



**On some properties combinatorics of Graphs in the d-dimensional FLS**

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**Abstract** This study is based on points, and some given properties and definitions about vertex, neighbours and independent or dependent definitions in the Graphs are extended to FLS. It is seen that  $S = (\mathfrak{N}, D)$  FLS is a complete graph. Arguments of graphs are transferred to FLS. Therefore, as the necessary preliminaries, concept and operations in graph relative to FLS are introduced. Their combinatoric properties are investigated. We know from [2] that FLS is a generalization of graph in the idea fuzzy. Some studies are performed on Health and Engineering sciences particularly in Dentistry and Mining at the FLU [5].  
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**1. Introduction**

A graph with vertex set  $\mathfrak{N}$  is said to be a graph on  $\mathfrak{N}$  for  $L_n$ . The vertex set of a graph  $S$  is referred to as  $\mathfrak{N}(S)$ , the set of edges is denoted by  $D(S)$ .

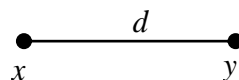
The elements of  $D$  are its edges (or lines). A vertex  $x$  is incident with an edge if the exists some  $d$  in  $D$  such that  $d(x) \neq \theta$ . Here  $d$  is an edge at  $x$ . The vertices  $x, y$  are endvertices or end if  $d(x) \wedge d(y) \neq \theta$  from FLS1, and an edge joins its ends. The set of all the edges in  $S$  at  $x$  is shown by  $D(x)$ . Thus;

$$D(x) = \{d \mid d(x) \neq \theta, d \in D\} \text{ for } x \in \mathfrak{N}.$$

**2. Neighbor In Theory Graph**

In  $S = (\mathfrak{N}, D)$  FLS  $[\mathfrak{N}]^2 \subseteq D \subseteq [\mathfrak{N}]^k$  for  $k > 3$ . The usual way to picture a graph is to draw a dot for each vertex and joining at least two of these dots by a line if the corresponding at least two vertices form an edge.

Any  $x, y \in \mathfrak{N}$  for  $I = \{\theta, I\}$ ;



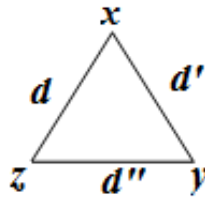
$d(x) = d(y) = I$ , the number of edges is one for  $I$ , but ; the number of edges are more than one for  $L_n$ .

The vertices  $x, y$  of  $S$  are adjacent, or neighbors, if  $\exists d \in D : d(x) \wedge d(y) \neq \theta$ . Two edges  $d \neq l$  are adjacent if  $d(x) \wedge l(x) \neq \theta$ . They have an endpoint in common. The set of neighbours of a vertex  $x$  in  $S$

is denoted by  $N(x)$ . More generally for  $U \subseteq \aleph$ , the neighbours in  $\aleph - U$  of vertices in  $U$  are called neighbours of  $U$ ; their set is denoted by  $\aleph(U)$ .

**Example**

$S = (\aleph, D)$  for  $\aleph = \{x, y, z\}$  is



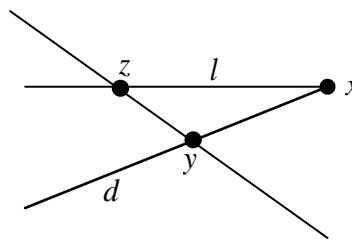
The vertex  $x$  has two neighbours  $y, z$ , and two edges  $d, d'$  are neighbours.

Any vertex  $x$  has  $k - 1$  neighbours if  $S$  has  $k$  vertices, where  $k \geq 3$ . And there are no pairwise non-adjacent edges, because of FLS2.

All the vertices of  $S$  are pairwise adjacent, because of FLS1, in any  $S$  FLS, then  $S$  is copmplete. Pairwise non-adjacent vertices or edges are called independent, otherwise dependent. There are no vertices independent for any two vertices  $x, y$  in FLS. There are three vertices that are not on the same line. A set of such points is independent if three vertices are not adjacent. Any two vertices on edges are dependent, because of FLS1.

**Lemma.** Let be a  $S = (\aleph, D)$  FLS. If  $v_j = |\{x | d(x) \neq \theta\}| d(x) = k, k \geq 3$  then at least set of independent vertices has three elements.

**Proof.** In any case, two vertices are neighbours. An edge has smallest two vertices in a  $S = (\aleph, D)$  FLS; the  $d \in D$  and  $d(x) \neq \theta, d(y) \neq \theta$ . 3th vertex  $z$  is on other edge  $l$ . And,



$d(x) \wedge d(y) \wedge d(z) = \theta$ , because of FLS3. The vertices  $x, y, z$  are not neighbours. They are independent.

Any  $x, y \in S = (\aleph, D)$  for  $L_1 = \{\theta, a, I\}$  ;

$$x = (s, d(s)) = \begin{cases} (s, 1), d(s) = 1 \\ (s, a), d(s) = a \end{cases}$$

$$y = (z, d(z)) = \begin{cases} (z, 1), d(z) = 1 \\ (z, a), d(z) = a \end{cases}$$

$$[d(s)]^m = \begin{cases} 1, & \text{if } d(s) = 1 \\ a^m, & \text{if } d(s) = a \\ a^k, \text{ if } k - \text{items}, d(s) = a, (m - k) - \text{items}, d(s) = 1 \end{cases}$$

This idea is generalized for  $L_n$  at FLS as follows.

Let  $S = (\aleph, D)$  be a FLS, and  $k, s, m \in \mathbb{N}^+$ . Then some combinatoric properties are below for vertex  $x$ ,  $d \in D$ ;



i.  $[d(x)]^m = 1$  and  $|[d(x)]^m| = 1$  if  $L_0 = \{\theta, I\}$ ,

ii.  $[d(x)]^m = \begin{cases} 1, & \text{if } d(x) = 1 \\ a_1^m, & \text{if } d(x) = a_1 \\ a_1^k, & \text{if } k\text{-items, } d(x) = a_1, (m-k)\text{-items, } d(x) = 1 \end{cases}$

and  $|[d(x)]^m| = 3$  if  $L_1 = \{\theta, a_1, I\}$ ,

iii.  $[d(x)]^m = \begin{cases} 1, & \text{if } d(x) = 1 \\ a_1^m, & \text{if } d(x) = a_1 \\ a_2^m, & \text{if } d(x) = a_2 \\ a_1^k, & \text{if } k\text{-items, } d(x) = a_1, (m-k)\text{-items, } d(x) = 1 \\ a_2^k, & \text{if } k\text{-items, } d(x) = a_2, (m-k)\text{-items, } d(x) = 1 \\ a_1^s a_2^k, & \text{if } s\text{-items, } d(x) = a_1, k\text{-items, } d(x) = a_2, (m-k-s)\text{-items, } d(x) = 1 \end{cases}$

and  $|[d(x)]^m| = 6$  if  $L_2 = \{\theta, a_1, a_2, I\}$ , therefore  $|[d(x)]^m| = 11$  if  $L_3 = \{\theta, a_1, a_2, a_3, I\}$ .

**Definition.** Let  $S = (\mathfrak{S}, D)$  be a FLS, and for  $\exists x, y \in \mathfrak{S}$  such that  $d(x) \neq \theta, d(y) \neq \theta$  then the number of neighbour lines  $d$  is called *fuzzy neighbour order* on  $x, y$  and denoted by  $|d(x, y)|$  or  $|d(x)|$  or  $|d(y)|$  or  $|d|$ .

**Lemma.** Let  $S = (\mathfrak{S}, D)$  be a FLS,  $m \in \mathbb{N}^+$ . and  $d(x) \in L_n - \{\theta\}$ . Then

$$|[d(x)]^m| = n + 1 + \binom{n}{2} + \dots + \binom{n}{n-1}.$$

**Proof.** It is clear for  $L_0$ . If  $d(x) \neq \theta$  for  $x \in \mathfrak{S}$  and  $L_n = \{\theta, a_1, a_2, \dots, a_n, I\}$  then

$$\overbrace{[d(x)]^m}^{|d(x)]^m| \text{-items}} = \underbrace{d(x) = 1}_{1\text{-item}} \text{, or } \underbrace{\begin{bmatrix} d(x) = a_1 \\ \vdots \\ d(x) = a_1 \end{bmatrix}}_{n\text{-items}} \text{, or } \underbrace{\begin{bmatrix} d(x) = a_1 a_2 \\ \vdots \\ d(x) = a_1 a_n \end{bmatrix}}_{\binom{n}{2}\text{-items}} \text{, } \dots \text{, } \underbrace{\begin{bmatrix} d(x) = a_1 a_2 \dots a_{n-2} \\ \vdots \\ d(x) = a_1 a_2 \dots a_n \end{bmatrix}}_{\binom{n}{n-2}\text{-items}}$$

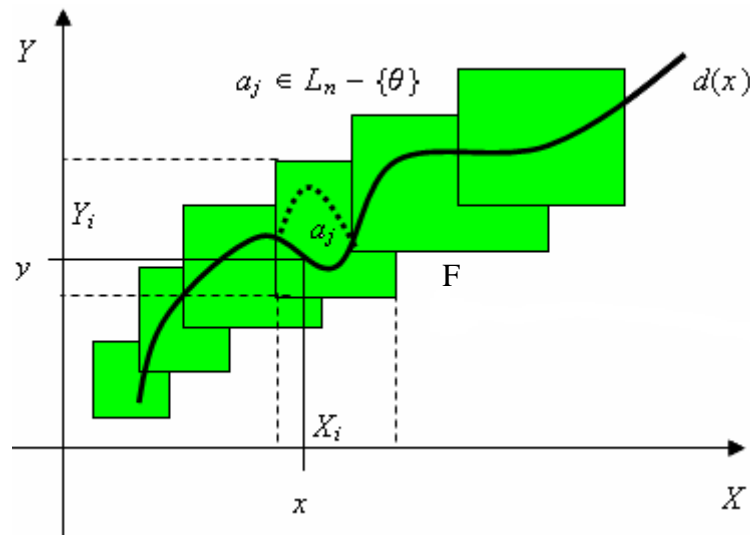
Thus,

$$|[d(x)]^m| = n + 1 + \binom{n}{2} + \dots + \binom{n}{n-1}.$$

A graph of fuzzy neighbours is follow in FLS as :

$$d : F = \bigvee_{i=1}^n (X_i \times Y_i) \rightarrow L_n$$

$$x \in F \Rightarrow x \rightarrow d(x) = a_j.$$



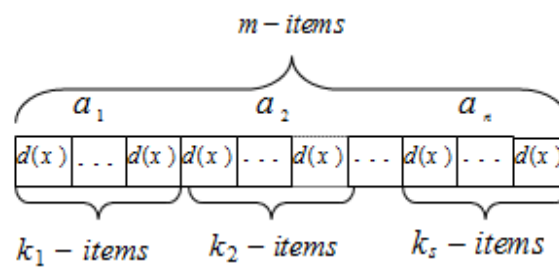
It is different or same in  $[X_i, Y_i]$   $i = 1, \dots, m$  if  $d(x) \neq \theta$ .

**Lemma.** Let  $S = (\aleph, D)$  be  $k_1, k_2, \dots, k_s, m, s \in \mathbb{N}^+$ . Then

for vertex  $x, d \in D$ ;

$$[d(x)]^m = \prod_{i=1, 1 \leq j \leq n}^s a_j^{k_i}$$

*Proof.* For vertex  $x, d \in D$ , if  $d(x) \neq \theta$ , then



$$[d(x)]^m = (a_1^{k_1}) \dots (a_s^{k_s}) = \prod_{i=1, 1 \leq j \leq n}^s a_j^{k_i}$$

**3. Results**

- i. Any two vertices  $x, y$  are neighbours. i.e there are no vertices which are not neighbours.
- ii. The number of independent vertices are at least three.
- iii. There are more than one graphs on  $xy$  – plane in FLS.

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