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Research Article

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Solution of the Problem of Stress-Strain State of Physically Non-Linear Hereditarily Plastic Infinite Plate with a Hole at the Action of Internal Pressure

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Abstract The stress-strain states of physically non-linear hereditarily plastic infinite plate with the hole at the action of internal pressure are determined. The known non-linear determining relations of V.V. Moskvitin are used. The problem is solved by method of successive approximations. The exact analytical solutions of problems of each approximation are found. It is introduced the arguments in favour of using convergence of considered approximations.

Keywords stress-strain state, deformation, visco-elasticity problem, creeping

Introduction

Consider the infinite plate with the circular hole of radius a, which is under the action of the uniform pressure $p(t)$ applied to the contour of the hole and it has a field of homogeneous and not depending on the time t temperature T. The material of plate is near to the mechanically incompressible and has physically nonlinear hereditarily elastic properties [1]. We also note that the known admissions, which are accepted in plate computations in limits of elasticity, remain valid in our case too.

Consequently in the considered plate the plane stress-state is realized. Use the cylindrical system of the coordinates (r,φ,z). At this we have:

$$\sigma_{zz}=\sigma_z=0;\ \sigma_{rz}=0;\ \sigma_{r\varphi}=0;\ \sigma_{\varphi z}=0;$$

$$\sigma_{rr}=\sigma_r\neq 0;\ \sigma_{\varphi\varphi}=\sigma_\varphi\neq 0;\ \varepsilon_{r\varphi}=0;\ \varepsilon_{rz}=0;$$

$$\varepsilon_{\varphi z}=0;\ \varepsilon_{rr}=\varepsilon_r\neq 0;\ \varepsilon_{\varphi\varphi}=\varepsilon_\varphi\neq 0;\ \varepsilon_{zz}=\varepsilon_z\neq 0.$$

Here σ_{ij} and $\varepsilon_{ij}(i,\ j=r,\varphi,z)$ are the components of stresses and deformations tensors respectively.

As determinative relations of physically non-linear sequential elasticity we shall use the V.V. Moskvitin's [1] known relations:

$$2G_0e_{ij}=f(\sigma_+)s_{ij}+\int_0^t\Gamma(t-\tau)f(\sigma_+)s_{ij}d\tau, \tag{1}$$

$$\theta = 3\alpha T. \tag{2}$$

Here G_0 is a momentary elasticity module; $e_{ij} = \varepsilon_{ij} - \varepsilon\delta_{ij}$ *and* ε_{ij} deformation deviator, $\varepsilon = \dfrac{\varepsilon_{ij}\delta_{ij}}{3}$ - is a mean deformation, δ_{ij} is Kronecker symbols; $\theta = 3\varepsilon$ is a fractional variation of capacity; α is a coefficient of linear expansion; $s_{ij} = \sigma_{ij} - \sigma\delta_{ij}$ deviator of the stresses of $\sigma_{ij}, \sigma = \dfrac{\sigma_{ij}\delta_{ij}}{3}$ is mean stress, $\sigma_+ = \left(\dfrac{3}{2} s_{ij}s_{ij}\right)^{\frac{1}{2}}$ is intensity of stresses; $\Gamma(t)$ is function of heredity of kernel creeping; $f(\sigma_+)$ is function of physically non-linearity.

In conformity to our problem $\varepsilon = \dfrac{1}{3}\theta = \dfrac{1}{3}(\varepsilon_r + \varepsilon_\varphi + \varepsilon_z) = \alpha T;\ \sigma = \dfrac{1}{3}(\sigma_r + \sigma_\varphi)$. For the intensity of the stress σ_+ we have

$$\sigma_+ = \left(\sigma_r^2 - \sigma_r\sigma_\varphi + \sigma_\varphi^2\right)^{\frac{1}{2}}. \tag{3}$$

Besides, $e_r = \varepsilon_r - \alpha T$; $e_\varphi = \varepsilon_\varphi - \alpha T$; $e_z = \varepsilon_z - \alpha T$; $s_r = \dfrac{1}{3}(2\sigma_r - \sigma_\varphi)$; $s_\varphi = \dfrac{1}{3}(2\sigma_\varphi - \sigma_r)$; $s_z = -\dfrac{1}{3}(\sigma_r + \sigma_\varphi)$.

At applying the last relations in (1) we shall get the following two independent equations:

$$2G_0(\varepsilon_\varphi - \varepsilon_r) = f(\sigma_+)(\sigma_\varphi - \sigma_r) + \int_0^t \Gamma(t-\tau) f(\sigma_+)(\sigma_\varphi - \sigma_r)\,d\tau, \tag{4}$$

$$6G_0(\varepsilon_\varphi + \varepsilon_r - 2\alpha T) = f(\sigma_+)(\sigma_\varphi + \sigma_r) + \int_0^t \Gamma(t-\tau) f(\sigma_+)(\sigma_\varphi + \sigma_r)\,d\tau. \tag{5}$$

For the statement of the problem we should add the relation (2) to the relations (4), (5) which we write in the form

$$\varepsilon_r + \varepsilon_\varphi + \varepsilon_z = 3\alpha T \tag{6}$$

the differential equation of the equilibrium

$$\frac{\partial\sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\varphi}{r} = 0, \tag{7}$$

compatibility condition

$$\frac{\partial\varepsilon_\varphi}{\partial r} + \frac{\varepsilon_\varphi - \varepsilon_r}{r} = 0, \tag{8}$$

and the boundary conditions

$$\sigma_r\big|_{r=a} = -p;\ \sigma_r\big|_{r\to\infty} \to 0. \tag{9}$$

We also note that the permutation u_r is connected with the deformations ε_φ and ε_r by Cauchy relations

$$\varepsilon_\varphi = \frac{u_r}{r}, \quad \varepsilon_r = \frac{du_r}{dr}. \tag{10}$$

We shall solve the problem (4)-(9) by the successive approximations method worked out in [2]. Following [2] represent the function $f(\sigma_+)$ in the form $f(\sigma_+) = 1 + f^0(\sigma_+)$. At initial approximation we accept $f^0(\sigma_+) = 0$. In this case the relations (4) and (6) take the form

$$2G_0\left(\varepsilon_\varphi^{(1)} - \varepsilon_r^{(1)}\right) = \left(\sigma_\varphi^{(1)} - \sigma_r^{(1)}\right) + \int_0^t \Gamma(t-\tau)\left(\sigma_\varphi^{(1)} - \sigma_r^{(1)}\right)d\tau, \tag{11}$$

$$6G_0\left(\varepsilon_\varphi^{(1)} + \varepsilon_r^{(1)} - 2\alpha T\right) = \sigma_\varphi^{(1)} + \sigma_r^{(1)} + \int_0^t \Gamma(t-\tau)\left(\sigma_\varphi^{(1)} + \sigma_r^{(1)}\right)d\tau. \tag{12}$$

To (11) and (12) add (5)-(9) with corresponding upper indices. The obtained problem is of physically nonlinear visco-elasticity problem. In case of linear elasticity the stress components σ_φ and σ_r are represented in the form:

$$\sigma_\varphi^{(1)} = p\left(\frac{a}{r}\right)^2, \quad \sigma_r^{(1)} = -p\left(\frac{a}{r}\right)^2. \tag{13}$$

As we see the components of material are not contained in the solution (13) are therefore the formulae (13) are also solutions of the problem (11), (12), (6-9). At this $\sigma_\varphi^{(1)} + \sigma_r^{(1)} = 0$ and from (12) follows that

$$\varepsilon_\varphi^{(1)} + \varepsilon_r^{(1)} = 2\alpha T. \tag{14}$$

Allowing for (14) from the conditions (6) we find $\varepsilon_z^{(1)} = \alpha T$. *At using (13) from (11) we determine*

$$\varepsilon_\varphi^{(1)} - \varepsilon_r^{(1)} = \frac{1}{G_0}\left(\frac{a}{r}\right)^2 p^*, \tag{15}$$

where $p^* = p(t) + \int_0^t \Gamma(t-\tau)p(\tau)d\tau$.

The deformation components $\varepsilon_\varphi^{(1)}$ *and* $\varepsilon_r^{(1)}$ *we find with using the relations (14), (15):*

$$\varepsilon_\varphi^{(1)} = \alpha T + \frac{1}{2G_0}\left(\frac{a}{r}\right)^2 p^*; \quad \varepsilon_r^{(1)} = \alpha T - \frac{1}{2G_0}\left(\frac{a}{r}\right)^2 p^*. \tag{16}$$

Consequently by solving the problem on initial approximation all unknown values are found which as it is easy to be convinced satisfy all necessary relations. Now following the formula (3) calculate the value

$\sigma_+^{(1)} : \sigma_+^{(1)} = \sqrt{3}p\left(\frac{a}{r}\right)^2$. Let $f^0(\sigma_+)$ be entire function $f^0(\sigma_+) = A(\sigma_+)^\gamma$.

Then $f^{0}\left(\sigma_{+}^{(1)}\right)=A3^{\frac{\gamma}{2}}\left(\frac{a}{r}\right)^{2\gamma}p^{\gamma}$, $f^{0}\left(\sigma_{+}^{(1)}\right)\left(\sigma_{\varphi}^{(1)}-\sigma_{r}^{(1)}\right)=2A3^{\frac{\gamma}{2}}\left(\frac{a}{r}\right)^{2(\gamma+1)}p^{\gamma+1}$;

$f^{0}\left(\sigma_{+}^{(1)}\right)\left(\sigma_{\varphi}^{(1)}-\sigma_{r}^{(1)}\right)=0$.

Here A and γ are the known material constants determined from the experiments.

At the next approximation the relations (4) and (5) will have the form

$$2G_0\left[\varepsilon_{\varphi}^{(2)}-\varepsilon_{r}^{(2)}-\frac{A}{G_0}3^{\frac{\gamma}{2}}\left(\frac{a}{r}\right)^{2(\gamma+1)}\left(p^{\gamma-1}\right)^{*}\right]=\sigma_{\varphi}^{(2)}-\sigma_{r}^{(2)}+$$

$$+\int_0^t\Gamma(t-\tau)\left(\sigma_{\varphi}^{(2)}-\sigma_{r}^{(2)}\right)d\tau, \tag{17}$$

$$2G_0\left(\varepsilon_{\varphi}^{(2)}+\varepsilon_{r}^{(2)}-2\alpha T\right)=\sigma_{\varphi}^{(2)}-\sigma_{r}^{(2)}+$$

$$+\int_0^t\Gamma(t-\tau)\left(\sigma_{\varphi}^{(2)}-\sigma_{r}^{(2)}\right)d\tau. \tag{18}$$

The problem statement of the considering approximation will be closed with addition to (17) and (18) the relations (6)-(9). Introduce the following notations:

$$\sigma_{r}^{(2)}=\bar{\sigma}_{r}^{(2)}+A3^{\frac{\gamma}{2}}\left(\frac{a}{r}\right)^{2(\gamma+1)}p^{1+\gamma}(t),$$

$$\sigma_{\varphi}^{(2)}=\bar{\sigma}_{\varphi}^{(2)}-A3^{\frac{\gamma}{2}}\left(\frac{a}{r}\right)^{2(\gamma+1)}p^{1+\gamma}(t), \tag{19}$$

$$\varepsilon_{r}^{(2)}=\bar{\varepsilon}_{r}^{(2)},\ \varepsilon_{\varphi}^{(2)}=\bar{\varepsilon}_{\varphi}^{(2)},\ \varepsilon_{z}^{(2)}=\bar{\varepsilon}_{z}^{(2)}.$$

With using the notations (19). the relations (17), (18), (6)-(9) turn into the relations:

$$2G_0\left(\bar{\varepsilon}_{\varphi}^{(2)}-\bar{\varepsilon}_{r}^{(2)}\right)=\bar{\sigma}_{\varphi}^{(2)}-\bar{\sigma}_{r}^{(2)}+\int_0^t\Gamma(t-\tau)\left(\bar{\sigma}_{\varphi}^{(2)}-\sigma_{r}^{(2)}\right)d\tau, \tag{20}$$

$$6G_0\left(\bar{\varepsilon}_{\varphi}^{(2)}+\bar{\varepsilon}_{r}^{(2)}-2\alpha T\right)=\bar{\sigma}_{\varphi}^{(2)}+\bar{\sigma}_{r}^{(2)}+\int_0^t\Gamma(t-\tau)\left(\bar{\sigma}_{\varphi}^{(2)}+\bar{\sigma}_{r}^{(2)}\right)d\tau, \tag{21}$$

$$\bar{\varepsilon}_{r}^{(2)}+\bar{\varepsilon}_{\varphi}^{(2)}+\bar{\varepsilon}_{z}^{(2)}=3\alpha T, \tag{22}$$

$$\frac{d\bar{\sigma}_{r}^{(2)}}{dr}+\frac{\bar{\sigma}_{r}^{(2)}-\bar{\sigma}_{\varphi}^{(2)}}{r}-2\gamma A3^{\frac{\gamma}{2}}a^{2(\gamma+1)}r^{-2\gamma-3}p^{1+\gamma}(t)=0, \tag{23}$$

$$\frac{\partial\bar{\varepsilon}_{\varphi}^{(2)}}{\partial r}+\frac{\bar{\varepsilon}_{\varphi}^{(2)}-\bar{\varepsilon}_{z}^{(2)}}{r}=0, \tag{24}$$

$$\bar{\sigma}_{r}^{(2)}\Big|_{r=a}=-p(t)-A3^{\frac{\gamma}{2}}p^{1+\gamma}(t);\ \bar{\sigma}_{r}^{(2)}\Big|_{r\to\infty}=0. \tag{25}$$

The problem (20)-(25) is of linear visco-elasticity problem where enter the volume and surface force determined by the characteristics of physical nonlinearity geometrical characteristics and the given pressure. Solving this problem write the analytical expression for the initial values:

$$\bar{\sigma}_r^{(2)} = -\left(\frac{a}{r}\right)^2 p(t) - \frac{3^{\frac{\gamma}{2}} A}{4(1+\gamma)}\left(\frac{a}{r}\right)^2 \left[3 + (1+4\gamma)\left(\frac{a}{r}\right)^{2\gamma}\right] p^{1+\gamma}(t), \tag{26}$$

$$\bar{\sigma}_\varphi^{(2)} = -\left(\frac{a}{r}\right)^2 p(t) + \frac{3^{\frac{\gamma}{2}} A}{4(1+\gamma)}\left(\frac{a}{r}\right)^2 \left[3 + (1-2\gamma)\left(\frac{a}{r}\right)^{2\gamma}\right] p^{1+\gamma}(t), \tag{27}$$

$$\bar{\varepsilon}_r^{(2)} = \alpha T - \frac{1}{2G_0}\left(\frac{a}{r}\right)^2 \left[p(t) + \int_0^t \Gamma(t-\tau) p(\tau) d\tau\right] -$$

$$- \frac{1}{2G_0}\left(\frac{a}{r}\right)^2 \frac{3^{\frac{\gamma}{2}} A}{4(1+\gamma)} \left[3 - 2\gamma\left(\frac{a}{r}\right)^{2\gamma}\right]\left[p^{1+\gamma}(t) + \int_0^t \Gamma(t-\tau) p^{1+\gamma}(\tau) d\tau\right], \tag{28}$$

$$\bar{\varepsilon}_\varphi^{(2)} = \alpha T + \frac{1}{2G_0}\left(\frac{a}{r}\right)^2 \left[p(t) + \int_0^t \Gamma(t-\tau) p(\tau) d\tau\right] +$$

$$+ \frac{1}{2G_0}\left(\frac{a}{r}\right)^2 \frac{3^{\frac{\gamma}{2}} A}{4(1+\gamma)} \left[3 + \left(\frac{a}{r}\right)^{2\gamma}\right]\left[p^{1+\gamma}(t) + \int_0^t \Gamma(t-\tau) p^{1-\gamma}(\tau) d\tau\right], \tag{29}$$

$$\bar{\varepsilon}_z^{(2)} = \alpha T - \frac{A 3^{\frac{\gamma}{2}}(1+2\gamma)}{8G_0(1+\gamma)}\left(\frac{a}{r}\right)^{2(1+\gamma)} \left[p^{1+\gamma}(t) + \int_0^t \Gamma(t-\tau) p^{1+\gamma}(\tau) d\tau\right]. \tag{30}$$

By immediate substitution we are convinced the solution (26)-(30) is the exact solution of the'problem (20)-(25) in case of arbitrary kernel of the creeping $\Gamma(t)$.

According to the notations (19) the deformation components $\varepsilon_r^{(2)}$, $\varepsilon_\varphi^{(2)}$ and $\varepsilon_z^{(2)}$ will be determined by the formulae (28)-(30) respectively the stress components of $\sigma_r^{(2)}$ and $\sigma_\varphi^{(2)}$ with using (26) and (27) will be written in the form

$$\sigma_r^{(2)} = -\left(\frac{a}{r}\right)^2 p(t) - \frac{A 3^{1+\frac{\gamma}{2}}\left(\frac{a}{r}\right)^2}{4(1+\gamma)}\left[1 - \left(\frac{a}{r}\right)^{2\gamma}\right] p^{1+\gamma}(t), \tag{31}$$

$$\sigma_\varphi^{(2)} = \left(\frac{a}{r}\right)^2 p(t) + \frac{A 3^{1+\frac{\gamma}{2}}\left(\frac{a}{r}\right)^2}{4(1+\gamma)}\left[1 - (1+2\gamma)\left(\frac{a}{r}\right)^{2\gamma}\right] p^{1+\gamma}(t). \tag{32}$$

Hence the solution of the problem of considering approximation became known in analytic form. The problems of the following approximation are constructed by the analogous way. If to the relations (20)-(25) apply the

Laplace transformation then the process of approximations in image will be analogous to the process of approximation in the method of elastic solutions A.A. Ilyushin in theory of elastic-plastic deformations [3], whose proof of convergence is known [4], [5]. Therefore at close to the real boundedness on the function $f^0(\sigma_+)$, we can talk about convergence applied in our problem of approximations. Following from this let be restricted considered in the last case of approximation and for the solution of initial problem (4)-(9) accept approximately:

$$\sigma_r \approx \sigma_r^{(2)},\ \sigma_\varphi \approx \sigma_\varphi^{(2)},\ \varepsilon_r \approx \varepsilon_r^{(2)},\quad \varepsilon_\varphi \approx \varepsilon_\varphi^{(2)},\ \varepsilon_z \approx \varepsilon_z^{(2)},$$

where $\sigma_r^{(2)},\ \sigma_\varphi^{(2)},\ \varepsilon_r^{(2)},\quad \varepsilon_\varphi^{(2)},\ \varepsilon_z^{(2)}$ axe determined by the analogous expressions (31), (32), (28), (29), (30), respectively.

Using the first relation (10) and (29) we can determine the unique component of the vector permutation u_r :

$$u_r = r\varepsilon_\varphi \approx r\varepsilon_\varphi^{(2)}.$$

In conclusion note that if in case of elastic or physically linear hereditarily elastic material of plate every point the state of clear shift is realized then as show the solution in case of physically non-linear hereditary elastic material such state of plate does not have.

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