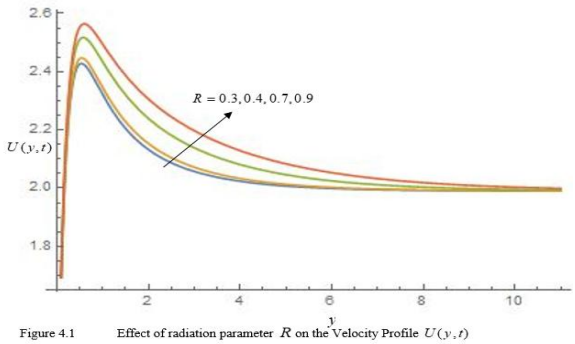

Figure 4.1 Effect of radiation parameter R on the Velocity Profile $U(y,t)$ for the Parameters $Kr = 1, M = 5, Pr = 0.71, Gr = 20, Gm = 5, Ec = .01, Sr = 5, Sc = 0.22, \chi = 0.2, \omega = 1.0, t = 0.2, \varepsilon = 1$

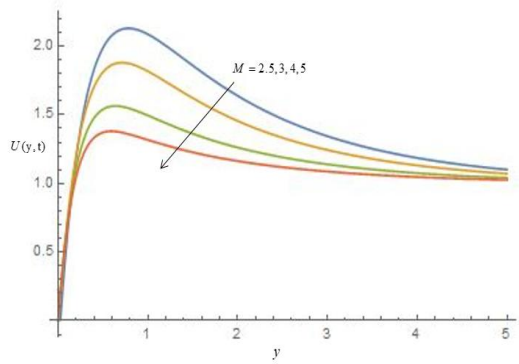

Figure 4.2 Effect of Magnetic Field parameter M on the Velocity Profile $U(y,t)$ for the parameter $R = 0.4, Kr = 2, Pr = 0.71, Gm = 5, Gr = 5, Ec = 0.01, Sr = 1.0, Sc = .22, \chi = 0.1, \omega = 1.0, t = 0.2, \varepsilon = 1$.

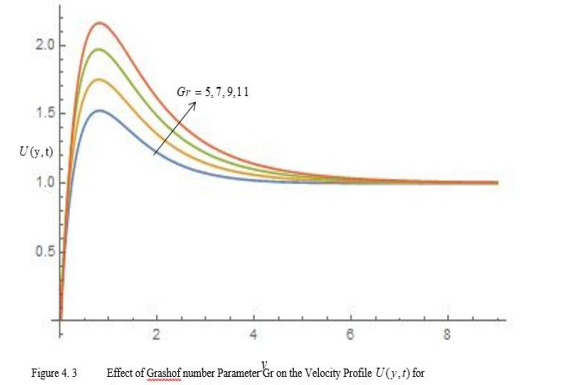

Figure 4.3 Effect of Grashof number Parameter Gr on the Velocity Profile $U(y,t)$ for the parameters $R = 0.4, Kr = 2, M = Pr = 0.71, Gm = 5, Ec = 0.01, Sr = 1.0, Sc = 0.22, \chi = 0.1, \omega = 1.0, t = 0.2, \varepsilon = 1$.

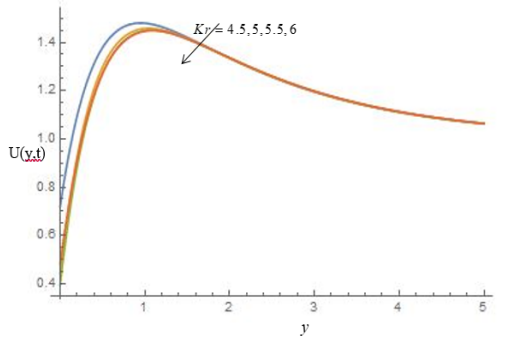

Figure 4.4 Effect of Chemical Effect Kr on Velocity profile for the other Parameters

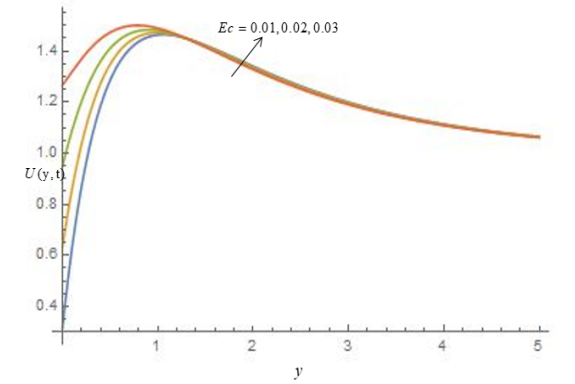

Figure 4.5 Effect of Eckert number Ec on velocity profile $U(y,t)$ for parameters $R = 0.5, Kr = 5, M = 2, Pr = 0.71, Gm = 5, Gr = 4, Sc = 0.22$ $Sr = 1.0, \chi = 0.1, \omega = 1.0, t = 0.2, \varepsilon = 1$.

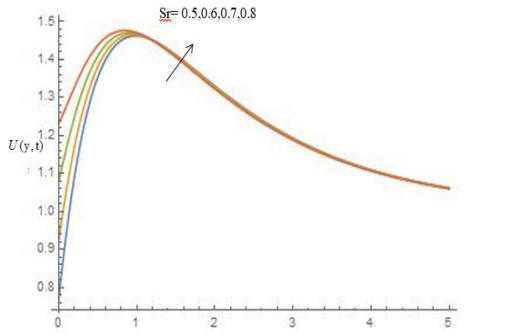

Figure 4. 6 Effect of Soret number parameter Sr on velocity profile $U(y,t)$ for the parameters $R = 0.5, Kr = 5, M = 2.0, Pr = 0.71, Gm = 5, Gr = 5, Ec = 0.01, \chi = 0.1, \omega = 1.0, t = 0.2, \varepsilon = 1, Sc = 0.22$.

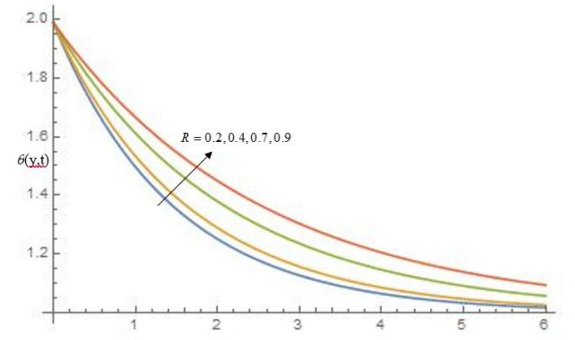

Figure 4.7 Effect of radiation parameter R on the temperature profile $\theta(y,t)$ for the Parameters $Kr = 0.1, M = 5, Pr = 0.71, Gr = 5, Gm = 5, Ec = 0.01, Sr = 1, Sc = 0.22, \chi = 0.1, \omega = 1.0, t = 0.2, \varepsilon = 1$.

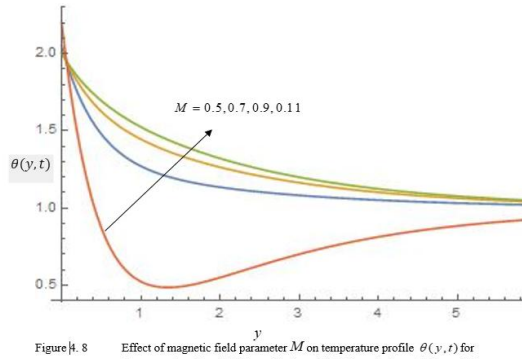

Figure 4. 8 Effect of magnetic field parameter M on temperature profile $\theta(y,t)$ for the parameters $R = 0.7, Kr = 0.1, Pr = 0.71, Gr = 5, Gm = 5, Ec = 0.01, Sr = 1, Sc = 0.22, \chi = 0.1, \omega = 1.0, t = 0.2, \varepsilon = 1$.

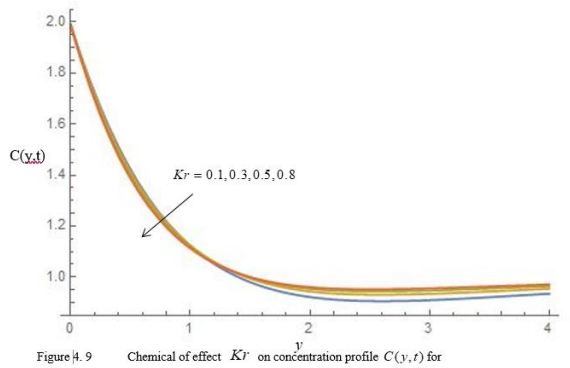

Figure 4. 9 Chemical of effect Kr on concentration profile $C(y,t)$ for the parameters $R = 0.4, M = 2, Pr = 0.71, Gr = 10, Gm = 5, Ec = 0.01, Sr = 1, Sc = 0.22, \chi = 0.1, \omega = 1.0, t = 0.2, \varepsilon = 1$.

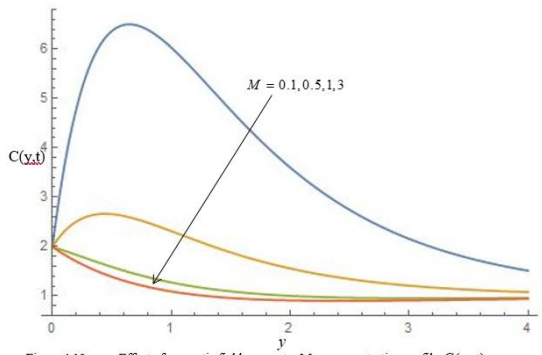

Figure 4.10 Effect of magnetic field parameter M on concentration profile $C(y,t)$ for the parameters $R = 02, Kr = 0.1, Pr = 0.71, Gr = 5, Gm = 5, Ec = 0.01, Sr = 1, Sc = 0.22, \chi = 0.1, \omega = 1.0, t = 0.2, \varepsilon = 1$.

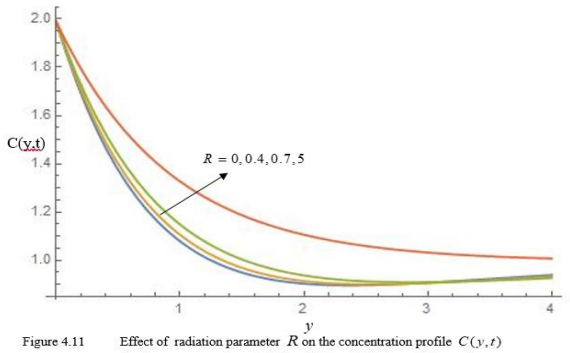

Figure 4.11 Effect of radiation parameter R on the concentration profile $C(y,t)$ for the parameters $Kr = 0.1, M = 5, Pr = 0.71, Gr = 5, Gm = 5, Ec = 0.01, Sr = 1, Sc = 0.22, \chi = 01, \omega = 1.0, t = 0.2, \varepsilon = 1$.

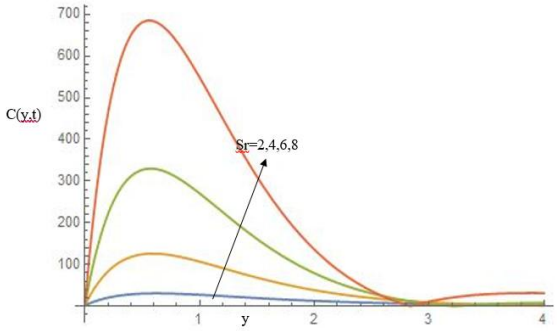

Figure 4.12 Effect of Soret parameter Sr on the concentration profile $C(y,t)$ for the parameters $Kr = 0.1, M = 5, Pr = 0.71, Gr = 5, Gm = 5, Ec = 0.01, Sc = 0.22, \chi = 0.1, \omega = 1.0, t = 0.2, \varepsilon = 1$.

Conclusion

In conclusion, the effects of thermal radiation on MHD free convection reactive flow with time-dependent suction have great imparts as increase in the values of radiation, Grashof, Eckert and Soret numbers lead to increase in velocity profiles but experience decline for different parameters values for magnetic field and chemical reaction. In addition, temperature rapidly increases for the increased values of radiation and magnetic field parameters. The applied magnetic field heats up the fluid thereby retarding the velocity flow due to Lorentz force which opposes the fluid motion. In the case of chemical reaction and magnetic field, it is discovered that they played a good role in specie concentration since increase in both parameters reduce the concentration in the boundary layer but increase in radiation increases the concentration.

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Appendix

$m_1 = -\frac{Pr}{1+R^2}$; $m_2 = \frac{-(1+\sqrt{Sc+4ScKr})}{2}$; $m_3 = \frac{-(1+\sqrt{1+4\delta_1})}{2}$;

$m_4 = -\frac{(Pr+\sqrt{Pr^2+4(1+R^2)(Pri\omega)})}{2(1+R^2)}$; $m_5 = -\frac{1}{2}\left(1+\sqrt{1+4\delta_2}\right)$;

$m_6 = \frac{-1}{2}\left(1+\sqrt{1+4\delta_2}\right)$; $\alpha_1 = -\frac{ScSr\,m_1^2}{m_1^2+Scm_1-ScKr}$; $\alpha_2 = 1$; $\alpha_3 = \frac{Gr+Gm\alpha_1}{m_1^2+m_1-\delta_1}$;

$\alpha_4 = \frac{Gm(1-\alpha_1)}{m_1^2+m_1-\delta_1}$; $\alpha_5 = \frac{-Prm_1^2\ \alpha_3^2}{4(1+R^2)m_1^2\ +2Prm_1}$; $\alpha_6 = \frac{-Prm_2^2\ \alpha_4^2}{4(1+R^2)m_2^2\ +2Prm_2}$;

$\alpha_7 = \frac{-Prm_3^2\ A_1^2}{4(1+R^2)m_3^2\ +2Prm_3}$; $\alpha_8 = \frac{-2Prm_1m_2\alpha_3\alpha_4}{(1+R^2)(m_1+m_2)^2+\Pr(m_1+m_2)}$;

$\alpha_9 = \frac{2Prm_1m_3\alpha_3A_1}{(1+R^2)(m_1+m_3)^2+\Pr(m_1+m_3)}$; $\alpha_{10} = \frac{2Prm_2m_3\alpha_4A_1}{(1+R^2)(m_2+m_3)^2+\Pr(m_2+m_3)}$;

$\alpha_{11} = \frac{-ScSrm_1^2A_2}{m_1^2+Scm_1-ScKr}$; $\alpha_{12} = \frac{-4ScSr\,m_1^2\alpha_5}{4m_1^2+2Scm_1-ScKr}$; $\alpha_{13} = \frac{-4ScSr\,m_1^2\alpha_6}{4m_2^2+2Scm_2-ScKr}$;

$\alpha_{14} = \frac{-4Scm_1^2\alpha_7}{4m_3^2+2m_3-Kr}$; $\alpha_{15} = \frac{-ScSr(m_1+m_2)^2\alpha_8}{(m_1+m_2)^2+Sc(m_1+m_2)-ScKr}$;

$\alpha_{16} = \frac{-ScSr(m_1+m_3)^2\alpha_9}{(m_1+m_3)^2+(m_1+m_3)Sc-ScKr}$; $\alpha_{17} = \frac{-ScSc(m_2+m_3)^2\alpha_{10}}{(m_2+m_3)^2+(m_2+m_3)Sc-ScKr}$;

$\alpha_{18} = \frac{-(GrA_2+Gm\alpha_{11})}{m_1^2+m_1-\delta_1}$; $\alpha_{19} = \frac{-GmA_3}{m_2^2+m_2-\delta_1}$; $\alpha_{20} = \frac{-(Gr\alpha_5+Gm\alpha_{12})}{4m_1^2+2m_1-\delta_1}$;

$\alpha_{21} = \frac{-(Gr\alpha_6+Gm\alpha_{13})}{4m_2^2+2m_2-\delta_1}$; $\alpha_{22} = \frac{-(Gr\alpha_7+Gm\alpha_{14})}{4m_3^2+2m_3-\delta_1}$; $\alpha_{23} = \frac{-(Gr\alpha_8+Gm\alpha_{15})}{(m_1+m_2)^2+(m_1+m_2)-\delta_1}$;

$\alpha_{24} = \frac{-(Gr\alpha_9+Gm\alpha_{16})}{(m_1+m_3)^2+(m_1+m_3)-\delta_1}$; $\alpha_{25} = \frac{-(Gr\alpha_{10}+Gm\alpha_{17})}{(m_2+m_3)^2+(m_2+m_3)-\delta_1}$; $\alpha_{26} = \frac{-\Pr m_1}{(1+R^2)m_1^2+\Pr m_1-Pri\omega}$;

$\alpha_{27} = \frac{-Sc(m_1\alpha_1+Scm_1^2A_5)}{m_1^2+Scm_1-\delta_3Sc}$; $\alpha_{28} = \frac{-m_2Sc(1-\alpha_1)}{m_2^2+Scm_2-\delta_3Sc}$; $\alpha_{29} = \frac{-(ScSr\,m_4^2\alpha_{26})}{m_4^2+Scm_4-\delta_3Sc}$;

$\alpha_{30} = \frac{(m_1\alpha_3-GrA_5-Gm\alpha_{27})}{m_1^2+m_1-\delta_2}$; $\alpha_{31} = \frac{(m_3\alpha_4-Gm\alpha_{28})}{m_2^2+m_2-\delta_2}$; $\alpha_{32} = \frac{-m_3A_1}{m_3^2+m_3-\delta_2}$;

$\alpha_{33} = \frac{-(Gr\alpha_{26}+Gm\alpha_{29})}{m_4^2+m_4-\delta_2}$; $\alpha_{34} = \frac{-(GmA_6)}{m_5^2+m_5-\delta_2}$; $\alpha_{35} = -1$;

$\alpha_{36} = \frac{-\Pr m_1A_2}{(1+R^2)m_1^2+m_1-Pri\omega}$; $\alpha_{37} = \frac{2\Pr(m_1^2\alpha_3\alpha_{30}-m_1\alpha_5)}{4(1+R^2)m_1^2+2Prm_1-Pri\omega}$; $\alpha_{38} = \frac{2Pr(m_2^2\alpha_4\alpha_{31}-m_2\alpha_6)}{4(1+R^2)m_2^2+2Prm_2-Pri\omega}$;

$\alpha_{39} = \frac{-2Pr(m_3^2A_1\alpha_{32}+m_3\alpha_7)}{4(1+R^2)m_3^2+2Prm_3-Pri\omega}$; $\alpha_{40} = \frac{\Pr(2m_1m_2(\alpha_3\alpha_{31}+\alpha_4\alpha_{30})-(m_1+m_2)\alpha_8)}{(1+R^2)(m_1+m_2)^2+\Pr(m_1+m_2)-Pri\omega}$;

$\alpha_{41} = \frac{-\Pr(2m_1m_3(A_1\alpha_{30}+\alpha_3\alpha_{32})+(m_1+m_3)\alpha_9)}{(1+R^2)(m_1+m_3)^2+\Pr(m_1+m_3)-Pri\omega}$; $\alpha_{42} = \frac{2Prm_1m_4\alpha_3\alpha_{33}}{(1+R^2)(m_1+m_4)^2+Pr(m_1+m_4)-Pri\omega}$;

$\alpha_{43} = \frac{2Prm_1m_5\alpha_3\alpha_{34}}{(1+R^2)(m_1+m_5)^2+Pr(m_1+m_5)-Pri\omega}$; $\alpha_{44} = \frac{2Prm_1m_6A_7\alpha_3}{(1+R^2)(m_1+m_6)^2+Pr(m_1+m_6)-Pri\omega}$;

$\alpha_{45} = \frac{-\Pr(2m_2m_3(A_1\alpha_{31}-\alpha_4\alpha_{32})+(m_2+m_3)\alpha_{10})}{(1+R^2)(m_2+m_3)^2+Pr(m_2+m_3)-Pri\omega}$; $\alpha_{46} = \frac{2Prm_2m_4\alpha_4\alpha_{33}}{(1+R^2)(m_2+m_4)^2+Pr(m_2+m_4)-Pri\omega}$;

$\alpha_{47} = \frac{2Prm_2m_5\alpha_4\alpha_{34}}{(1+R^2)(m_2+m_5)^2+Pr(m_2+m_5)-Pri\omega}$; $\alpha_{48} = \frac{2Prm_2m_6A_7\alpha_4}{(1+R^2)(m_2+m_6)^2+Pr(m_2+m_6)-Pri\omega}$;

$\alpha_{49} = \frac{-2Prm_3m_4A_1\alpha_{33}}{(1+R^2)(m_3+m_4)^2+\mathrm{Pr}(m_3+m_4)-Pri\omega}$; $\alpha_{50} = \frac{-2Prm_3m_5A_1\alpha_{34}}{(1+R^2)(m_3+m_5)^2+Pr(m_3+m_5)-Pri\omega}$;

$\alpha_{51} = \frac{-2Prm_3m_6A_1A_7}{(1+R^2)(m_3+m_6)^2+Pr(m_3+m_6)-Pri\omega}$; $\alpha_{52} = \frac{-Sc(Srm_1^2\alpha_{36}+m_1\alpha_{11})}{m_1^2+Scm_1-Sc\delta_3}$;

$\alpha_{53} = \frac{-m_2A_3Sc}{m_2^2+Scm_2-Sc\delta_3}$; $\alpha_{54} = \frac{-ScSrm_4^2A_8}{m_4^2+Scm_4-Sc\delta_3}$; $\alpha_{55} = \frac{-Sc(4Srm_1^2\alpha_{37}+2m_1\alpha_{12})}{4m_1^2+2Scm_1-Sc\delta_3}$;

$\alpha_{56} = \frac{-Sc(4Srm_2^2\alpha_{38}+2m_2\alpha_{13})}{4m_2^2+2Scm_2-Sc\delta_3}$; $\alpha_{57} = \frac{-Sc(4Srm_3^2\alpha_{39}+2m_2\alpha_{14})}{4m_3^2+2Scm_3-Sc\delta_3}$;

$\alpha_{58} = \frac{-Sc(Sr(m_1+m_2)^2\alpha_{40}+(m_1+m_2)\alpha_{15})}{(m_1+m_2)^2+Sc(m_1+m_2)-Sc\delta_3}$; $\alpha_{59} = \frac{-Sc(Sr(m_1+m_3)^2\alpha_{41}+(m_1+m_3)\alpha_{16})}{(m_1+m_3)^2+Sc(m_1+m_3)-ScT_3}$;

$\alpha_{60} = \frac{-ScSr(m_1+m_4)^2\alpha_{42}}{(m_1+m_4)^2+Sc(m_1+m_4)-Sc\delta_3}$; $\alpha_{61} = \frac{-ScSr(m_1+m_5)^2\alpha_{43}}{(m_1+m_5)^2+Sc(m_1+m_5)-Sc\delta_3}$;

$\alpha_{62} = \frac{-ScSr(m_1+m_6)^2\alpha_{44}}{(m_1+m_6)^2+Sc(m_1+m_6)-Sc\delta_3}$; $\alpha_{63} = \frac{-Sc(Sr(m_2+m_3)^2\alpha_{45}+(m_2+m_3)\alpha_{17})}{(m_2+m_3)^2+Sc(m_2+m_3)-Sc\delta_3}$;

$\alpha_{64} = \frac{-ScSr(m_2+m_4)^2\alpha_{46})}{(m_2+m_4)^2+Sc(m_2+m_4)-Sc\delta_3}$; $\alpha_{65} = \frac{-ScSr(m_2+m_5)^2\alpha_{47}}{(m_2+m_5)^2+Sc(m_2+m_5)-Sc\delta_3}$;

$\alpha_{66} = \frac{-ScSr(m_2+m_6)^2\alpha_{48}}{(m_2+m_6)^2+Sc(m_2+m_6)-Sc\delta_3}$; $\alpha_{67} = \frac{-ScSr(m_3+m_4)^2\alpha_{49}}{(m_3+m_4)^2+Sc(m_3+m_4)-Sc\delta_3}$;

$\alpha_{68} = \frac{ScSr(m_3+m_5)^2\alpha_{50}}{(m_3+m_5)^2+Sc(m_3+m_5)-Sc\delta_3}$; $\alpha_{69} = \frac{ScSr(m_3+m_6)^2\alpha_{51}}{(m_3+m_6)^2+Sc(m_3+m_6)-Sc\delta_3}$;

$\alpha_{70} = \frac{-(m_1\alpha_{18}+Gr\alpha_{36}+Gm\alpha_{52})}{m_1^2+m_1-\delta_2}$; $\alpha_{71} = \frac{-(m_2\alpha_{19}+Gm\alpha_{53})}{m_2^2+m_2-\delta_2}$; $\alpha_{72} = \frac{-m_3A_4}{m_3^2+m_3-\delta_2}$;

$\alpha_{73} = \frac{-(GrA_8+Gm\alpha_{54})}{m_4^2+m_4-\delta_2}$; $\alpha_{74} = \frac{-GmA_9}{m_5^2+m_5-\delta_2}$; $\alpha_{75} = \frac{-(2m_1\alpha_{20}+Gr\alpha_{37}+Gm\alpha_{55})}{4m_1^2+2m_1-\delta_2}$;

$\alpha_{76} = \frac{-(2m_2\alpha_{21}+Gr\alpha_{38}+Gm\alpha_{56})}{4m_2^2+2m_2-\delta_2}$; $\alpha_{77} = \frac{-(2m_3\alpha_{22}+Gr\alpha_{39}+Gm\alpha_{57})}{4m_3^2+2m_3-\delta_2}$;

$\alpha_{78} = \frac{-((m_1+m_2)\alpha_{23}+Gr\alpha_{40}+Gm\alpha_{58})}{(m_1+m_2)^2+(m_1+m_2)-\delta_2}$ $\alpha_{79} = \frac{-((m_1+m_3)\alpha_{24}+Gr\alpha_{41}+Gm\alpha_{59})}{(m_1+m_3)^2+(m_1+m_3)-\delta_2}$;

$\alpha_{80} = \frac{-(Gr\alpha_{42}+Gm\alpha_{60})}{(m_1+m_4)^2+(m_1+m_4)-\delta_2}$; $\alpha_{81} = \frac{-(Gr\alpha_{43}+Gm\alpha_{61})}{(m_1+m_5)^2+(m_1+m_5)-\delta_2}$;

$\alpha_{82} = \frac{-(Gr\alpha_{44}+Gm\alpha_{62})}{(m_1+m_6)^2+(m_1+m_6)-\delta_2}$; $\alpha_{83} = \frac{-((m_2+m_3)\alpha_{25}+Gr\alpha_{45}+Gm\alpha_{63})}{(m_2+m_3)^2+(m_2+m_3)-\delta_2}$;

$\alpha_{84} = \frac{-(Gr\alpha_{46}+Gm\alpha_{64})}{(m_2+m_4)^2+(m_2+m_4)-\delta_2}$; $\alpha_{85} = \frac{-(Gr\alpha_{47}+Gm\alpha_{65})}{(m_2+m_5)^2+(m_2+m_5)-\delta_2}$; $\alpha_{86} = \frac{-(Gr\alpha_{48}+Gm\alpha_{66})}{(m_2+m_6)^2+(m_2+m_6)-\delta_2}$;

$\alpha_{87} = \frac{-(Gr\alpha_{49}+Gm\alpha_{67})}{(m_3+m_4)^2+(m_3+m_4)-\delta_2}$; $\alpha_{88} = \frac{-(Gr\alpha_{50}+Gm\alpha_{68})}{(m_3+m_5)^2+(m_3+m_5)-\delta_2}$; $\alpha_{89} = \frac{-(Gr\alpha_{51}+Gm\alpha_{69})}{(m_3+m_6)^2+(m_3+m_6)-\delta_2}$;

$\alpha_{90} = -1$; $A_1 = \alpha_3 + \alpha_4 - 1$; $A_2 = -(\alpha_5 + \alpha_6 + \alpha_7 + \alpha_8 + \alpha_9 + \alpha_{10})$;

$A_3 = -(\alpha_{11} + \alpha_{12} + \alpha_{13} + \alpha_{14} + \alpha_{15} + \alpha_{16} + \alpha_{17})$;

$A_4 = -(\alpha_{18} + \alpha_{19} + \alpha_{20} + \alpha_{21} + \alpha_{22} + \alpha_{23} + \alpha_{24} + \alpha_{25})$; $A_5 = -\alpha_{26}$;

$A_6 = -(\alpha_{27} + \alpha_{28} + \alpha_{29})$; $A_7 = -(\alpha_{30} + \alpha_{31} + \alpha_{32} + \alpha_{33} + \alpha_{34} + \alpha_{35})$;

$A_8 = -(\alpha_{36} + \alpha_{37} + \alpha_{38} + \alpha_{39} + \alpha_{40} + \alpha_{41} + \alpha_{42} + \alpha_{43} + \alpha_{44} + \alpha_{45} + \alpha_{46} + \alpha_{47} + \alpha_{48} + \alpha_{49} + \alpha_{50} +$ *α51;*

$A_9 = -(\alpha_{52} + \alpha_{53} + \alpha_{54} + \alpha_{55} + \alpha_{56} + \alpha_{57} + \alpha_{58} + \alpha_{59} + \alpha_{60} + \alpha_{61} + \alpha_{62} + \alpha_{63} + \alpha_{64} + \alpha_{65} + \alpha_{66} + \alpha_{67} + \alpha_{68} + \alpha_{69})$;

$A_{10} = -(\alpha_{70} + \alpha_{71} + \alpha_{72} + \alpha_{73} + \alpha_{74} + \alpha_{75} + \alpha_{76} + \alpha_{77} + \alpha_{78} + \alpha_{79} + \alpha_{80} + \alpha_{81} + \alpha_{82} + \alpha_{83} + \alpha_{84} + \alpha_{85} + \alpha_{86} + \alpha_{87} + \alpha_{88} + \alpha_{89} + \alpha_{90})$;