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## A Novel Controller Design to Quench Chaotic Behavior in Magnetic Levitation System

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**Abstract** This article presents a novel control algorithm to quench the chaotic behavior in nonlinear magnetic levitation system. This algorithm not only mitigates the chaotic vibration and enables system to track reference command. System dynamic modeling is presented, the open loop system is simulating and chaotic behavior is analyzed using system state trajectories. A control algorithm is proposed and closed loop system is simulated and analyzed using its state trajectories. The results show that the proposed control algorithm not only has simpler structure to be implemented but also it is optimal with respect to the convergence time. This algorithm is very successful in eliminating the chaotic vibration as well as allows the system to synchronize with an external reference signal.

**Keywords** Chaos, Magnetic Levitation System, Feedback linearization, Chaotic Synchronization

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### 1. Introduction

The study of chaos in the perspective of control theory has been in boom over the last few decades and the theory of chaos is one of the most stirring and quickly growing research topics [1, 2]. The response of system with chaos can be tolerable in some of the scenarios, however most of the mechatronic systems suffer the deterioration owing to chaotic behaviors hence limiting the operable region of the system to retain the performance index within permissible bounds. Therefore, it can be inferred that when dealing with chaotic systems the objective is to mitigate chaotic behavior in majority of the cases [3]. In many cases the objective is the synchronization of two chaotic systems which is achieved by synthesizing a controller that forces the chaotic system to follow a given reference signal [4, 5]. Many strategies are developed and tested to subdue the chaotic responses of the systems. Now-a day the application of chaotic theory for magnetic suspension system has gained a pace and many strategies have been proposed [6].

Magnetic Levitation (Maglev) is a very stimulating system. It consists of suspending a ferromagnetic body in the space against the force of gravity. It is characteristically nonlinear and unstable. The feedback is inevitable to stabilize this system [7, 8]. This setup acts as a standard for research of trains that work on the principle of Magnetic levitation. The vehicles based in magnetic levitation run more smoothly and somewhat more quietly than wheeled mass transit systems. The Maglev vehicles do not depend on traction and track friction, which means that acceleration and deceleration can outstrip that of wheeled transports, and they are unaffected by weather. The power required for levitation is normally not a big percentage of the total energy intake [9]. The conveyor belts based on Maglev principle are used for materials transport systems.

This paper considers a prototype, externally excited magnetic suspension system. The open loop system is modelled and chaotic behavior is analyzed. A digital controller is designed and simulated to quench the chaotic behavior followed by real time implementation of the controller. The Rapid Control Prototyping is implemented for real time response monitoring and controller parameter tuning. The feasibility of the proposed control algorithm is proved experimentally by the excellent transient performance of the system and reference tracking capability, despite a simpler structure of the proposed control algorithm and ease of its digital implementation.



## 2. Overview of the Hardware Setup

Figure 1 shows the schematic diagram of the hardware setup. It consists of two annular permanent magnets (PM) and they are free to slide over a vertical nonmagnetic guide pipe. The similar poles of annular magnets face each other, which causes upper magnet to be levitated above the lower magnet. The vertical displacements of both magnets are monitored by non-contact proximity technique using ultrasonic sensors. The displacement of lower magnet is controlled by an electromechanical actuator. The displacement of upper magnet can be controlled by an electromagnetic force imparted on it by an electromagnet coil, whose excitation current is controllable.

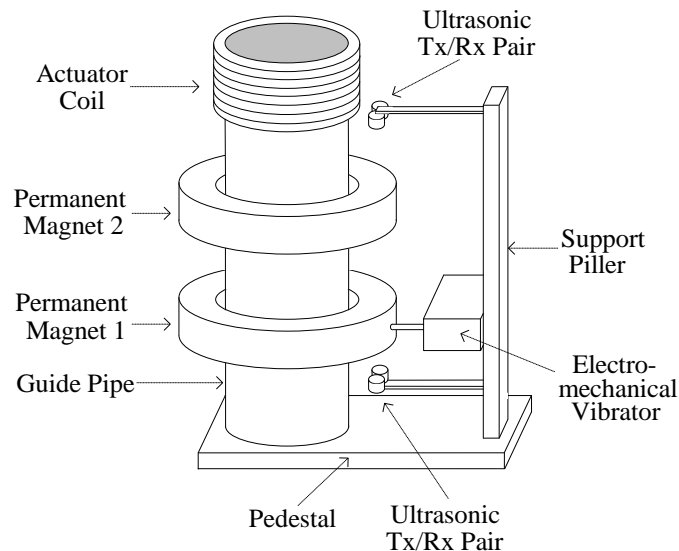


Figure 1: An overview of the hardware setup

## 3. The Control Problem Formulation and System Dynamics

Let us consider Figure 2. The problem is to analyze the chaotic behavior of displacement response  $p_1$  of PM2, subjected to the periodic sinusoidal vibration in the displacement  $p_0$  of PM1. These vibrations are caused by electrotechnical actuator. The vertical position  $p_3$  of coil is fixed. The control objective is to quench the chaotic behavior of  $p_1$  using electromagnetic actuation of the coil. The current in the coil is controlled in a manner that not only the chaotic behavior is subdued but also  $p_1$  tracks a reference command.

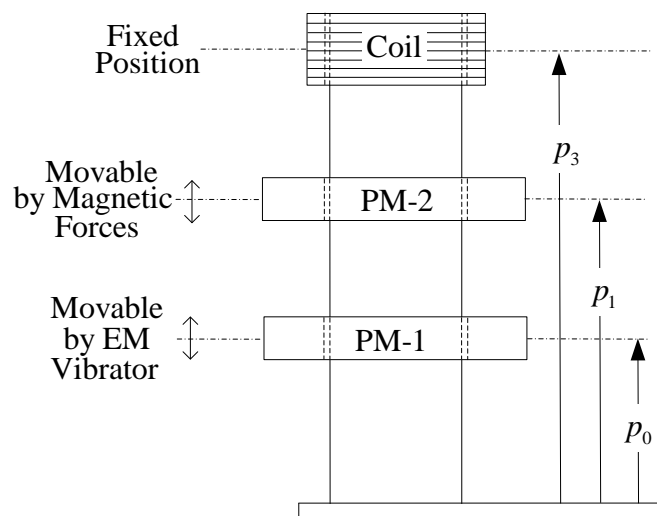


Figure 2: Illustration of the control problem



The free body diagram of the system is shown in Figure 3.  $F_c$  is the upward force generated by electromagnetic coil on PM2.  $F_g$  is the downward force of gravity on PM2.  $F_{10}$  is the force of repulsion exerted on PM2 by PM1.  $m_1$  is the mass of PM2.  $B$  is the frictional constant.

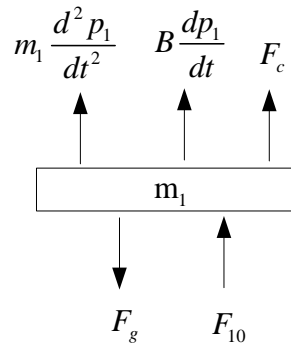


Figure 3: Free body diagram of hardware

The force balance equation gives us following system dynamic equation.

$$m_1 \frac{d^2 p_1}{dt^2} + B \frac{dp_1}{dt} = F_c + F_{10} - F_g \quad (1)$$

The expression for the force  $F_{10}$  is given by,

$$F_{10} = \frac{k_{10}}{(p_1 - p_0)^2} \quad (2)$$

Here  $k_{10}$  is a constant. The force of gravity  $F_g$  on PM-2 is given by,

$$F_g = m_1 g \quad (3)$$

let  $i$  and  $L_0$  being the current and inductance of coil respectively, and defining  $d = p_3 - p_1$ , the force  $F_c$  that electromagnet imparts on PM-2 is given by,

$$F_c = \frac{i^2 L_0 d_0}{2d^2} = \frac{k_c i^2}{(p_3 - p_1)^2} \quad (4)$$

Here  $k_c = \frac{L_0 d_0}{2}$  is a constant.

The value of constant  $k_{10}$  can be obtained from nominal steady state operating conditions, when  $i = 0$  and derivatives of  $p_1$  vanish in Equation 1 as,

$$0 = \frac{k_{10}}{(p_{10} - p_{00})^2} - m_1 g \quad (5)$$

Equation 5 gives us,

$$k_{10} = m_1 g (p_{10} - p_{00})^2 \quad (6)$$

Using Equations 2 through Equation 6 in Equation 1, we get,



$$\frac{d^2 p_1}{dt^2} = \frac{1}{m_1} F_c + \frac{g(p_{10} - p_{00})^2}{(p_1 - p_0)^2} - g - \frac{B}{m_1} \frac{dp_1}{dt} \quad (7)$$

Defining the state variables as,

$$\begin{aligned} x_1 &= p_1 \\ x_2 &= \dot{p}_1 \end{aligned} \quad (8)$$

Using Equation 8 and Equation 7, we get the following state space model.

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{g(x_{10} - p_{00})^2}{(x_1 - p_0)^2} - g - \frac{B}{m_1} x_2 + u \end{aligned} \quad (9)$$

Here  $p_0 = p_{0\max} \sin(188.5t)$  is the case study excitation provided by the electromechanical actuator to simulate chaotic behavior in PM2 vibration. The system in Equation 9 can be described by standard form given by,

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(x_1, x_2, t) + u \\ y &= x_1 \end{aligned} \quad (10)$$

#### 4. The Control Algorithm Design

Let  $r$  be the reference signal. We define error signal as,

$$e = x_1 - r \quad (11)$$

Defining the control input as,

$$u = -f - cx_2 + \ddot{r} + c\dot{r} \quad (12)$$

Using Equation 11 and Equation 12 in Equation 10 gives us the error dynamics,

$$\ddot{e} + c\dot{e} = 0 \quad (13)$$

If control is turned on at time  $t=T$ , then the convergence time under the boundary condition  $\dot{e}(0) = x_2(T) - r'(T)$  for Equation 13 is given by,

$$t_{con} = \frac{1}{c} \ln \frac{|\dot{e}(0)|}{c\mathcal{G}}, \quad (14)$$

Where  $\mathcal{G}$  and  $c$  are constants. We can find the optimal value of  $c$  by minimizing the convergence time in Equation 14. This value is given by,

$$c_{opt} = \frac{|\dot{e}(0)|}{\mathcal{G}} \quad (15)$$

Using Equation 15 and Equation 14 in dynamic Equation 13 leads  $e(t) \rightarrow A$  as  $t \rightarrow t_{con}$ , where  $A$  is a constant and  $x_1(t) \rightarrow r(t) + A$ . By appropriately offsetting the reference command, the steady state error  $A$  in  $x_1$  can be mitigated. Using Equation 4, Equation 9 and Equation 12 and using feedback linearization, the value of current in electromagnetic coil is given by,



$$i = \sqrt{\frac{m_1(p_3 - p_1)^2}{k_c} (-f - cx_2 + \ddot{r} + c\dot{r})} \tag{16}$$

**5. Simulation**

The actual values of the system parameters in the system modelling equations are given in Table 1. The open loop system has been simulated in Simulink package as shown in Figure 4.

**Table 1:** Numerical values of the system parameters.

Parameter	Value
$m_1$	0.025Kg
$B$	$0.218 \times 10^{-3}$ Kg/s
$g$	9.8m/s <sup>2</sup>
$x_{10}$	0.02m
$p_{00}$	0.01m
$p_3$	0.035m
$L_0$	0.177H
$d_0$	0.015m
$k_c$	$1.3275 \times 10^{-3}$ Nm <sup>2</sup> /A <sup>2</sup>

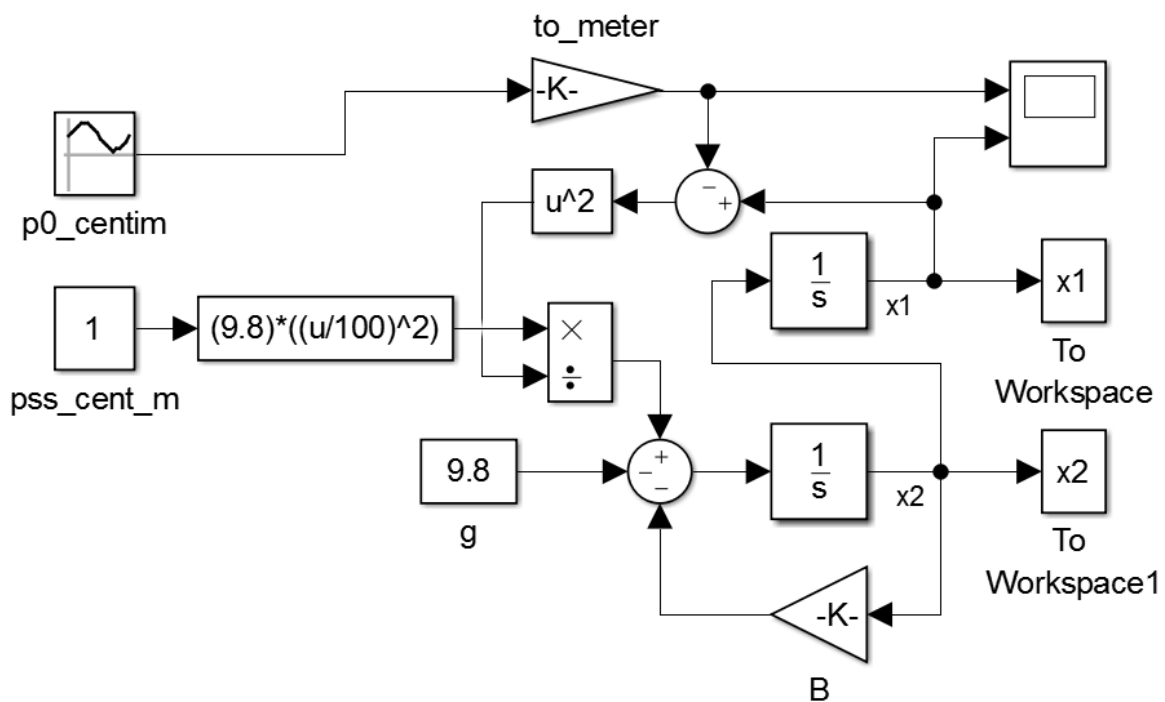


Figure 4: Open loop system simulation in Simulink

The displacement of PM2 is plotted in Figure 5. It is clear that response proceeds chaotically with time. Figure 6 shows the velocity response of PM2, which is equally chaotic.

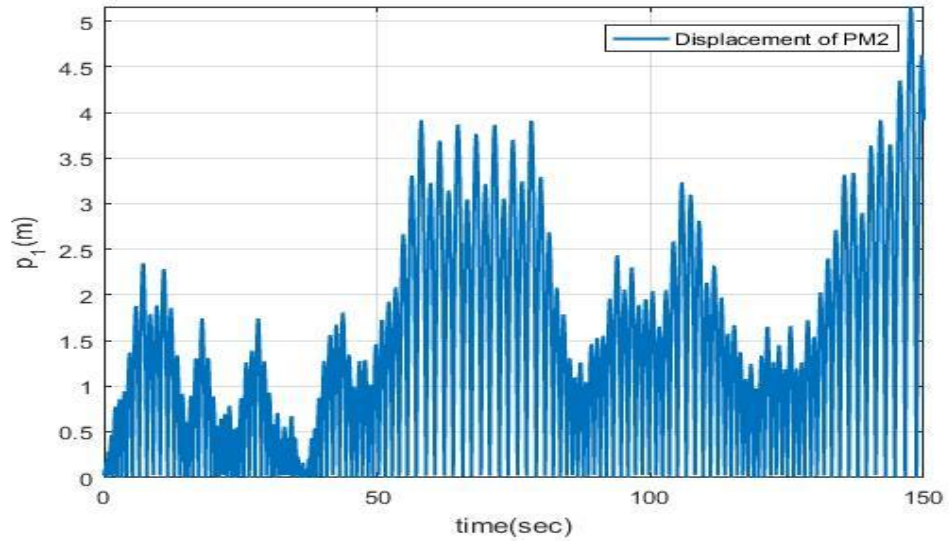


Figure 5: Open loop response of PM2 displacement

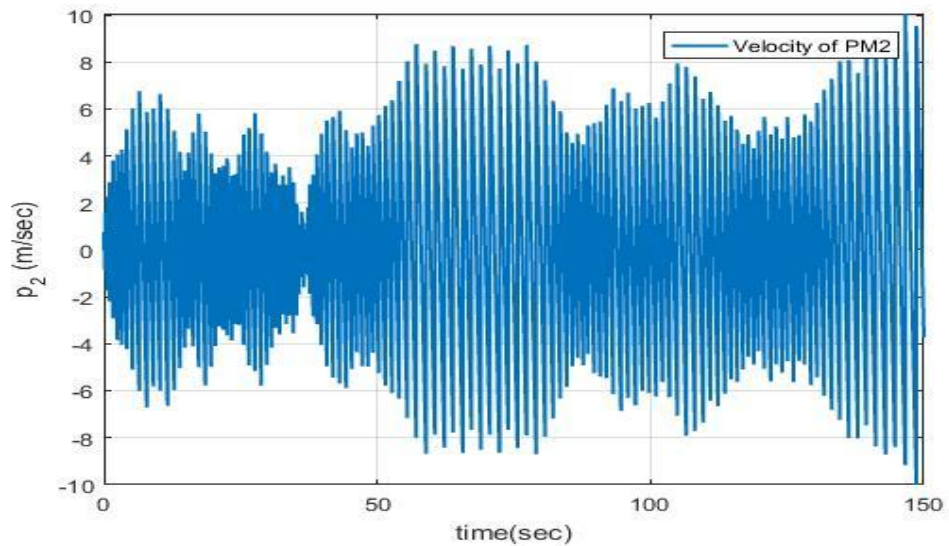


Figure 6: Open loop response of PM2 velocity

Figure 7 and Figure 8 shows the chaotic system trajectories over a short and long time span respectively. The chaotic system maps are clearly visible.

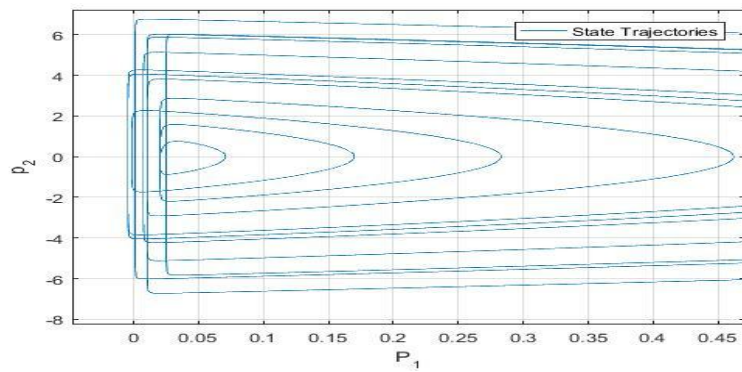


Figure 7: Open loop system state trajectories over a short time span

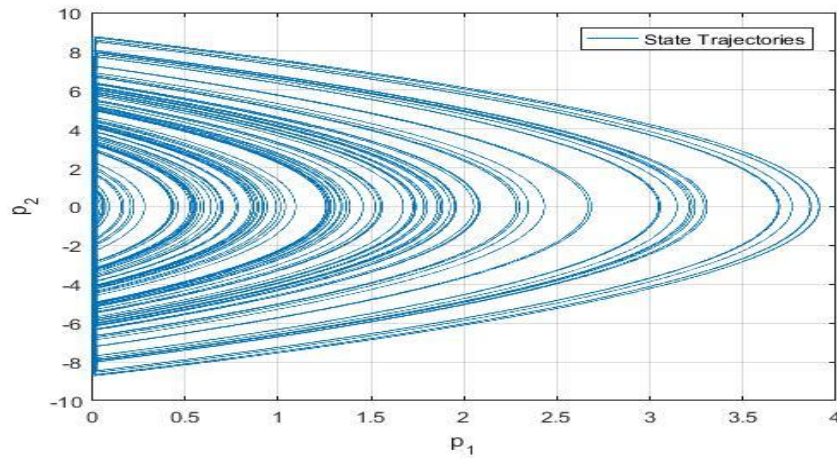


Figure 8: Open loop system state trajectories over a long-time span

Figure 9 shows closed loop system simulation, considering the reference command to be of form  $r(t) = r_{\max} \sin(\omega t)$ . The boxes with grey background correspond to the proposed controller.

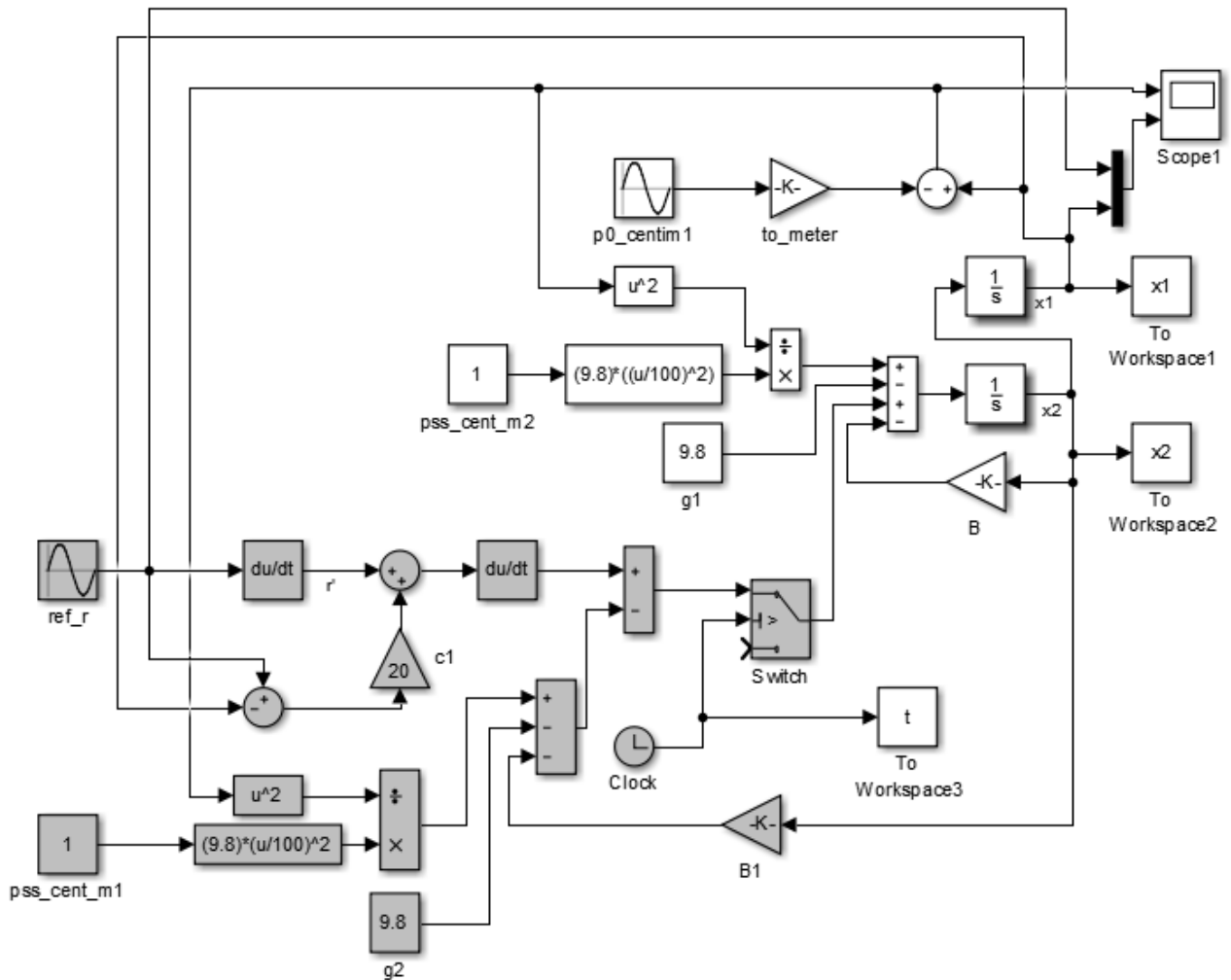


Figure 9: Closed loop system simulation in Simulink



Figure 10 shows the closed loop response of PM2 displacement. The controller is turned on at  $t=9$ sec. It is clearly visible that controller is not only able to quench the chaotic behavior but has also enabled the system to follow the commanded reference sinusoidal signal. This is effectively equivalent to synchronization of the system with an external commanded reference.

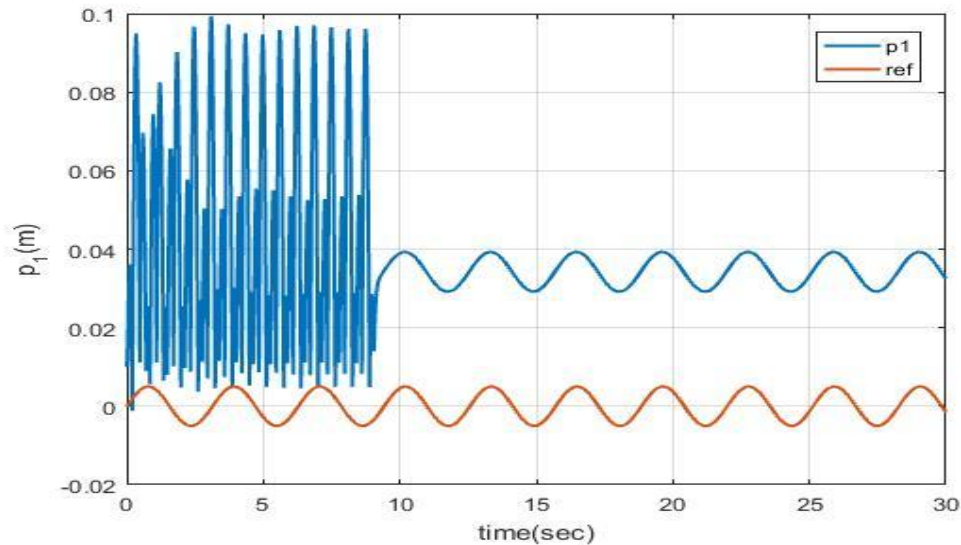


Figure 10: Closed loop response of PM2 displacement with controller turned on at  $t=9$  sec

Figure 11 shows the closed loop system state trajectories with controller turned on at  $t=9$ sec. Without the controller turned on, the system state trajectories are chaotic, however, as the controller turns on the system state trajectories converge to a predefined reference trajectory. This trajectory is showed zoomed-in in Figure 11.

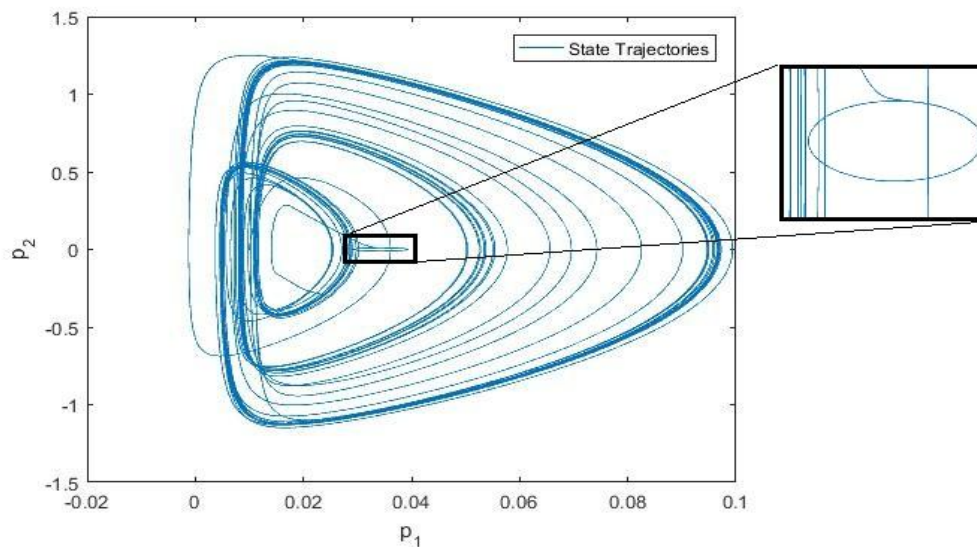


Figure 11: Closed loop system state trajectory

Figure 12 shows the plant input signal. As the controller is turned on at  $t=9$ sec, the actuating signal becomes active and stays bounded.



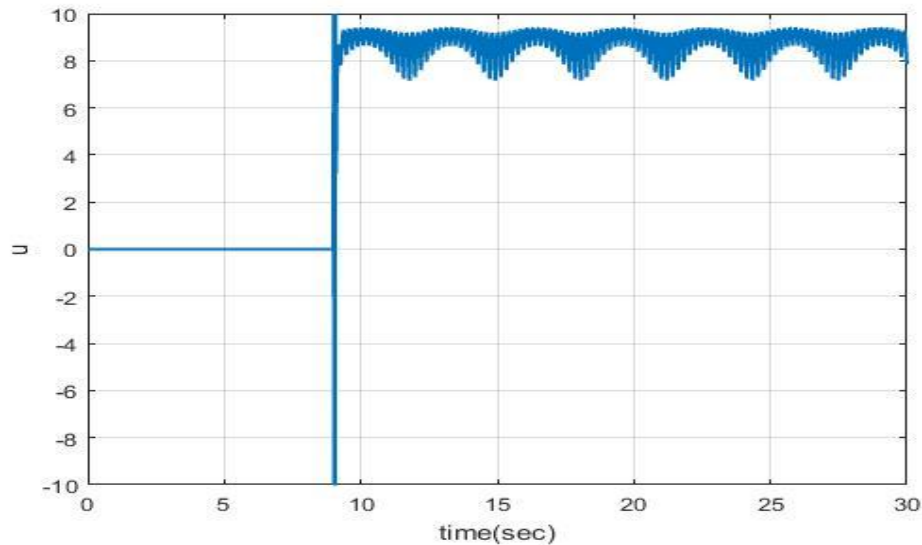


Figure 12: Closed loop plant input with controller turned on at  $t=9\text{sec}$

## 6. Discussion and Conclusions

A control algorithm is presented to suppress the chaotic behavior of a nonlinear magnetic levitation system. The open loop system dynamics are presented followed by control algorithm design. The open loop system response is simulated and analyzed for the chaotic behavior. The closed loop system is simulated and its response is analyzed. The responses clearly manifest the success of designed control algorithm to quench the chaotic behavior in system state trajectories and allow the system trajectories to converge to a predefined reference trajectory

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