



An Algorithm for Finding Critical Path, Activities and Total Duration of Multiple Projects (Multiple Sources to Multiple Destinations)

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Abstract In this work, we examine the critical path of multiple sources to multiple destinations project. A systematic approach was adopted to derive the critical path from given multiple sources to multiple destinations. In doing this, the algorithm of the well-known Bellman functional relation for Dynamic programming problem was modified to enable the relation solve multiple sources to multiple destinations project problems directly without splitting such network problems into either: single source to single destination, multiple sources to single destination or single source to multiple destinations as has been the case with the existing algorithms. The proposed algorithm in this paper reduces the mathematical computations involved in the solution process and obtains the optimal shortest path like other algorithm. The proposed algorithm was used to solve the PW Nig Ltd construction multiple projects of roads and obtained the critical path. The result was not compromised.

Keywords multiple sources, multiple destinations, project and Modified dynamic algorithm

Introduction

Network is a graphical representation of arrows and nodes for showing the logical sequence of various activities to be performed to achieve project objectives. Its applications cannot be enumerated due to its wide use and demand in several sphere of life, particularly in engineering telecommunication computer sciences, transportation, project management and others.

As companies are growing larger and their sales are becoming nationally and internationally recognized, sometimes they are awarded with more than one contract, and trying to analyze the critical activities of several projects (contracts) individually, takes a lot of time, therefore we proposed an algorithm in this paper to handle multiple projects at once without splitting them into single project like the existing algorithms does. The essence for this is to reduce the time taken by network analyze in solving such problems.

Planning of a project involves several activities some of which must be completed before others can begin. The duration of each activity is known in advance. The contractor will want to find the time required to complete the whole project as well as the critical activities (those that if slightly delayed will result in a corresponding delay of completion of the overall project).

Problem Formation

According to Bertsekas (1998) [1], the problem of this kind can be represented by a graph (network) where nodes represents completion of some phases of the project. An arc (i,j) represents an activity that starts once phase i is completed and has known duration $t_{ij} > 0$. A phase (node) j is completed when all activities or arcs (i,j) that are incoming to j are completed. Two special nodes 1 and N represent the start and end of the project respectively. Node 1 has no incoming arcs while node N has no outgoing arc. Furthermore, there is at least one path from node 1 to every other node as show in fig 1 below:



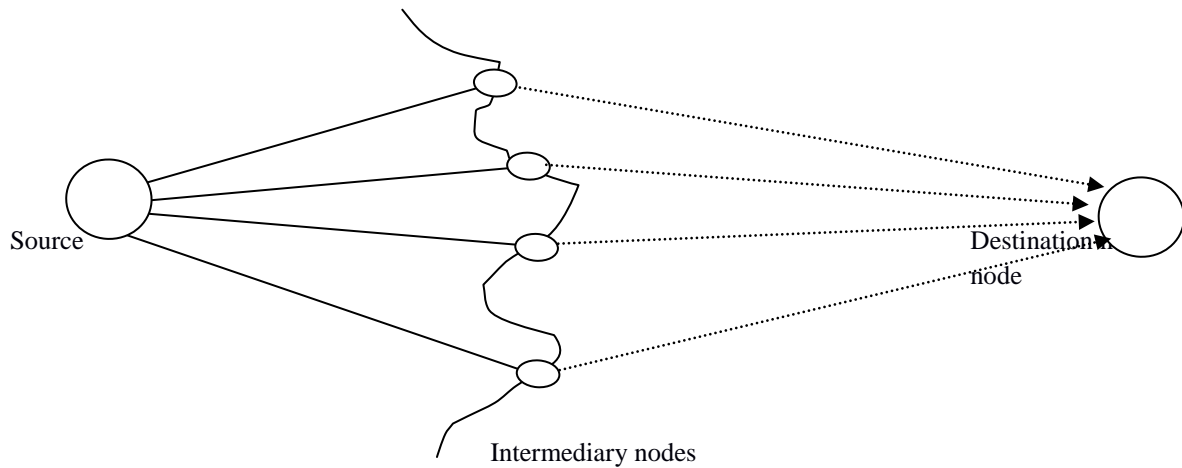


Figure 1: A Network of a Single Project

For any path:

$$P = (I, j_1), (j_1, j_2), \dots, (j_k, i)$$

from node I to a node i , let D_p be the duration of the path define as the sum of durations of its activities; that is

$$D_p = t_{ij_1} + t_{j_1j_2} + \dots + t_{j_ki}$$

Then the time t_i required to complete phase I is

$$T_i = \text{Max}_{\text{Paths } p \text{ From } I \text{ to } i} D_p$$

The maximum above is obtained by some path, because there can be only finite number of paths from I to i , since the network is acyclic. This to find T_i , we should find the longest path from node I to i

The algorithms developed in Bertsekas (1998) [1], Gupta and Hira (2013) [2] and Sharma (2013) [3] only guaranteed the critical path of a single project in a network, which may not be versatile in handling multiple project of the nature of fig 2. This is the reason for the proposed algorithm.

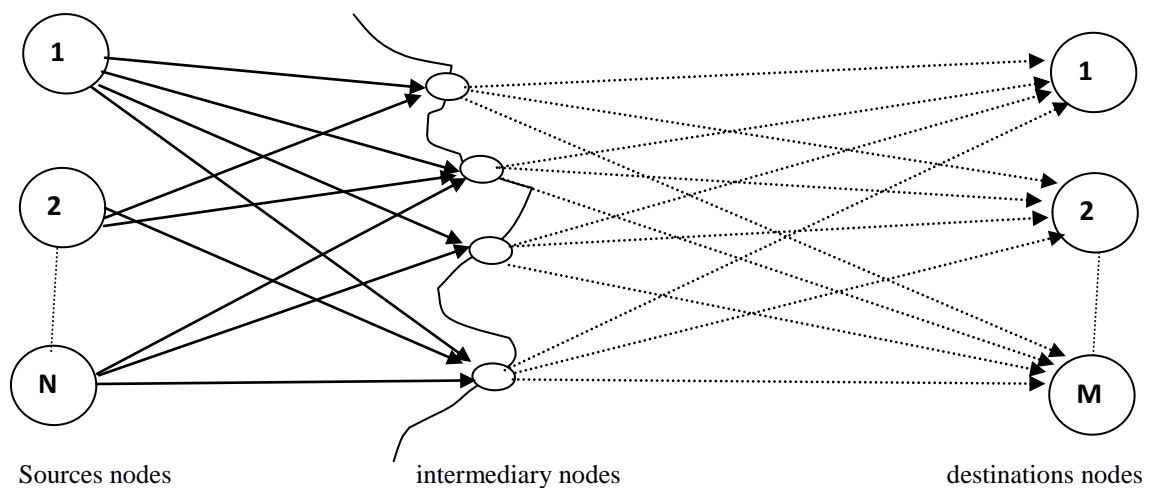


Figure 2: A network of multiple projects in one network

Propose Algorithm

To solve the kind of problem of in fig 2 above cannot be handled by the existing algorithms therefore we proposed an algorithm with a recursive function:



$$D_{id}^{(m)} = \text{optimized } \{d_{ij}^{(m-1)} + d_{kj}\}$$

Where d_{ij}^m = optimal distance at m

$D_{ij}^{(m-i)}$ = optimal distance at the previous stage($m-i$)

d_{kj} = distance (weight) at current stage

m = Edges

k = Vertex before j

Algorithm

Step 1: Add a dummy node to the sources nodes, if there are more than one source, if not proceed to step ‘2’

Step 2: Pick and remove a location from the frontier

Step3: Mark the location as visited so that we will not process it again

Step 4: Expand it by looking at neighbours, any neighbours we haven’t seen yet we add to the frontier

Step 5: Repeat step ‘2’, ‘3’ and ‘4’ until the frontier

Step 6: Truncate dummy vertex and all its edges from the graph (network)

Algorithm Implementation

P.W. Nig. Ltd were awarded a contract to construct three roads of 120km, 180km and 150km respectively. Table1 shows the detail activities required in the construction.

Table 1: Detail of constructing three roads by P.W. Nig. Ltd

	Hire personnel (months)	Order material (months)	Trains personnel (months)	Transport materials (months)	Road construction (months)	Road distance
Road 1	2	3	2	3	4	120km
Road 2	4	5	2	3	7	180km
Road 3	3	3	2	3	5	15km

This can be represented in a network as follows:

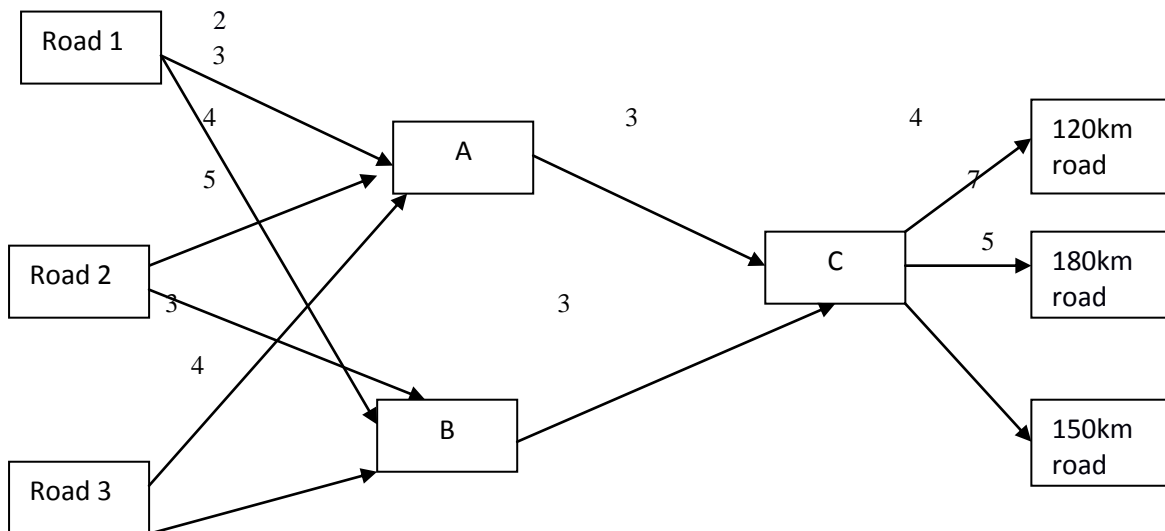


Figure 3: Network of three roads construction activities by P.W Nig Ltd

Truncating the dummy node and its vertexes we have:

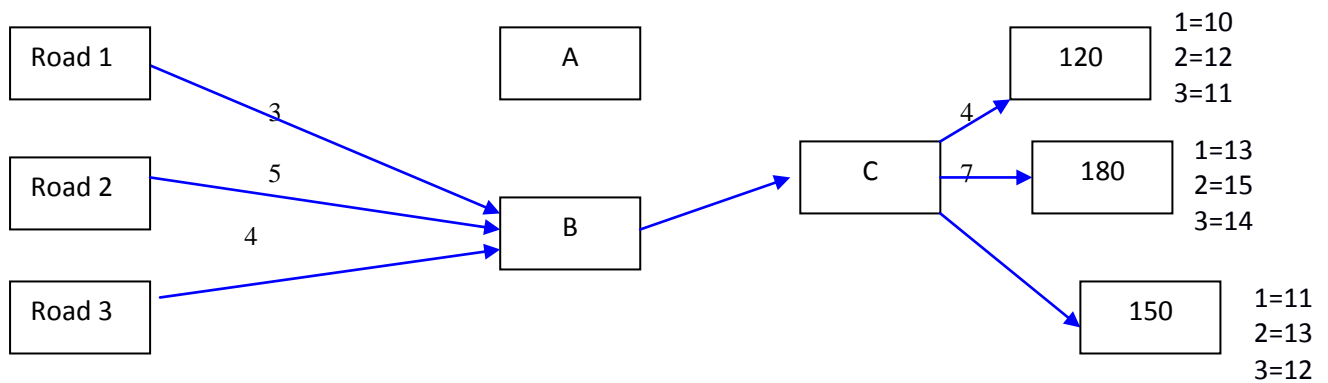


Figure 4: Results of the Critical Activities and Durations of the Network of the three Roads Construction Activities by P.W Nig Ltd

Discussion

From the result obtained, the contractor (P.W. Nig. Ltd) is supposed to handle the following critical activities in order to complete the project in good time:

- Road1: Order materials, transport materials, then construct the road of 120km = 11
 Order materials, transport materials, then construct the road of 180km= 11
 Order materials, transport materials, then construct the road of 150km= 11
- Road2: Order materials, transport materials, then construct the road of 120km =13
 Order materials, transport materials, then construct the road of 180km = 11
 Order materials, transport materials, then construct the road of 150km = 11
- Road3: Order materials, transport materials then construct the road of 120km =12
 Order materials, transport materials, then construct the road of 180km = 11
 Order materials, transport materials, then construct the road of 150km = 11

The critical (Activities) longest path (time) shows that, when the activities with the longest time (path) are been handled, the ones with less time can still be going on in order to optimize time can still be going on in order to optimize time.

Findings

1. This kinds of problems can also be viewed as a shortest path problem with the length of each arc (i, j) being $-d_{ij}^m$. In particular, finding the duration of the project is equivalent to finding the shortest path from multiple sources to multiple destinations.
2. This algorithm can also be used to find the longest path (critical path) as well as the alternate (next) longest path.
3. This algorithm can also be used to find the shortest path (stagecoach) as well as the alternate (next) shortest path if the other one fails.

References:

- [1]. Bertsekas D. P. (1998), Network Optimization: Continuous and Discrete Models. Athena Scientific, Belmont, Massachusetts. ISBN 1-886529-02-7 www.athenasc.com
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