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Research Article

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Encircled Energy for Array of Circular Synthetic Apertures

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Abstract In this research, an equation was derived for the encircled energy of point spread function for optical system of circular synthetic apertures. Also, to employ the above equation to determine the value of encircled energy for a set of circular synthetic apertures (N=1,4,8,12) .This study was done for the case of diffraction limited system using Math CAD program, the results show that there is a decrease in the values of encircled energy with increasing the apertures number.

Keywords Encircled Energy E(R), Point Spread Function (PSF), Circular Synthetic Apertures, Fourier Optics

1. Introduction

Artificial Aperture methods are commonly used to obtain high resolution from the purpose low quality resolution sensors. Also, they have been used in modern sonar and radar schemes, being selected by Synthetic Aperture Sonar (SAS) and by Synthetic Aperture Radar (SAR) systems, this type of schemes are currently employed in municipal and armed applications [1]. The aforementioned Aperture can be explained as a structure which employed to split optic systems of large specific aperture function, this is known called "Mosaic" or "Segmented Mirror". Synthetic aperture is an imaging system for independent optical systems which are including the entire image field [2]. The most important corollary of the "Point Spread Function" is the "encircled energy" [3]. The "encircled energy" function measures the element of the total energy in the "point spread function" which within the radius R. [4]. The importance of the consequence of the "point spread function" is the encircled energy that measures part of total energy in point spread function which lies inside a specified radius in the plane of observation. It can be considered as one of the significant parameters that can serve as an index of the of the performance of an optical system [5-6]. The "encircled energy" in the "point spread function "inside a circle of stated radius centered on the Gaussian image point may be intended methodically from an orthogonal series symbol for the "point spread function" [7]. Measurement of attentiveness of energy in an optical image is represented by the term "encircled energy". The image will be small when a single star is carried to its sharpest focus through the used lens, and this picture is named "point spread function "or" PSF", the "encircled energy" is summation of distribution of the energy for the PSF [8]. Encircled energy can be defined as the Intensity energy enclosed under the envelope of PSF as a function of off-axis angle [9].

2. Material and Methods

Although the point spread function is one of the important means in evaluating the performance of optical systems, but loses its importance and characteristics when increasing the aberrations and then cannot be relied upon in determining the efficiency of the system. Therefore, another criterion based on the point spread function is used, but it loses its general shape when increasing the aberrations. This standard is the enclosed energy which is defined as part of the total energy of the system image.

For the purpose of finding a relationship to the enclosed energy function we perform the numerical integration of the point spread function equation for a row of circular synthetic apertures.

$$E(z,m) = n.f \int_{-m}^{m} \int_{-z}^{z} G(\bar{z},\bar{m})d\bar{z}d\bar{m}$$

$E(z,m)$: Encircled energy to circular synthetic apertures.

$$PSF = G(\bar{z},\bar{m}) = |F(\bar{z},m)|^2$$

$$E(z,m) = n.f \int_{-m}^{m} \int_{-z}^{z} \int_{y}\int_{y_1}\int_{x}\int_{x_1} f((x,y)f^*(x_1,y_1)$$

$$e^{i(zx+my)}e^{-i(zx+my)}dx1dxdy1dyd\bar{z}d\bar{m}$$

Where

$$f^*(x_1,y_1) = \tau(x_1,y_1)e^{-ikw(x_1y_1)}$$

$$E(z,m) = n.f$$

$$\int_{y}\int_{y_1}\int_{x}\int_{x_1} f(x,y)f^*(x_1,y_1)dx_1dxdy_1dy \int_{-z}^{z} e^{i\bar{z}(x-x1)}d\,\bar{z}$$

$$\int_{-m}^{m} e^{i\bar{m}(y-y1)}d\bar{m}$$

By Using

$$\sin\emptyset = \frac{e^{i\emptyset} - e^{-i\emptyset}}{2i}$$

And

$$\int_{-z}^{z} e^{i\bar{z}(x-x_1)}\,d\bar{z} = 2\frac{\sin z(x-x_1)}{(x-x_1)}$$

$$\int_{-m}^{m} e^{i\bar{m}(y-y_1)}\,d\bar{m} = 2\frac{\sin m(y-y_1}{(y-y_1)}$$

Getting

$$E(z,m) = n.f \int_{y}\int_{y_1}\int_{x}\int_{x_1} f(x.y)f^*(x_1,y_1)$$

$$\frac{\sin\ z(x-x_1)}{(x-x_1)}\frac{\sin\ m(y-y_1)}{(y-y_1)}dx_1dxdy_1dy$$

By deriving the equation for the calculation of the Normalizing factor, we obtain

$$n.f = \frac{1}{\pi 3}$$

By substituting the value of normalizing factor, we obtain

$$E(R) = \frac{1}{\pi 3}\left[\int_{\frac{-1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}}\int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}}\int_{-\sqrt{\frac{1}{N}-Y^2}}^{\sqrt{\frac{1}{N}-Y^2}}\int_{-\sqrt{\frac{1}{N}-Y_1^2}}^{\sqrt{\frac{1}{N}-Y_1^2}} f(x,y)f^*(x_1,y_1)\frac{\sin(R(x-x_1))}{(x-x_1)}\frac{\sin(R(y-y_1))}{(y-y_1)}dx_1dxdy_1dy\right]$$

For simplification the complex calculations,

Equation of encircled energy for array of circular synthetic apertures,

Give as follows.

$$E(R) = \frac{1}{\pi 3}\left[\int_{\frac{-1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}}\int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}}\int_{-\sqrt{\frac{1}{N}-Y^2}}^{\sqrt{\frac{1}{N}-Y^2}}\int_{-\sqrt{\frac{1}{N}-Y_1^2}}^{\sqrt{\frac{1}{N}-Y_1^2}} [\cos(2\pi w_{20}[(x^2+y^2)-(x_1^2+y_1^2)]]\frac{\sin[R(x-x_1)]}{(x-x_1)}\frac{\sin[R(y-y_1)]}{(y-y_1)}dx_1dxdy_1dy\right]$$

Results and Discussion

The purpose of using the obscured apertures is to make images better and reduce the effect of aberration. That also increases the dealing with power of the optic system; one the other side of the or may be hand, obscured aperture can be used in expression telescopes. In 2011, the "encircled energy" was studied as an image quality assessment parameter [5].While in 2013, studying excluded energy and relative "encircled energy" that play an important role in the optical systems as image quality assessment parameters [10]. In 2013, studying "encircled energy" in the "point spread function" of amplitude of the optical system [11]. In 2001, calculate the "encircled energy" in the Image of Point Object [12]. Encircled Energy function of the optical system is a fundamental criterion for determining the efficiency of the optical systems and the quality of the images created by these systems. Figures (1, 2, 3,and 4) represent curves of encircled energy in the point spread function for diffraction limited system for individual circular aperture (N=1) and when number of apertures (N= 4, 8 and 12). Math CAD programs were used to simulate the equation of the encircled energy, we noticing the figures above decrement of encircled energy with increment number of synthetic apertures and increasing the distance. Note that the encircled energy function is flow function compared to point spread function is oscillatory function.

The encircled energy curves decay to values (R) which is very few values and approach from one for values (R) which reaches (∞). The slope of the enclosed energy curves of the system includes circular apertures higher than the system including synthetic circular apertures, because the synthetic circular apertures are dispersing a quantity of light, which affects the efficiency of the image. Low value of encircled energy containing a number of synthetic circular apertures is evidence for that the central spot in the central loop group contains a small part of the total energy and therefore the entire set of central loops is an effective image element.

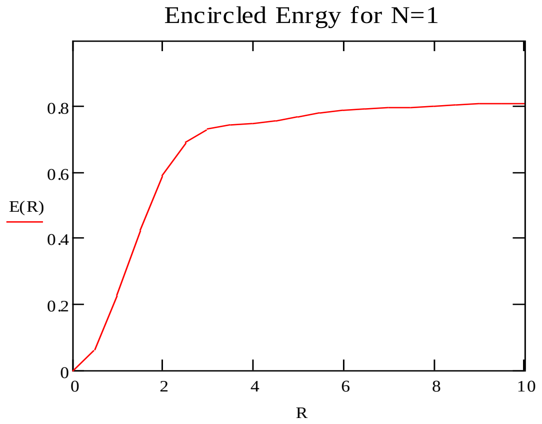

Figure 1: Encircled energy for one aperture of diffraction limited system.

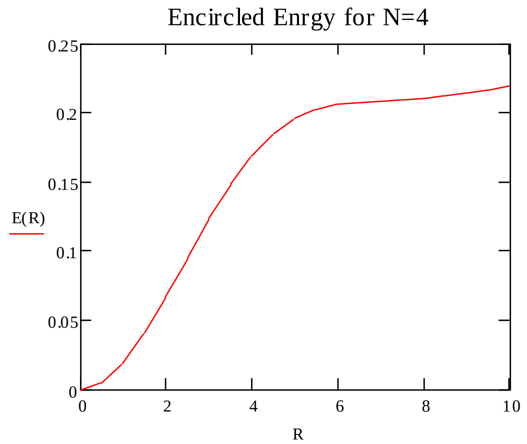

Figure 2: Encircled energy for (N=4) apertures of diffraction limited system

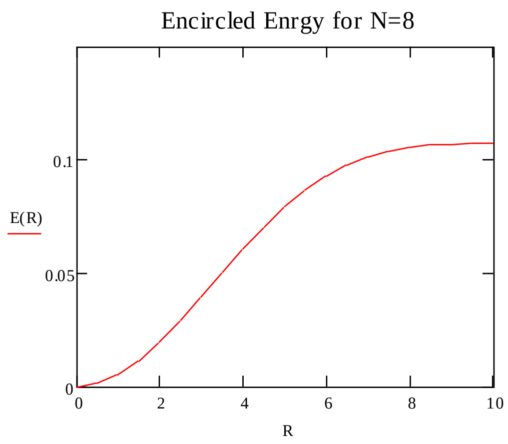

Figure 3: Encircled energy for (N=8) apertures of diffraction limited system

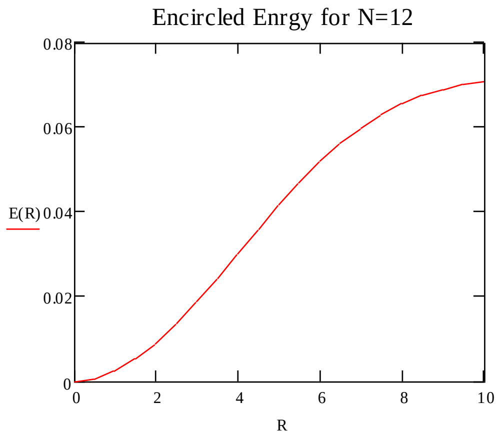

Figure 4: Encircled energy for (N=12) apertures of diffraction limited system

Conclusion

Calculation of the values of encircled energy by using Math CAD program and array of circular synthetic apertures in this research gives more accurate results, comparison with theoretical values. We notice decreasing the value of encircled energy E(R) with increasing the number of synthetic apertures (N), while increment encircled energy E(R) with increment the distance (R).

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