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A novel algorithm for sarima model fitting: modelling pound-euro exchange rates as case study

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Abstract Many economic and business time series exhibit seasonal tendencies. Analytical techniques for such series which take into account these tendencies have engaged the attention of researchers of recent. One such modelling technique is the Box- Jenkins seasonal autoregressive integrated moving average (SARIMA) technique. A novel algorithm is hereby proposed. This algorithm which is based on autoregressive-moving average duality arguments is applied to model daily exchange rates of the British pound sterling and the European Euro currencies. The data analyzed are 178 daily pound/euro exchange rates 13th December 2015 to 7th June 2016. Application of the algorithm using the SARIMA(1,1,1)x$(1,1,1)_7$ *template* as proposed yields a SARIMA(1,1,1)x$(0,1,1)_7$ model. Further 8 values from 8th June to 15th June 2016 are used for out-of-sample comparison of observations with forecasts. The adequacy of the chosen model is not in doubt since the residuals are uncorrelated and are normally distributed. Moreover out-of-sample forecasts closely agree with the observed values. This *additive-multiplicative* model may be used for forecasting and simulation purposes.

Keywords Foreign Exchange Rates, the euro, the pound sterling, Sarima modelling

Introduction

Global economy rests on foreign exchange amongst countries. This work involves the modelling of the daily exchange rates of the British pound (GBP) and the European Euro (EUR). The pound sterling is the world's oldest currency that is still in use since its introduction on October 8, 1990 [1]. Official users of the pound sterling include nine British colonies: British Antarctic Territory, Falkland Islands, Gibraltar, Saint Helena, Ascension and Tristan da Cunha, South Georgia and the South Sandwich Islands, British Indian Ocean Territory, Guernsey, Isle of Man and Jersey. Unofficial users include Uganda, Zimbabwe, Zambia, Sierra Leone, Tanzania, Rwanda, Malawi and Botswana.

On the other hand the euro was officially adopted on 16 December 1995 and introduced to the world markets as an accounting currency on 1 January 1999. Official users of the currency are countries within the eurozone: Austria, Belgium, Cyprus, Estonia, Finland, France, Germany, Greece, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherlands, Portugal, Slovakia, Slovenia,and Spain. Unofficial users include Kosovo, Montgenero, Northern Cyprus and Zimbabwe [1].

This paper examines the possibility of modelling the daily exchange rates of the GBP and the EUR with a view to proposing a model for their forecasting and simulation. This can go a long way to help in management and planning. The approach which shall be adopted is the seasonal autoregressive integrated moving average (SARIMA) approach.

The SARIMA approach is an adaptation of the general autoregressive integrated moving average (ARIMA) modelling both proposed by Box and Jenkins [2]. It has been extensively applied to model time series especially those which show evidence of seasonality. For instance, Wongkoon *et al.* [3] modelled monthly DHF incidence in Northern Thailand as a SARIMA$(2,0,1)x(0,2,0)_{12}$ model, Shitan *et al.* [4] modelled Bangladesh export values by a SARIMA$(1,1,0)x(0,1,1)_{12}$, Cuhadar [5] forecasted tourism demand to Istanbul using a SARIMA$(2,0,0)x(1,1,0)_{12}$, Asamoah-Boaheng [6] fitted a SARIMA$(2,1,1)x(1,1,2)_{12}$ model to monthly mean surface air temperature in the Ashanti Region of Ghana and Alvarez-Diaz and Gupta [7] proposed a SARIMA$(3,1,1)x(2,1,2)_{12}$ forecasting model for the United States Consumer Price Indices . Some others who have applied SARIMA modelling in recent times are Papmichail and Georgiou [8], Brida and Garrido [9], Khajavi *et al.* [10], Mombeni *et al.* [11] and Maarof *et al.* [12], to mention but a few.

ARIMA and SARIMA modelling have been applied of recent to model foreign exchange rates. For instance, Appiah and Adetunde [13] fitted an ARIMA(1,1,1) to the exchange rates of Ghana cedi and the US Dollar. Osarumwense and Waziri [14] modelled Nigerian Naira and the US Dollar exchange rates as an ARIMA(1,1,1). Martinez and Gaw [15] modelled the Philippine/ US Dollar exchange rate as an ARIMA(1,1,2). Etuk *et al.* [16] have fitted an *additive* SARIMA model to daily Ugandan shilling – Nigerian Naira exchange rates.

Materials and Methods

Data: The data for this work are daily exchange rates between British Pound and the European Euro from 13 December, 2015 to 7 June, 2016 obtained from the website www.exchangerates.org.uk/GBP-EUR-exchange-rate-history.html . The website was accessed on 8 June, 2016 for the sample and on 16 June, 2016 for the out-of-sample observation/forecast comparison. It is interpreted as the amount of the Euro in one pound.

Sarima Methodology

A time series $\{X_t\}$ is said to follow a SARIMA $(p,d,q)x(P,D,Q)_s$ if

$$A(L)\Phi(L^s)\nabla_s^D\nabla^d(X_t) = B(L)\Theta(L^s)\varepsilon_t \quad (1)$$

where $\{\varepsilon_t\}$ is a white noise process, A(L) is the non-seasonal autoregressive operator – a p-order polynomial in L, B(L) is the non-seasonal moving average operator – a q-order polynomial in L, Φ(L) is the seasonal autoregressive operator – a P-order polynomial in L, Θ(L) is the seasonal moving average operator – Q-order polynomial in L, ∇ is the non-seasonal difference operator and ∇_s is the seasonal difference operator, d is the non-seasonal differencing order, D is the seasonal differencing order, L is the backshift operator defined by $L^kX_t = X_{t-k}$ and s is the seasonality period(Box and Jenkins, 1976).

Suhartono [17] has proposed an algorithm for SARIMA modelling. This is given by

1. Fit the moving average (MA) model

   $$X_t = \varepsilon_t + \beta_1\varepsilon_{t-1} + \beta_S\varepsilon_{t-s} + \beta_{s+1}\varepsilon_{t-s-1} \quad (2)$$

2. If $\beta_{s+1} = 0$, then the model is said to be *additive*. If not,

   if $\beta_1\beta_s = \beta_{s+1}$, then the model is said to be *multiplicative*, otherwise it is said to be *subset*.

On the basis of autoregressive-moving average duality properties, Etuk and Ojekudo [18] proposed an algorithm based on the autoregressive dual of Suhartono's [17] algorithm.

That is,

1. Fit the autoregressive (AR) model

   $$X_t + \alpha_1X_{t-1} + \alpha_sX_{t-s} + \alpha_{13}X_{t-s-1} = \varepsilon_t \quad (3)$$

2. If $\alpha_{s+1} = 0$, then the model is said to be *additive*. If not,

   If $\alpha_1\alpha_s = \alpha_{s+1}$, then the model is said to be *multiplicative*, otherwise it is said to be *subset*.

The Novel Algorithm:

A stationary time series may be represented as a *general linear process* which may be defined as a moving average model of infinite dimension. It may be shown that both an AR(p) and an MA(q) are finite approximations of the process. The ARMA model resulted as a more parsimonious representation of the process.

A merger of the two algorithms is hereby proposed as the novel algorithm. That is:

Fit the SARIMA$(1,0,1)x(1,0,1)_s$ model:

$$X_t + \alpha_1X_{t-1} + \alpha_sX_{t-s} + \alpha_{s+1}X_{t-s-1} = \varepsilon_t + \beta_1\varepsilon_{t-1} + \beta_s\varepsilon_{t-s} + \beta_{s+1}\varepsilon_{t-s-1} \quad (4)$$

The left hand side (lhs) of (4) is the AR component and the right hand side (rhs) the MA component. If both components are subset in the respective sense of (2) and (3), the resultant model may be called a *subset-subset* SARIMA model. If the lhs is additive and the rhs subset then the model may be called an *additive-subset* SARIMA model. Similarly, a *multiplicative-subset,* a *subset-additive,* an *additive-additive,* a *multiplicative-additive,* a *subset-multiplicative,* an *additive-multiplicative* and a *multiplicative-multiplicative* SARIMA model may be defined. It is also possible that the AR or MA component is not statistically significant. The word *nil* may be used to denote this possibility. For instance, by a *nil-additive* SARIMA model it is meant the model

$X_t = \varepsilon_t + \beta_1\varepsilon_{t-1} + \beta_s\varepsilon_{t-s}$

and by a *multiplicative-nil* SARIMA model it is meant the lhs of (3) such that $\alpha_1\alpha_s = \alpha_{s+1}$. Similarly a *nil-multiplicative,* a *nil-subset,* an *additive-nil* or a *subset-nil* SARIMA model may be defined.

Sarima Modelling

Stationarity of time series shall be tested by Augmented Dickey Fuller (ADF) Test. It is often enough to difference the series such that D + d is not more than 2, for stationarity to be achieved.

The autocorrelation structure associated with the model (4) is such that there is a significant spike in the autocorrelation function at lag s and comparable spikes at lags s-1 and s+1. Similarly, the partial autocorrelation function is such that there is a significant spike at lag s and comparable ones at lags s-1 and s+1.

The reviews package shall be used to fit the model (4) by the least error sum of squares criterion. Model discrimination and selection shall be done by automatic model selection criteria like Akaike's Information Criteria (AIC) [19], Schwarz Criterion [20] and Hannan-Quinn criterion [21].

Results and Discussion

The time-plot of Figure 1 shows an overall slightly negative trend up to the first week of April, 2016 after which it shows an overall positive trend. This may be interpreted to mean that the GBP relatively depreciated from December 2015 to March 2016 after which it appreciated. The ADF test statistic for the pound-to-euro exchange rate series, is equal to -2.50 and the 1%, 5% and 10% critical values are respectively -3.47, -2.88 and -2.58. Therefore the rates are non-stationary. A weakly differencing of the series yields another series which has a slightly upward overall trend (See Fig. 2). With an ADF test statistic of -3.27, it is non-stationary at 1% level of significance. Besides, its correlogram of Fig. 3 shows evidence of seasonality. Therefore a non-seasonal differencing of the seasonal differences was done to obtain a series which shows no trend (See Fig. 4) and a correlogram showing suggestive of seasonality with s = 7 (See Fig. 5).

Estimation of the SARIMA(1,1,1)x(1,1,1)$_7$ model in Table 1 yields

$$X_t - 0.5075X_{t-1} + 0.1083X_{t-7} + 0.0337X_{t-8} = \varepsilon_t - 0.4659\varepsilon_{t-1} - 0.9483\varepsilon_{t-7} + 0.4143\varepsilon_{t-8} \quad (5)$$

(±0.1252) (±0,0791) (±0.0943) (±0.1596) (±0.0179) (±0.1408)

Clearly for model (5) only the lags 7 and 8 coefficients of the lhs are not statistically significant.

The resultant model is an *additive-multiplicative* SARIMA model. Traditionally it is a SARIMA(1,1,1)x(0,1,1)$_7$ model. It is as estimated in Table 2 by

$$X_t - 0.4760X_{t-1} = \varepsilon_t - 0.4698\varepsilon_{t-1} - 0.9417\varepsilon_{t-7} + 0.4115\varepsilon_{t-8} \quad (6)$$

(±0.0552) (±0.0883) (±0.0195) (±0.0852)

The information criteria AIC, Schwarz Criterion and Hannan-Quinn criterion unanimously choose model (6) in preference to (5).

It is noteworthy that the *additive-multiplicative* model (6) has residuals which are uncorrelated as evident from Figure 6 and are normally distributed as evident from the Jarque-Bera test of Figure 7. In addition the Table 3 results show that there is a close enough agreement between the observations and forecasts of the exchange rate values of 8th June 2016 through 15th June 2016 which are out-of-sample.

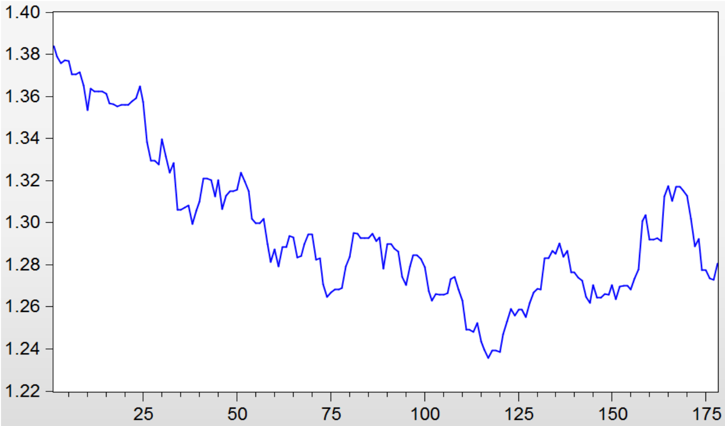


Figure 1: Time Plot of the Series

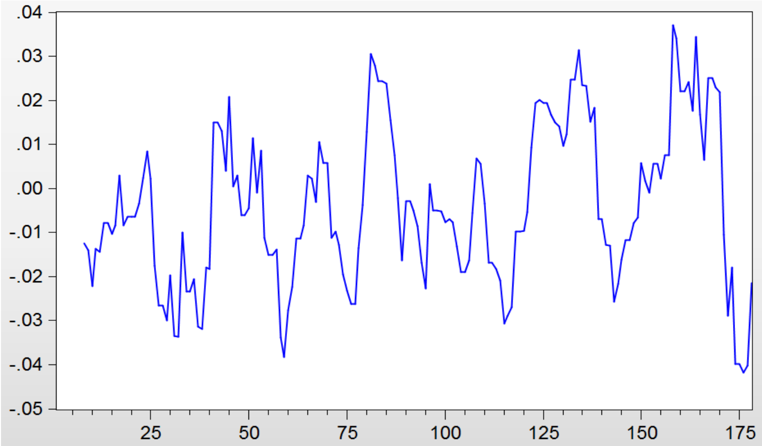
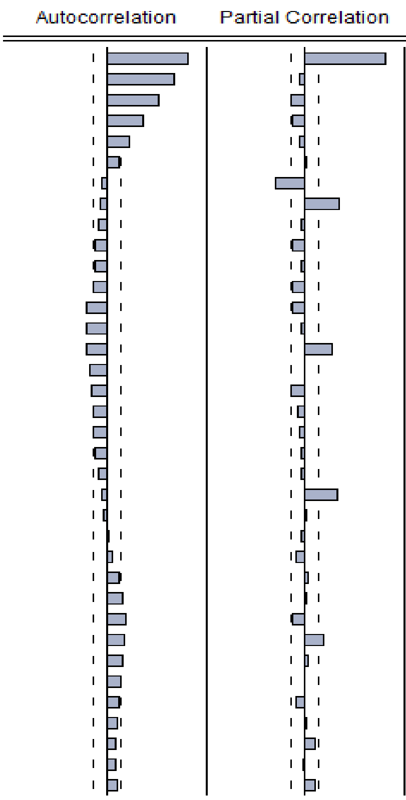


Figure 2: Seasonal Differences



Lag	AC	PAC	Q-Stat	Prob
1	0.846	0.846	124.60	0.000
2	0.700	-0.056	210.43	0.000
3	0.539	-0.139	261.58	0.000
4	0.375	-0.119	286.43	0.000
5	0.228	-0.053	295.69	0.000
6	0.115	0.009	298.07	0.000
7	-0.051	-0.300	298.54	0.000
8	-0.078	0.357	299.65	0.000
9	-0.095	-0.025	301.29	0.000
10	-0.116	-0.125	303.76	0.000
11	-0.125	-0.039	306.66	0.000
12	-0.153	-0.137	311.01	0.000
13	-0.221	-0.116	320.14	0.000
14	-0.229	-0.035	330.05	0.000
15	-0.209	0.293	338.31	0.000
16	-0.185	-0.005	344.81	0.000
17	-0.157	-0.144	349.55	0.000
18	-0.150	-0.080	353.91	0.000
19	-0.148	-0.054	358.15	0.000
20	-0.115	-0.041	360.73	0.000
21	-0.095	-0.044	362.53	0.000
22	-0.061	0.342	363.28	0.000
23	-0.028	0.022	363.44	0.000
24	0.014	-0.035	363.47	0.000
25	0.052	-0.095	364.01	0.000
26	0.115	0.034	366.70	0.000
27	0.168	0.017	372.53	0.000
28	0.197	-0.135	380.58	0.000
29	0.182	0.191	387.45	0.000
30	0.158	0.039	392.69	0.000
31	0.133	-0.005	396.43	0.000
32	0.122	-0.093	399.59	0.000
33	0.101	0.018	401.76	0.000
34	0.090	0.112	403.52	0.000
35	0.097	-0.021	405.57	0.000
36	0.108	0.108	408.11	0.000

Figure 3: Correlogram of the seasonal differences

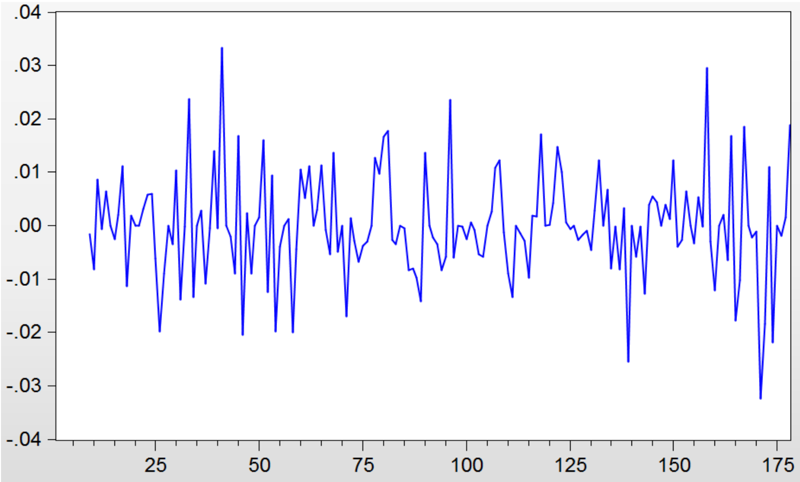
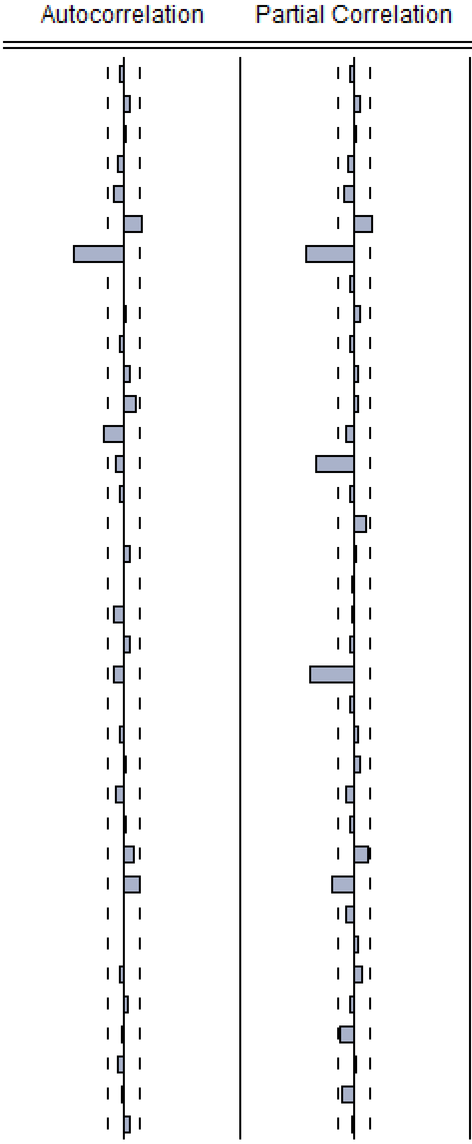


Figure 4: Difference of the Seasonal Differences



	AC	PAC	Q-Stat	Prob
1	-0.047	-0.047	0.3872	0.534
2	0.045	0.043	0.7396	0.691
3	0.017	0.021	0.7899	0.852
4	-0.060	-0.060	1.4175	0.841
5	-0.086	-0.093	2.7145	0.744
6	0.166	0.165	7.5991	0.269
7	-0.443	-0.437	42.815	0.000
8	-0.001	-0.025	42.815	0.000
9	0.023	0.056	42.913	0.000
10	-0.044	-0.037	43.259	0.000
11	0.063	0.039	43.980	0.000
12	0.114	0.033	46.389	0.000
13	-0.187	-0.072	52.886	0.000
14	-0.076	-0.341	53.966	0.000
15	-0.027	-0.044	54.103	0.000
16	-0.001	0.105	54.103	0.000
17	0.063	0.010	54.867	0.000
18	0.004	-0.021	54.870	0.000
19	-0.091	-0.014	56.486	0.000
20	0.048	-0.035	56.926	0.000
21	-0.082	-0.385	58.246	0.000
22	0.007	-0.043	58.256	0.000
23	-0.039	0.033	58.565	0.000
24	0.015	0.061	58.608	0.000
25	-0.070	-0.075	59.610	0.000
26	0.018	-0.042	59.674	0.000
27	0.085	0.124	61.150	0.000
28	0.154	-0.199	66.053	0.000
29	0.008	-0.076	66.065	0.000
30	0.002	0.027	66.066	0.000
31	-0.042	0.072	66.438	0.000
32	0.033	-0.044	66.674	0.000
33	-0.024	-0.119	66.799	0.000
34	-0.062	0.019	67.624	0.001
35	-0.019	-0.104	67.703	0.001
36	0.042	-0.020	68.094	0.001

Figure 5: Correlogram of the difference of the seasonal differences

Table 1: Estimation of the sarima(1,1,1)X(1,1,1)$_7$ model

Dependent Variable: DSDPEUR1
Method: Least Squares
Date: 06/08/16 Time: 17:56
Sample (adjusted): 17 178
Included observations: 162 after adjustments
Convergence achieved after 24 iterations
MA Backcast: 9 16

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	0.507523	0.125212	4.053304	0.0001
AR(7)	-0.108317	0.079121	-1.369006	0.1730
AR(8)	0.033697	0.094264	0.357476	0.7212
MA(1)	-0.465932	0.159644	-2.918576	0.0040
MA(7)	-0.948280	0.017852	-53.11995	0.0000
MA(8)	0.414262	0.140774	2.942753	0.0037

R-squared	0.523826	Mean dependent var	-8.09E-05
Adjusted R-squared	0.508564	S.D. dependent var	0.010002
S.E. of regression	0.007011	Akaike info criterion	-7.046207
Sum squared resid	0.007669	Schwarz criterion	-6.931852
Log likelihood	576.7428	Hannan-Quinn criter.	-6.999777
Durbin-Watson stat	2.034634		

Inverted AR Roots	.70-.30i	.70+.30i	.31	.19+.70i
	.19-.70i	-.43+.56i	-.43-.56i	-.71
Inverted MA Roots	1.00	.62-.77i	.62+.77i	.44
	-.22-.97i	-.22+.97i	-.89-.43i	-.89+.43i

Table 2: Estimation of the *additive-multiplicative* model

Dependent Variable: DSDPEUR1
Method: Least Squares
Date: 06/08/16 Time: 18:03
Sample (adjusted): 10 178
Included observations: 169 after adjustments
Convergence achieved after 18 iterations
MA Backcast: 2 9

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	0.476029	0.055180	8.626840	0.0000
MA(1)	-0.469752	0.088332	-5.318030	0.0000
MA(7)	-0.941714	0.019506	-48.27926	0.0000
MA(8)	0.411494	0.085219	4.828638	0.0000

R-squared	0.504452	Mean dependent var	-4.38E-05
Adjusted R-squared	0.495442	S.D. dependent var	0.009850
S.E. of regression	0.006997	Akaike info criterion	-7.063337
Sum squared resid	0.008078	Schwarz criterion	-6.989257
Log likelihood	600.8520	Hannan-Quinn criter.	-7.033274
Durbin-Watson stat	2.012919		

Inverted AR Roots	.48			
Inverted MA Roots	1.00	.62+.77i	.62-.77i	.44
	-.22+.97i	-.22-.97i	-.89-.43i	-.89+.43i

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.018	-0.018	0.0558	
		2	0.052	0.051	0.5178	
		3	-0.040	-0.039	0.8001	
		4	-0.078	-0.082	1.8679	
		5	-0.046	-0.045	2.2375	0.135
		6	0.121	0.128	4.8527	0.088
		7	-0.102	-0.102	6.7173	0.081
		8	-0.008	-0.036	6.7290	0.151
		9	0.040	0.057	7.0235	0.219
		10	-0.054	-0.041	7.5537	0.273
		11	0.024	0.009	7.6620	0.363
		12	0.079	0.066	8.8226	0.357
		13	-0.133	-0.113	12.090	0.208
		14	-0.146	-0.174	16.043	0.098
		15	-0.010	-0.004	16.061	0.139
		16	-0.023	0.024	16.159	0.184
		17	0.061	0.018	16.864	0.206
		18	-0.037	-0.097	17.124	0.250
		19	-0.048	-0.022	17.573	0.286
		20	0.022	0.048	17.663	0.344
		21	-0.064	-0.107	18.464	0.360
		22	0.009	0.004	18.482	0.424
		23	-0.047	-0.057	18.921	0.462
		24	0.028	0.021	19.073	0.517
		25	-0.086	-0.081	20.557	0.486
		26	0.019	-0.004	20.630	0.544
		27	0.056	0.066	21.266	0.565
		28	0.160	0.111	26.529	0.327
		29	0.027	0.005	26.675	0.372
		30	-0.009	-0.016	26.690	0.426
		31	-0.039	-0.003	27.013	0.463
		32	0.021	0.010	27.108	0.512
		33	0.041	0.050	27.473	0.546
		34	-0.053	-0.090	28.083	0.566
		35	0.052	0.055	28.662	0.587
		36	0.046	0.063	29.121	0.613

Figure 6: Correlogram of Sarima(1, 1, 1)X(0, 1, 1)$_7$ Resiiduals

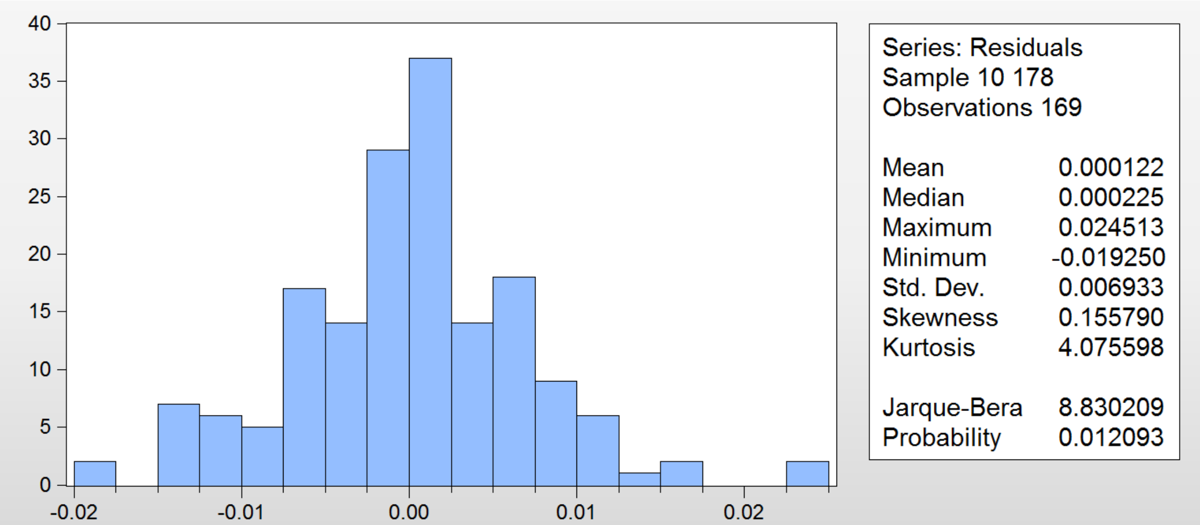


Figure 7: Histogram of the sarima(1,1,1)X(0,1,1)$_7$ residuals

Table 3: Out-of-sample comparison of forecasts with observations

Time	Observation	Forecast
8th June, 2016	1.2730	1.2635
9th June, 2016	1.2783	1.2619
10th June 2016	1.2675	1.2446
11th June 2016	1.2675	1.2392
12th June 2016	1.2646	1.2329
13th June 2016	1.2573	1.2333
14th June 2016	1.2587	1.2408
15th June 2016	1.2605	1.2237

Conclusion

The algorithm (4) is a merger of algorithms (2) and (3). It is therefore associated with a wider scope than either (2) or (3). This means that more time series would be analysed using this new algorithm than either earlier one. Corresponding notations and nomenclatures have been introduced.

The *additive-additive* SARIMA model fitted to the pound-euro exchange rates is shown to be adequate. The correlogram of its residuals in Figure 6 shows that the residuals are all uncorrelated. Besides their histogram in Figure 7 shows that at 1% level of significance the residuals are normally distributed.

The model (6) may therefore be used for the simulation and forecasting of the exchange rate series.

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