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## A brief discussion of characteristics of erosion in multi scale as well as iterative environment

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**Abstract** This paper discusses erosion in multiscale as well as iterative environment. The morphological operations are main source for defining composite morphological operations. These are applied in various image processing operations. In addition to them morphological operations are having special applications also. So the study of morphological operations in various environments will provide broad look of these operations, which throws light on understanding of these basic principles which will help in further new applications of this mathematical morphology. So in this paper a morphological operation is discussed in a new dimension.

**Keywords** erosion, dilation, open, close, multi scale, mathematical morphology, iterative morphology, valley, hole, island, peak.

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### Introduction

If we observe carefully, the human beings have the desire of recording incidents, through images. Their view may be for the purpose of future generation. Images also, played the role of symbols of languages, for communication purpose.

The early cavemen documented some of the incidents through images in the caves. They documented some of the incidents of their routine life, on stones, by using primitive tools. Important incidents such as battles, routine incidents such as food habits were recorded by them, on stones. These provide record, which is historically very important, of early human civilization. The images drawn by primitive tools by Egyptians, Indians, have provided a lot of valuable information, for historians, about civilizations.

After this, paints or inks were invented. The human beings started to record scenes, incidents through these paints and inks. Letter on J. B. Porta, an Italian Philosopher, during the II half of 18<sup>th</sup> century, by mean of an accidental discovery, was able to assemble a camera like equipment by mirrors and lens, which is the first step towards the modern day photography. At the same time a France scientist observed silver chloride characteristics with respect to light. After two centuries Alexander Charles extended above concept, and produced simple photo graphs.

After one century, at around 1835 Henry Fox Talbot extended above concepts, using silver nitrate, extended the design of camera, and modern photography was born from this experiment, which is presented in royal society.

This technology is used to record incidents of U.S. civil war, or, to record incidents of wealthy people, but not reached to a common man, due to complex chemical process, for the development of photographs till "KODAK" has entered in 1884. Later on research is done on motion pictures by Thomas A. Edison & William Kennedy Laurie Dickson, which is foundation for modern movie technology. Actually the first step for images processing was laid during Second World War. Technical experts, who are trained specially, are used to improve quality of image. They are specially trained in object recognition, they used to identity targets, manually. So, it is first step in image processing. After invention of digital computer, digital image processing came into existence. NASA, in early 1960's, got images from Space Crafts, Ranger 7, of the Lunar Surface, in thousands. These images were processed to minimize distortions. This is initial digital I.P. work, using a computer. This work was done in NASA's JET propulsion laboratory (JPL), in California.



This initial digital images processing work was very satisfactory. So, NASA continued it's funding, resulting in the development of digital image processing area.

The reduction in Hardware cost, mass production of chips, reduction in memory cost, reduction in size of computers, boosted the development of Digital Image Processing area.

So, researches in general have been showing interest and developed algorithms for image smoothening, edge enhancement, image compression, image segmentation, 2D to 3D conversion etc., Now a day, it is having applications from entertainment area to medical area. The detailed explanation is given in author's papers.

At the same time mathematical morphology emerged and developed separately, with some other interests and motivations. The purpose of this area is different. But later on, it is identified that the mathematical morphology is having very important applications in image processing. So, mathematical morphology is considered now, a very important branch of image processing.

Actually J. SERRA [1] and MATHERON [2] are founders of mathematical morphology. They have explained all the fundamentals of mathematical morphology in their books.

Actually the primitive operations are EROSION & DILATION. The composite operations are open and close. All these are explained in chapters 1 and 2. There are some more composite operations, like thinning, skeletonization etc. But the work is limited to erosion, dilation, open, close.

Mr. H.J.A.M. HEIJMANS has given a detailed discussion of these operations in 4. Till now the light is thrown on the fundamentals of mathematical morphology [1-4].

The morphological operations are suitable to apply on binary images only. Actually, applications of morphological operations were extended by SERRA also. Later STERNBERG concentrated in this area. In depth study was done (the theoretical analysis) by J.A.M HEIGMANS in this area. PETROS MARAGOS has discussed about morphology also. PETROS MARAGOS has discussed about morphology and given theoretical analysis.

For elimination or minimization of noise in the images a lot of research is done. The researchers developed algorithms for smoothening with detail preservation and for edge enhancement also. Some reserschers developed morphological algorithms for elimination of salt and pepper noise ,and impulse noise also. It has entered into medical area also the detailed references are available in the other papers of author [6-9].

The mathematical morphology has entered in to some more areas like soft morphology, fuzzy morphology, flat morphology etc. some of the work done by the author in soft morphology is referred in references.

### Definitions

The primitive morphological operations are dilation and erosion. By means of these operations only, all the remaining morphological operations may be defined. These two morphological operations play the role of bricks, for a house. Because this paper deal with erosion, in this section the definition of erosion is only given.

**Erosion:** This morphological operation also defined in so many ways, by different researchers.

Def 1):- The erosion of A by B is denoted by  $A \ominus B$ , and is defined by

$A \ominus B = \{x/x + b \in A \text{ for every } b \in B\}$  Here  $x \in E^N$  when  $E^N = N$  space.

Def 2):-  $A \ominus B = \{x/ \text{for every } b \in B, \text{ there exists and } a \in A, \text{ such that } x = a - b\}$

Def 3):-  $A \ominus B = \{x/ (B) x \subseteq A\}$ . Here A is image, B is S.E.

Here  $x \in E^N(B)_x$ : Translation of B by "x"

Def4):-  $A \ominus B = \bigcap_{b \in B} (A)_b$  Here "A" is the image and B is the S.E.

(A)  $(A)_b$ : Translation of A by b

(B) Def 5):-  $(I \ominus S)[x, y] = 1$  If  $|I \cap S_{(x,y)}| = |S|$

(C) = 0 other wise

(D) Here I is image and S is S.E.  $I(x, y)$  denotes image value at coordinate (x, y)

(E)  $|Z|$  denotes the cardinality, of the set Z.

(F)  $S_{(x,y)}$ : S translated by the displacement (x, y)

### Properties of Erosion

A) Erosion is Non – Commutative

$$A \ominus B \neq B \ominus A$$

B) Erosion is Non – Associative

$$A \ominus (B \ominus C) \neq (A \ominus B) \ominus C$$

C) Erosion is translation invariant

$$(A)_x \ominus B = (A \ominus B)_x$$

D) Erosion is having "INCREASING" property.

$$A \subseteq B \Rightarrow A \ominus D \subseteq B \ominus D.$$

$$A \subseteq B \Rightarrow D \ominus A \supseteq D \ominus B.$$

E) Erosion is anti-extensive when origin belongs to S.E.



[Original image contains the Eroded IM when origin  $\in$  S.E].

$$F) (A \cup B) \ominus C \supseteq (A \ominus C) \cap (B \ominus C)$$

$$G) A \ominus (B \cup C) = (A \ominus B) \cap (A \ominus C)$$

$$H) (A \cap B) \ominus C = (A \ominus C) \cap (B \ominus C)$$

$$I) (A \ominus (B \cup C)) = (A \ominus B) \cap (A \ominus C)$$

#### Characteristics of Erosion

- A) By applying Erosion, the size of the peaks will be reduced.
- B) The Erosion will increase the size of the valleys.
- C) If there are any holes, in the image, the size of the holes will be increased.
- D) The size of the Islands will be reduced.
- E) The shape of the corners will not be changed.
- F) By Erosion, the size of the image will be reduced.
- G) It will disconnect some particles (islands)

#### Multi Scale Environment

##### Discussion on Multi Scale Soft Morphology

In the process of understanding the objective world, the appearance of an object does not depend only on the object itself, but also on the scale that the observer used. It seems that appearance under a specific scale does not give sufficient information about the essence of the percept, we want to understand. If we use a different scale, to examine this percept, it will usually have a different appearance. So, this series of images and its changing pattern over scales reflect the nature of the percept.

The S.E. dimension can be anything. It depends upon situation, requirement, and context etc. It can be

$$\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \frac{5}{5}, \frac{6}{6}, \frac{7}{7}, \dots$$

In some situations, particularly square grid is chosen, it can be  $\frac{3}{3}, \frac{5}{5}, \frac{7}{7}, \frac{9}{9}, \frac{11}{11}, \frac{13}{13}, \dots$

The S.E.'s, having series, and in increasing size [like mentioned above] is called multi scale S.E.'s and the morphological approach (operations) dealing with multi scale S.E.'s is called multi scale morphology. As the size of the S.E. is more, its impact upon image will be more. For example, amount of expansion by applying dilation operation is more on an image, if we apply  $\frac{5}{5}$  S.E., compared to amount of expansion of image, by

dilating by  $\frac{3}{3}$  S.E.

##### Review on Multi Scale Soft Morphology

Till now, some amount of research is done in this area, and it is applied in so many areas. In mathematical morphology also, a new area multi scale mathematical morphology is developed, and applied in so many areas like smoothening, edge enhancement, analysis of radar imagery, remote sensing, medical image processing etc. PETROS MARAGOS entered into multi scale morphology, in addition to other areas. He explained about changes of shapes, as the scale is changed. He explained the applications of MSMM, and back ground mathematics. He explained about application of MSMM in skeletonization also. He extended these concepts to gray scale also.

MING – HUA CHEN & PING – GAN YAN explained Erosion, Dilation, Open, Close in multi scale environment, with diagrams (results), mathematical analysis, as well as symbolic conventions.

PAUL. T. JACKWAY etc. provided one type of analysis in MSMM. They discussed how to relate the results of one scale with the results at different scale. They have provided this analysis with good examples, using Erosion/Dilation morphological operations. KUN WANG etc. proposed an algorithm, for edge detection in the presence of Gaussian noise & salt – pepper noise in multi scale morphological environment. The experimental results are better than that of conventional algorithms. The same authors KUNWANG etc. proposed another algorithm for edge detection which will function better in Gaussian, salt - paper noise environment, in MS morphological approach.

KIM WANG and others discussed an edge detection algorithm, in multi scale environment, which is suitable to apply on brain MRI, in noisy environment.

ZENG PINGPING etc. proposed another algorithm, for edge enhancement (112) in multi scale morphological approach, using order morphology also, which is suitable to apply in noisy environment also. ZHEANHUA LI; & others discussed another technique for edge enhancement, in MS morphological environment.

PANCHAO WU & others proposed another algorithm, for edge detection in noisy environment using MS MM & WAVELET transforms.



GAO LI etc proposed an adaptive algorithm for edge detection of a color image (In HIS space) in MSMM environment. CHEN JIN LONG, etc. proposed another methodology for edge detection in multi structure and multi scale mathematical morphology environment.

HAI LONG HUANG etc. proposed an algorithm for suppression of noise and preserve edges using multi share and multi scale mathematical morphology environment. HAI LONG HUANG etc. proposed an algorithm for suppression of noise and preserve edges using multi share and multi scale structure elements using different directions and sizes of S.E.'s.

These MSMM techniques are extended to segmentation also. DEBAY LE, J. etc extended MSMM for segmentation using adaptive technique and MARC DROSKE etc. also used MSMM for segmentation. H UANG, R. etc. discussed extension of MSMM to 3D. They discussed and designed algorithm for volume segmentation. For this purpose, they have designed spherical S. E.'s at various sizes. LETITIA, S; etc. applied MSMM for road segmentation from satellite aerial images.

PAUL. T. JACKWAY etc. provide another type of analysis in MSMM. Naturally a few questions arise in MSMM, like how to relate the results of one scale with the results of other scale. This type of analysis is provided in this paper by Erosion/Dilation operations with good examples.

MSMM is having, application in medical area also. DA WEI QI etc. shown an application in medical I.P. for edge detection in noisy environment, which gives better results, compared to traditional pictures. FEI ZHANG etc., given another algorithm, suitable for ECG analysis, in impulse noise environment using MSMM. DAWEI QI proposed another algorithm, for medical analysis environment. ZA BI HI, S.M etc. discussed application of MSMM for retinal vessel segmentation. DAWEI QI etc HAI YAN GU; etc WEIPING HOU etc discussed the applications of MSMM in wood analysis they have done wood decay estimations, defect identification of wood, etc. RUJIANG HAO etc. used MSMM open operation for identification of defects of the rolling beatings. YING ZHANG etc. used MSMM to do analysis of results of turbine rotor experiment. In noise environment also, it provides good results [strong edges].The detailed references and exolanations are available in my other papers, which are given in reference.

#### **Iterative Environment:**

Iterative morphology means, applying one morphological operator, on an image a few no of times. These morphological operators may have same S.E or different S.E's or same S.E with different dimensions. Iterative morphology is having its own importance. It is having so many applications in so many areas.

Iterative morphology appears in skeletonization process. In an algorithm for skeletonization erosion has to be applied, a few no of times. In thinning also, iterative morphology will appear. A Structuring Element has to be applied so many times, on an image; [Each time the Structuring Element, will be rotated]. Same case in thickening also. Thickening also uses iterative morphological concept.

In some situations, multi scale iterative concept will appear. In multi scale skeletonization, S.E. will be applied at various dimensions, each time upon an image, to get skeletons at various dimensions.

In the previous section, S.E. decomposition is discussed. A S.E. will be divided into series of mini S.E,'s. All these S.E.'s will be applied on the image one after the other as a series or these can be applied on the image simultaneously in parallel computing environment. Any way structuring element decomposition deal with iterative morphology. The S.E. may be decomposed into mini S.E's, with dimensions in increasing order. So, S.E decomposition can be in iterative environment and multi scale environment also.

CHANF – CHEF CHAOUNI, M etc developed a process for convergence criteria, in iterative morphology. In iterative morphology, the system has to go towards a better solution. It is discussed in this paper [10]. XIA – YONG and others proposed an algorithm in iterative morphological environment, for segmentation, using multi fractal estimation concept, which FS suitable to be applied in remote sensing data. (Of course, this algorithm can be applied, in other images [11] also). ROBIN, F. & others, designed H.W. SYSTEM, to implement morphological filters (operations) iteratively [12]. It provides a real time processing environment.

ONGWATTANA KUL. S. & others also developed implementation of iterative morphology, in H.U. point of view [13]. SHIH, F.Y. & others [14] proposed an algorithm in iterative morphological environment (Applying iterative erosion) for EUCLIDEAN distance transformations. AMAYEH, G & others [15] proposed algorithm in iterative morphological environment for hand verification.

JIWEI YUAN & others, [16] discussed methodology for segmentation technique, which is suitable to apply on sequence of images of traffic scenes. YONG XIA & others [17], discussed methodology using multi fractal estimation technique for image segmentation using iterative dilation scheme, (with a series of cubic S.E's) with reduction in computational complexity. SAARINEN, K discussed [18] segmentation, in color environment, using watershed technique in iterative morphological environment.



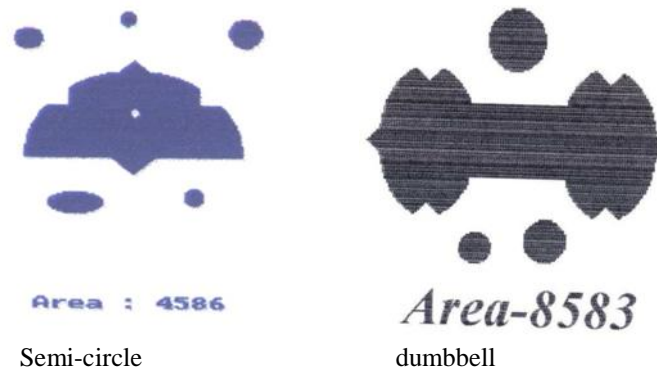
AUPIGITER. R. also discussed [19] segmentation using iterative watershed algorithm in 3D environment, which is suitable for medical image processing. ZHANG XIAO – JING & others discussed [20] segmentation using watershed algorithm, to be applied in medical area, using iterative erosion technique. ZHUANG, H; & others – [21] discussed methodology for smoothing (for the treatment of impulse Gaussian noises) using iterative close – open technique. SKOLNICK, M.M etc [22] discussed determination of centroids using iterative morphology. J.G.POSTAIRE & others [23] discussed the role of iterative morphology in cluster analysis. CHIVERTON, J.P. & others, discussed [24] about applications. Of iterative morphology in medical image processing, regarding neurological analysis which is very important. Some researchers like JIEKANG used iterative morphology, in CB morphological environment, on SAR images, for image Smoothing.

### Results and Discussions

The importance of multi scale as well as iterative morphology is explained in the above sections. So, the primitive operations of mathematical morphology--erosion and dilation are taken and they are discussed in multi scale morphology point of view as well as iterative morphology point of view. The concept is very simple. But some of the important points are elaborated practically with the help of a few images and some important observations are given.

In this section, the results of experiments are presented. Actually two diagrams are taken, a Semi circle shape and a dumbbell shape. On these images various morphological operations are applied. The output is got in the form of tables, diagrams and graphs, around 1000 pages. But here some important as well as samples outputs are presented, relevant to this work.

Images: In this section, the Semi circle and dumbbell images are presented.



### Erosion in multi scale as well as iterative environment:

The erosion operation is applied in multi scale environment as well as iterative environment on semi circle and dumbbell images. In this paper the results applied on semi circle are only presented.

In this section the results are given in the form of images and tables.

Fig – 1 will give eroded images of semi-circle in 3/3 window environment.

Fig – 2 will give eroded images of semi-circle in 5/5 window environment.

Fig – 3 will give eroded images of semi-circle in 7/7 window environment.

Fig – 4 will give eroded images of semi-circle in 9 /9 window environment.

Fig – 5 will give eroded images of semi-circle in 11/11 window environment.

Fig – 6 will give eroded images of semi-circle in 13/13 window environment.

Fig – 7 will give eroded images of semi-circle in 15/15 window environment.

**Table–A** will provide the information of above eroded images in multi scale environment. The G–1 will provide above information graphically.





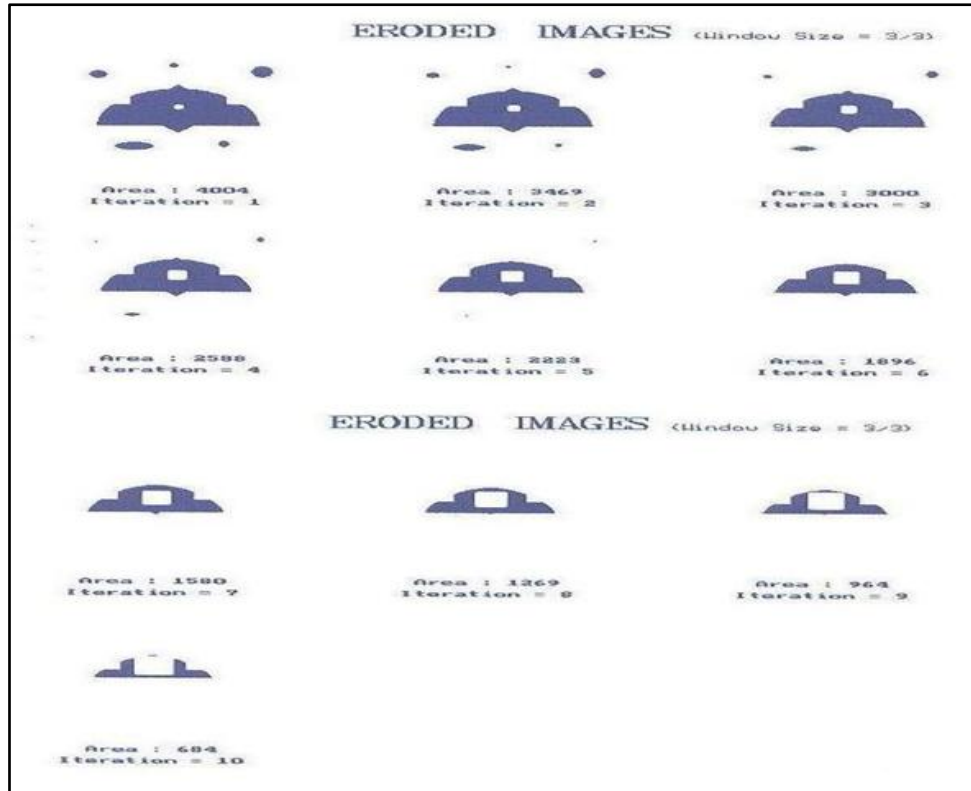


Figure: 1



Figure: 2

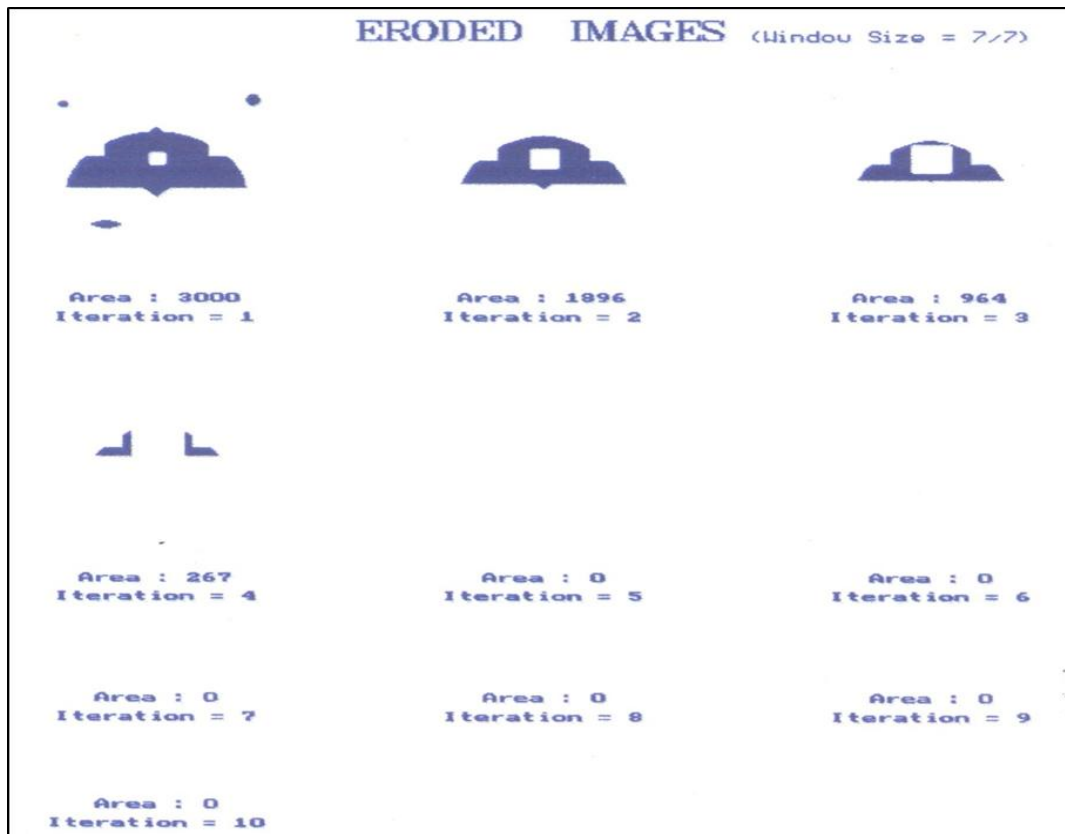


Figure: 3

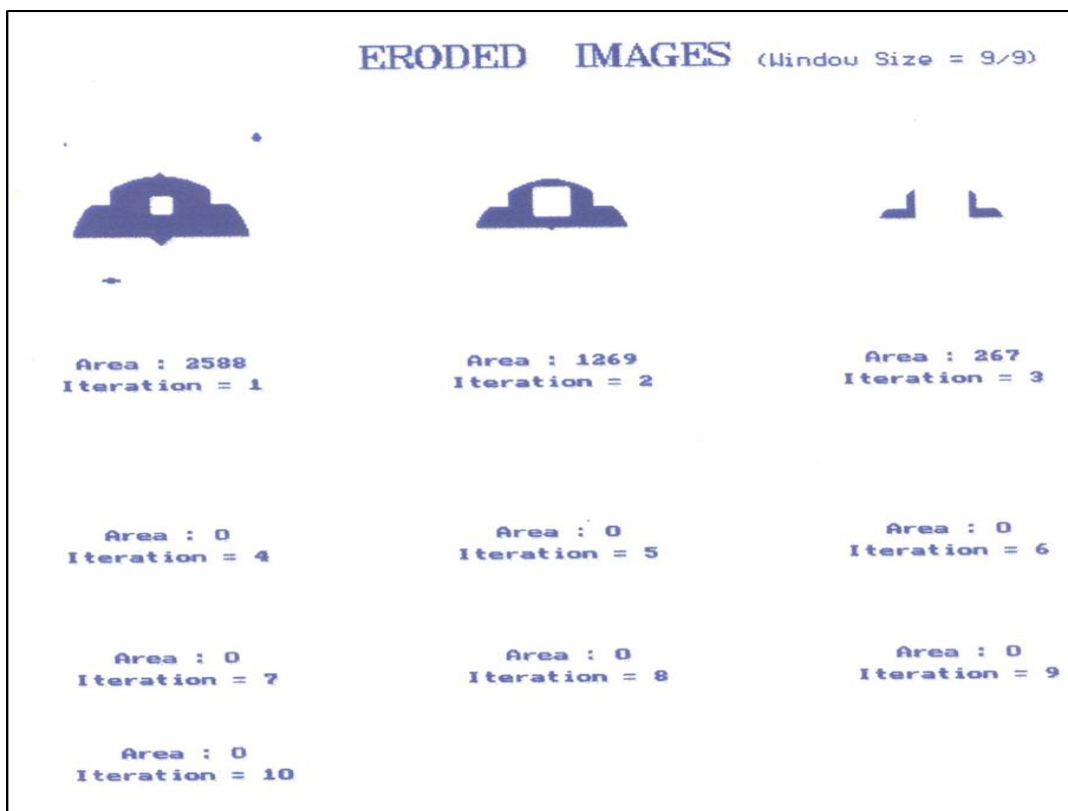
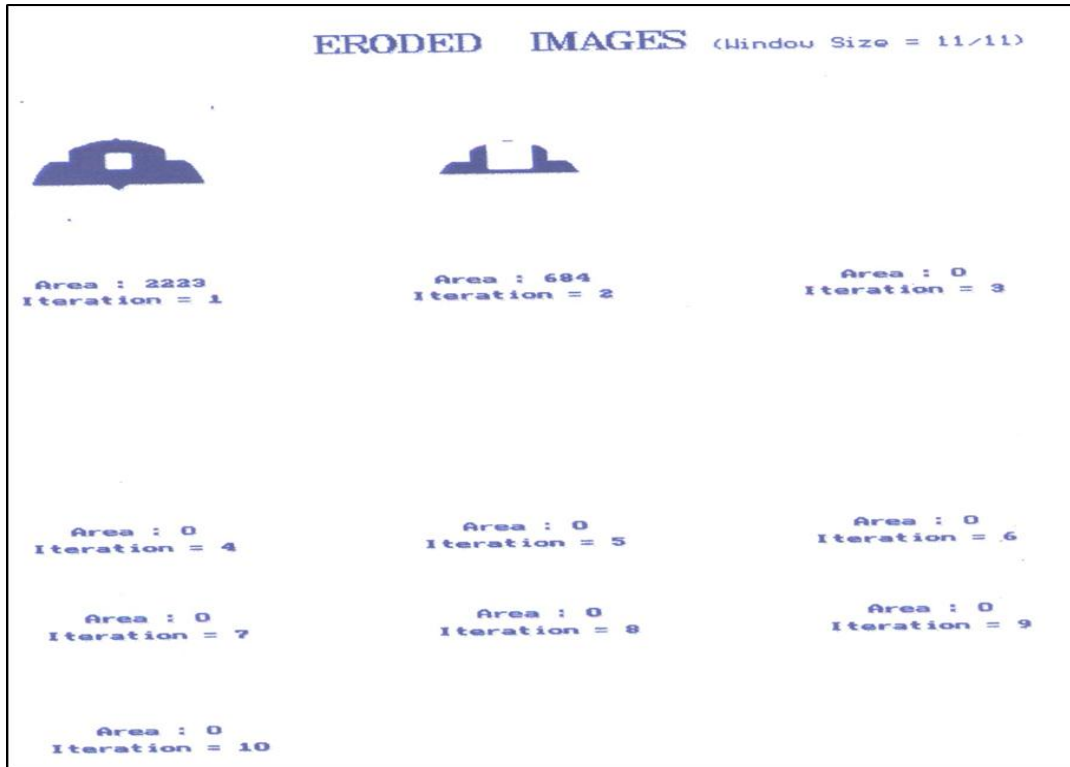


Figure: 4



Figur: 5

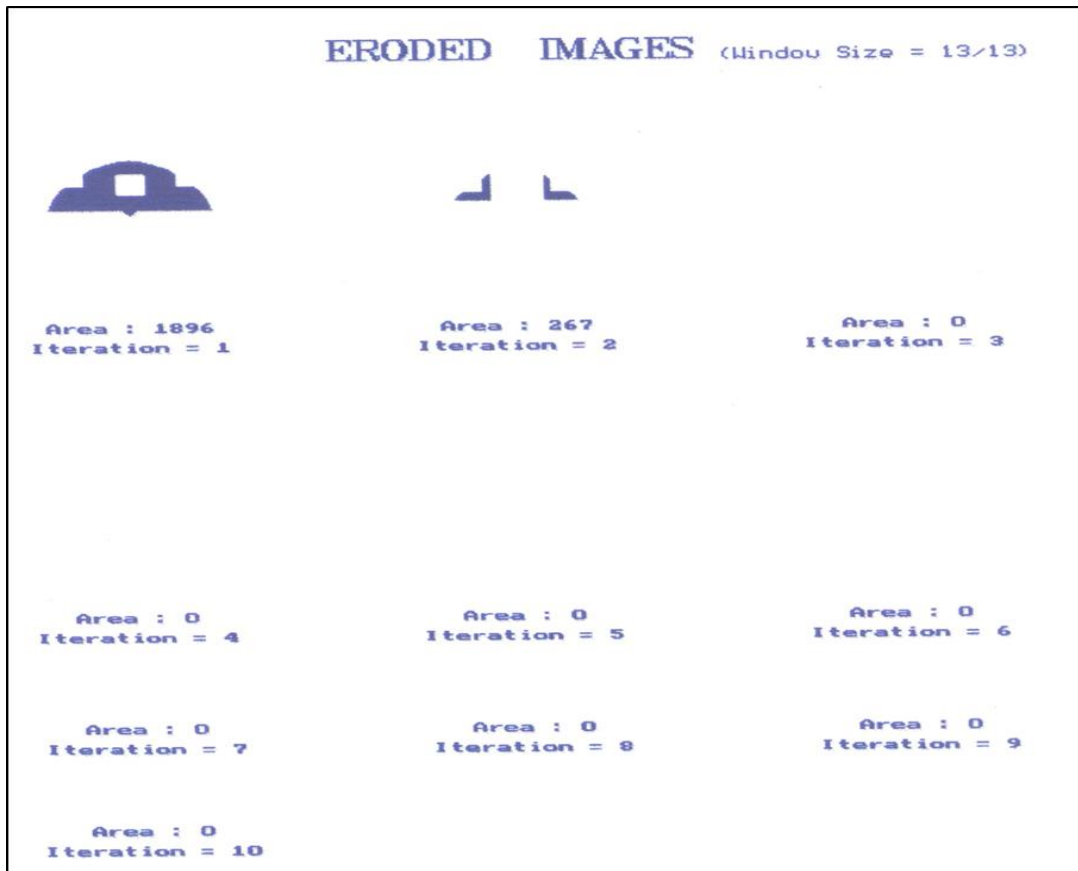


Figure: 6



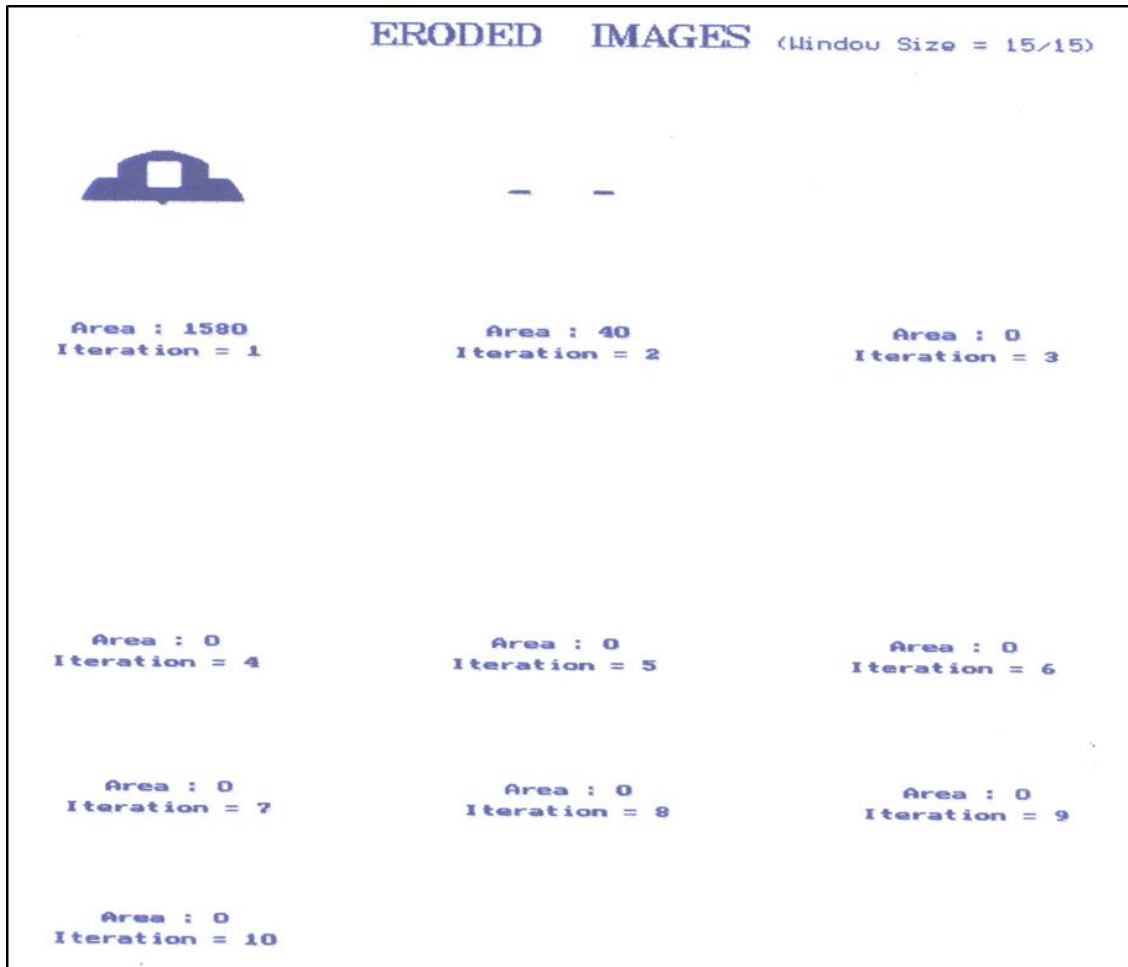


Figure: 7

**Table: A**  
**Areas for Eroded Images**

Iteration	Area						
	Window Sizes						
	3/3	5/5	7/7	9/9	11/11	13/13	15/15
1	4004	3469	3000	2588	2223	1896	1580
2	3469	2588	1896	1269	684	267	40
3	3000	1896	964	267	0	0	0
4	2588	1269	267	0	0	0	0
5	2223	684	0	0	0	0	0
6	1896	267	0	0	0	0	0
7	1580	40	0	0	0	0	0
8	1269	0	0	0	0	0	0
9	964	0	0	0	0	0	0
10	684	0	0	0	0	0	0

The above results are presented in the form of a graph (Figure: 8).

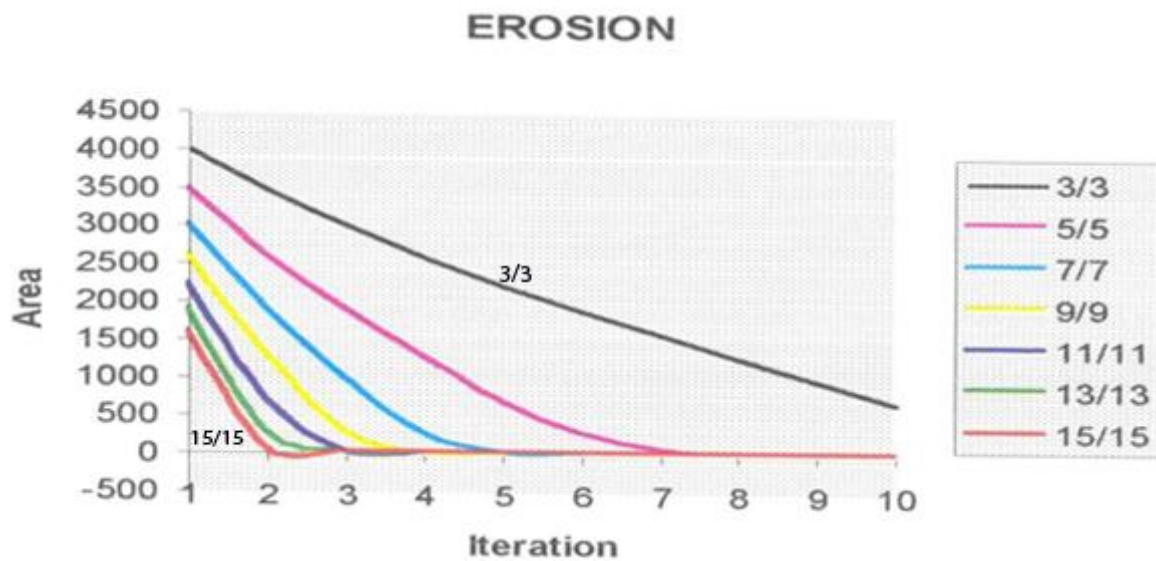


Figure: 8

#### Observations:

- As the size of the S.E. is enhanced, the size of the image is decreased. If the table is observed carefully, the image may vanish at 21/21 or 23/23.
- The amount of decrease is almost same as the S.E. size is enhanced.
- In the case of iteration also, the rate of decrease is almost same, for each iteration.
- If S.E. size is fixed, and iterations are enhanced, at a particular iteration, the image will vanish. That particular iteration depends upon the size of the image. This given semi circle may vanish, around 13th iteration for S.E. size 3/3. But for 5/5 S.E. size, it is going to vanish at iteration eight. And for 7/7 it is going to vanish at iteration 5. For 9/9, it is going to vanish at iteration 4. In this way the no of iterations are decreased for vanishing of images, as the S.E. size is enhanced.
- As the no of iteration is enhanced, the size of the hole of the image will be enhanced.
- As the size of the S.E. is enhanced, the size of the hole of the image also will be enhanced.
- As the no of iteration is enhanced, the size of the peak also will be reduced.
- The reduction rate will be enhanced, as the size of the S.E. is enhanced.
- The size of the valley will be enhanced, as the no of iteration is enhanced.
- The size of the valley is enhanced as the S.E. size is enhanced.
- The size of island will be decreased, as the no of iteration is increased. They may vanish after a few iterations. Any way, it depends upon size of islands also.
- The size of island will be decreased as the size of S.E. is enhanced. If the size of the island is small it will vanish also as the size of the S.E. is enhanced.
- For the bigger size of the S.E., the island will vanish even for one iteration.
- The bigger islands may continue, even though, the size of the S.E. is enhanced. It is up to some size of S.E. only. For a bigger size of S.E., even the bigger island also has to vanish.

#### Conclusions

In this paper erosion is studied with reference to size of the image, peaks, valleys, holes etc. in multi scale as well as iterative environment. This paper will discuss characteristics of erosion in some particular environment. This type of thorough discussion is not done, by the researchers, especially in iterative environment. There are so many applications of iterative morphology. The understanding of the characteristics of erosion will help in this application. New applications also may be invented by researchers, by understanding of these characteristics. For the understanding of any concept or device or an operation, the understanding of characteristics is very important. This paper will serve this important purpose.

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