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Some results on the Nadir's operator $N = AB^* - BA^*$

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Abstract In this paper, we present some new results for the operator $N = AB^* - BA^*$ and study the invertibility of $I - N$ in the algebra $B(H)$ of all bounded linear operators on a complex separable Hilbert space H.

Keywords Hilbert space, Nadir's operator

1. Introduction

Nadir's operator $N = AB^* - BA^*$ of two operators acting on a Hilbert spaceis a central concept in quantum mechanics, since it quantifies how well the two observables described by these operators can be measured simultaneously. From this definition, we can see that if the operator N is zero, then $AB^* = BA^*$ represents in a sense the degree to which (AB^*) is equal to its adjoint $(AB^*)^*$ so, the order of which the two corresponding measurements are applied to the physical system does not matter. On the contrary, if it is non-zero, then the order does matter also we reveal that the identity of any such algebrais never a Nadir's operator. This translates in the Banach algebra $B(H)$ with unit element that the operator $I + K$ where K is a compact operatorisnon Nadir's operator.

2. Main Results

Theorem 1

Let $B(H)$ be a Banach algebra with unit element I, then for all operators A and B in $B(H)$. The operator $N = AB^* - BA^*$ is a Skew self-adjoint operator and never equal to the identity I In other words

$$N = AB^* - BA^* \neq I$$

Proof Indeed, we have

$$\begin{aligned} N^* &= (AB^* - BA^*)^* \\ &= (AB^*)^* - (BA^*)^* \\ &= BA^* - AB^* \\ &= -(AB^* - BA^*) \\ &= -N. \end{aligned}$$

Assume that, $N = AB^* - BA^* = I$, it follows $N^* = (AB^* - BA^*)^* = I^* = I$. Hence, from the relation $N^* = -N$, we get
$I = -I$.

Contradiction

Proposition

The operator $N = AB^* - BA^*$ is normal

Indeed, it follows from the theorem 1

$$NN^* = \left(AB^* - BA^*\right)\left(AB^* - BA^*\right)^*$$
$$= N(-N)$$
$$= (-N)N$$
$$= N^*N$$

Corollary

The operator N^2 is negative, that is to say $\left\langle N^2x, x\right\rangle \leq 0$ for all non-zero vectors x in H.

Indeed, it is known that the operator NN^* is always positive and from the proposition 1 $NN^* = (-N)N = -N^2$

Theorem 2

Let A, B be a bounded operator on the Hilbert space H where one of the operators A and B is compact, then the operator $N = AB^* - BA^*$ is compact with the operator $I - N$ is invertible.

- **First case** A is compact B bounded

A compact $\Rightarrow A^*$ compact $\Rightarrow AB^*$ and BA^* are compacts. Hence N is compact.

- **Second case** B is compact A bounded

B compact $\Rightarrow B^*$ compact $\Rightarrow AB^*$ and BA^* are compacts. Hence N is compact.

Besides, it is known that, the operator N is never equal to the identity I, then $N - I \neq 0$ and so, $N - I$ is injective. Hence $N - I$ is injective.

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