



A new Quarterwave Phase Retarder with Novel Characteristics

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Abstract Design of a two-reflection, non-deviating beam total internal reflection quarterwave phase retarder is presented. The device treats the two main drawbacks of the Fresnel rhomb which are the lateral shift of the emergent beam and sensitivity of the retardance value to errors in the external angle of incidence. In addition, it is of compact size and the path length of the beam inside the device is 41.37 mm. The device is designed to introduce quarterwave retardance at 632.8 nm.

Keywords Phase retarder; Quarterwave retardance; Sensitivity to input angle

1. Introduction

The Fresnel rhomb (FR) makes use of the phenomenon of phase retardance introduced by total internal reflection (TIR). The input beam falls normal to the entrance face (the external angle of incidence $i = 0^\circ$) and suffers two equal internal reflections with the resulting retardances add together to provide $\lambda/4$ phase shift at the working wavelength, Fig.1. Due to each reflection, a phase shift δ is introduced such that [1]

$$\tan(\delta/2) = \cos \theta (n^2 \sin^2 \theta - 1)^{1/2} / n \sin^2 \theta \quad (1)$$

where θ is the reflection angle and n is the refractive index of the rhomb material. Obviously, δ varies with n (or equivalently with the wavelength λ).

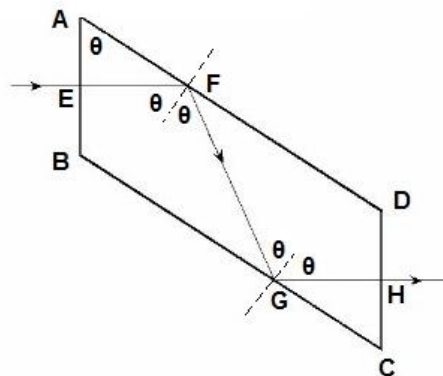


Figure 1: Quarterwave Fresnel rhomb made of glass with refractive index $n = 1.5505$.

If the incident beam is plane polarized at azimuth $\pm 45^\circ$, the emergent beam will be circularly polarized. For other azimuths, it will be elliptically polarized but with the same retardance value $\lambda/4$. The ratio between long to short sides of the rhomb is $2 \sin \theta \tan \theta$ where θ is each of the two the reflection angle (equal to the acute angle of the rhomb). The class of TIR retarders is generally referred to as achromatic retarders. Actually, the retardance varies with wavelength, but the variation is very weak as compared to birefringent phase plates. Typically, the retardance of a FR varies by about 2.5° throughout the visible spectrum. The FR has two main disadvantages. The incident and emergent beams are not collinear and the device is sensitive to errors in the external angle of incidence i . The first problem was treated in the two devices AD-1 and AD-2 (achromatic



devices 1 and 2) presented by Oxley [2]. The first device is also weakly sensitive to errors in the external incidence angle, but is of exceedingly large size and is difficult to align in optical systems while the second is of large size and is highly sensitive to errors in the external angle of incidence.

In this work, a new design for a TIR retarder is presented that treats the drawbacks of the FR. In our design, the incidence beam still falls normal to the entrance face which means that the reflection angles are of fixed values regardless of the value of n (or λ). But, the entrance and exit faces are not parallel. Also, the two reflection angles are not equal as in FR to reduce the sensitivity to errors in the external angle of incidence as will be described below. Finally, the incident and emergent beams are collinear.

2. Phenomenological treatment

Figure 2 shows the dispersion of the retardance δ with internal reflection angles θ for $n = 1.55$. The peak retardance δ' of the curve is [3]

$$\tan(\delta'/2) = (n^2 - 1) / 2n(2)$$

which corresponds to a reflection angle θ' given as

$$\sin \theta' = [2 / (n^2 + 1)]^{1/2}(3)$$

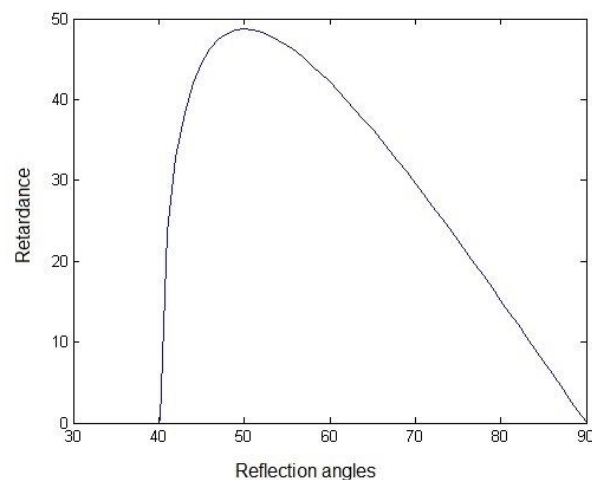


Figure 2: Dispersion relation of the retardance with reflection angles for $n = 1.55$.

Rhomb retarders usually make use of reflection angles θ on the descending side of the $(\theta - \delta)$ relation to avoid the strong dispersion of δ values on the ascending side. This reduces errors in retardance values for non-correct external angle of incidence. We selected our two reflection angles close to the peak of the $\theta - \delta$ relation such that they fall on different sides of the peak. This has the advantage that for errors in the incident angle i , both of the two reflection angles will either increase or decrease. As the two angles lie on different sides of the peak, one of them will increase while the other decreases. Accordingly, one of the two introduced retardances will increase while the other will decrease. This compensating effect reduces the error in the retardance value due to incorrect external angle of incidence provided that the reflection angles are properly selected.

To provide collinearity between the incident and emergent beams, the device is constructed from two pieces cemented together. The exit face is not parallel to the entrance face to allow for the required collinearity. This has no effect on the retardance value, but in order to obtain circularly polarized light, the azimuth of the incident plane polarized beam must be adjusted in accordance with Fresnel's transmission formulas [4].

3. The device

As mentioned above, the suggested device consists of two pieces cemented together. The first piece is part of a triangular prism ABCD with the short side AB representing the entrance face. The second piece is a parallelepiped DEFG. Dimensions and angles are calculated to provide $\lambda/4$ retardance at 632.8 nm, collinearity of incident and emergent beams, maximum use of the entrance face and weak sensitivity to errors in i . The device is designed using the data of optical glass SCHOTT-PSK with refractive index 1.5505 at the wavelength $\lambda = 632.8$ nm [5]. Modifications of the device parameters could be made for work at other wavelengths or using



other optical glasses. Figure 3 shows the suggested device with angles and dimensions for an entrance aperture of 10 mm presented in Table 1.

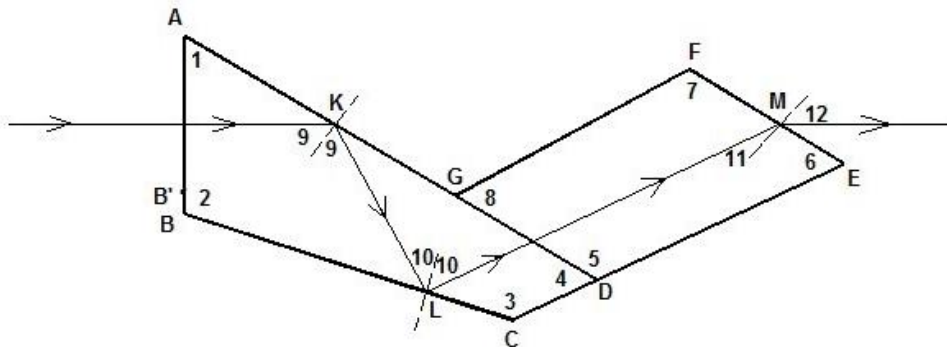


Figure 3: Design of the proposed device

Table 1: Angles and lengths of the device

Angle	degree	length	mm
1	59.4	AB	10
2	108	BC	19.504
3	136.8	CD	5.262
4	55.8	DA	27.081
5	124.2	DE	15.458
6	56.6	EF	10.221
7	123.4	FG	15.631
8	56.8	GD	10.316
9	59.4	HK	8.455
10	46.8	KL	10.763
11	33.4	LM	22.154
12	58.597	AB'	8.533

He-Ne Laser beam ($\lambda = 632.8$ nm) falling normal to the entrance face will undergo two internal reflections at angles $\theta_1 = 59.4^\circ$ and $\theta_2 = 46.8^\circ$. After reflection, the beam crosses the second piece and emerges from the device. The inclination of the exit face allows the emergent beam to be collinear with the incident beam provided that the incident beam falls at the center of the entrance face. The retardances introduced by reflections at θ_1 and θ_2 are $\delta_1 = 42.77^\circ$ and $\delta_2 = 47.28^\circ$ which add to give a retardance $\Delta = 90.05^\circ$. An error in the external angle of incidence i (ideally $i = 0^\circ$) will cause both of the two reflection angles to increase or decrease by the same amount. Thus, for an error of 1.5° in i , the reflection angles are changed by about $\pm 1^\circ$ and the device retardance will be 89.80° in both cases. For a FR made of the same glass, the two reflection angles θ_1 and θ_2 are both equal and each is 57.10° . A similar change of $\pm 1^\circ$ in the reflection angles will result retardances of 88.14° or 91.70° . Considering the total variation in Δ -value, the sensitivity of the retardance to an error of 1.5° in the external angle of incidence in our device is then eighteen times less than that for FR. Table 2 summarizes the characteristics of the presented device.

Table 2: Characteristics of the device

Characteristic	Device
Material	SCHOTT-PSK3
Refractive index	1.5505
Working wavelength	632.8 nm
Incident and emergent beams	Collinear
Sensitivity to errors in incident angle	Weakly sensitive
Reflection angles	59.4° and 46.8°
Introduced retardance	90.05°
Entrance face length	10 mm



Useful entrance aperture	8.533 mm
Exit aperture	5.326 mm
Achromaticity	Less achromatic than FR

The compensating effect resulting from selecting the two reflection angles on different sides of the $\theta - \delta$ relation makes the device suitable for use at a single wavelength. In fact, for use at other wavelengths, the device is less achromatic than FR because of the strong dispersion of the retardance on the ascending side of the $\theta - \delta$ relation. Note also that as a result of the inclination of the exit face, a focusing effect is introduced such that the length of the emergent beam is smaller than that of the incident beam. For an input aperture of 10 mm, the output aperture is 0.62 mm. This focusing effect makes a beam incident above the center of the entrance face emerges at a lower level and vice versa. Note that the useful aperture of the device is AB' and is equal to 8.533 mm (Fig.3). The parameters of the device were adjusted to allow for a beam incident at the center of the entrance face AB to exit concentric with the incident one. The path length of a beam falling at the center of the entrance face inside the device is 41.37 mm as compared to 30.93 mm for FR of equal aperture and made of the same glass.

4. Discussion

We have presented a design for a TIR phase retarder that treats the two main drawbacks of the FR, namely, the lateral shift of the emergent beam and sensitivity to errors in the external angle of incidence. The incident beam must fall at the center of the entrance face AB for exact collinearity of the incident and emergent beams. The retarder is designed for use at a single wavelength (632.8 nm) and could be constructed using different optical glasses to provide $\lambda/4$ retardance at the same (or other) wavelengths.

References

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