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Effect of variable suction on unsteady MHD oscillatory flow of jeffrey fluid in a horizontal channel with heat and mass transfer

Joseph KM, Magaji AS, Peter A, Tijani NZ

Department of Mathematical Sciences, Kaduna State University, Nigeria

Abstract The effect ofvariable suction on unsteady MHD oscillatory flow of Jeffrey fluid in a horizontal channel with heat and mass transfer has been studied. The temperature prescribed at plates is uniform and asymmetric. A perturbation method is employed to solve the momentum and energy equations. The effects of various dimensionless parameters on velocity and temperature profiles are considered and discussed in details through graphs. It is found that, the velocity u increases with increase in $h_2, \alpha_1, \lambda_1, Gr, Gc, N, Re\ and\ Sc$. The velocity also increases with decrease in $h_1, \alpha_2, Ha, and\ K_c$. It is also observed that the temperature θ increases with increase in $N\ and\ Pe$. Increase in Schmidt number Sc and chemical parameter K_c respectively increase and decrease the species concentration or the concentration boundary layer thickness of the flow field.

Keywords Heat and Mass Transfer, Unsteady, MHD, Jeffrey Fluid, Oscillatory Flow, Variable Suction

1. Introduction

The effect of heat and mass transfer on unsteady MHD oscillatory flow of Jeffrey fluid in horizontal media with variable suction are encountered in a wide range of engineering and industrial applications such as molten iron flow, recovery extraction of crude oil, geothermal systems. Many chemical engineering processes like metallurgical and polymer extrusion processes involve cooling of a molten liquid being stretched in cooling systems; the fluid mechanical properties of penultimate product depend mainly on the cooling liquid used and the rate of stretching. Some polymers fluids like polyethylene oxide and polysobutylene solutions in a cetane, having better electromagnetic properties are normally used as cooling liquid as their flow can be regulated by external magnetic fields in order to improve the quality of the final product. Also, the radiative heat transfer is an important factor of thermodynamics of very high temperature systems such as electric furnaces, solar collectors, storage of nuclear wastes packed bed catalytic reactors, satellites, steel rolling, cryogenic engineering etc. Mass transfer processes are evaporation of water from a pond to the atmosphere, the diffusion of chemical impurities in lakes, rivers and ocean from natural or artificial sources.

The study of such flow under the influence of magnetic field and heat transfer has attracted the interest of many investigators and researchers.

Asadullah et al (2013) consider the MHD flow of a Jeffrey fluid in converging and diverging channels [1]. The flows between non parallel walls have a very significant role in physical and biological sciences.

Kavita et al (2012) investigated the influence of heat transfer on MHD oscillatory flow of Jeffery fluid in a Channel [2]. They found out that an analysis of first order homogeneous chemical reaction and heat source on MHD oscillatory flow of viscous–elastic fluid through a channel filled with saturated porous medium are reported by Devika et al (2013) [3].

An oscillatory flow of a Jeffrey Fluid in an elastic tube of variable cross – section has been investigated at low Reynolds number by Badari et al (2012) [4]. Their main concentration is on the excess pressure of the tube. The equation has been solved numerically and investigations are made for different cases on the tube.

Kumari et al (2012) studied the effect of heat transfer on MHD oscillatory flow of Jeffrey fluid in a channel with slip effect at a lower wall where the expressions for the velocity and temperature are obtained analytically.

Unsteady Flow of a Jeffrey Fluid in an elastic tube with stenosis was investigated by Sreenadh *et al* [5]. The governing equation for the excess pressure is obtained for Jeffrey model. The governing equations was solved numerically and investigations are made for different cases for strength, tapered and constricted tubes.
An oscillatory flow of a Jeffrey fluid in an elastic tube of variable cross section at low Reynolds number has been investigated by Badari et al (2012) [4]. The main concentration is on the excess pressure of the tube. The equations have been solved numerically.
Hayat et al (2012) analyzed the effects of Newtonian heating and magnetohydrodynamic (MHD) in a flow of Jeffrey fluid in stagnation point of view over radially stretching surface [6].
The effect of heat transfer through radiation on velocity, magnetic and temperature fields in the case of two dimensional hydromagnetic oscillatory flow of a viscous incompressible and electrically conducting fluid past a porous, a limiting surface, subjected to variable suction and moving impulsively with a constant velocity in the presence of transverse magnetic field has been analyzed.
Idowu et al (2013) studied the effect of heat and mass transfer on unsteady MHD oscillatory flow of Jeffrey fluid in a horizontal channel with chemical reaction. They found that the velocity is more of Jeffrey fluid [7].
The effect of heat and mass transfer on MHD oscillatory flow of Jeffrey fluid with variable viscosity through porous medium was investigated. He found out that the velocity for Jeffrey fluid with variable viscosity is less than velocity of Jeffrey fluid zith constant velocity. This result is in agreement with Kavita (2012) [2].
This present work focused oneffect ofvariable suction on unsteady MHD oscillatory flow of Jeffrey fluid in a horizontal channel with heat and mass transfer.This study is expected to help in understanding the concept of Jeffrey fluid as a non – Newtonian fluid and the effect of heat and mass on unsteady oscillatory flow of Jeffrey fluid with variable suction and slip flow parameters.
The study have potential applications in oil recovery, filtration systems, geophysical, astrophysical, cosmical studies and in medicine especially in finding remedy for atherosclerosis and several applications.

2. Problem Formulation

We consider the flow of Jeffrey fluid in horizontal infinite parallel plates channel. The channel width is h.
The constitute equation for S Jeffrey fluid [2] is

$$S = \frac{\mu}{1+\lambda_1}\left(\frac{d\eta}{dt'} + \lambda_2 \frac{d^2\eta}{dt'^2}\right) \tag{2.1}$$

Where μ is the dynamic viscosity, λ_1 is the ratio of relaxation to retardation times, λ_2 is the retardation time and $\frac{d\eta}{dt'}$ is the shear rate.
Now, let u' be the velocity of the fluid in x' direction taken along the horizontal infinite parallel plate channel under a chemical reaction with species concentration $C'$and $y' - axis$ is taken normal to the direction of flow. The radiative heat term in x' direction is considered negligible in comparison with y' direction. The upper and lower plates are kept at $T' = T'_\infty$ and $T' = T'_w$ respectively. The Cartesian coordinate system and the flow configuration are shown in figure 1. At a time $t' > 0$, the plate is given an impulsive motion in a horizontal direction with uniform mean velocity U. Moreover at this stage an unsteady component $\varepsilon T' e^{iwt'}$ where $\varepsilon \ll 1$ is the amplitude of oscillation, is assumed to be superimposed on the temperatures of the plates.

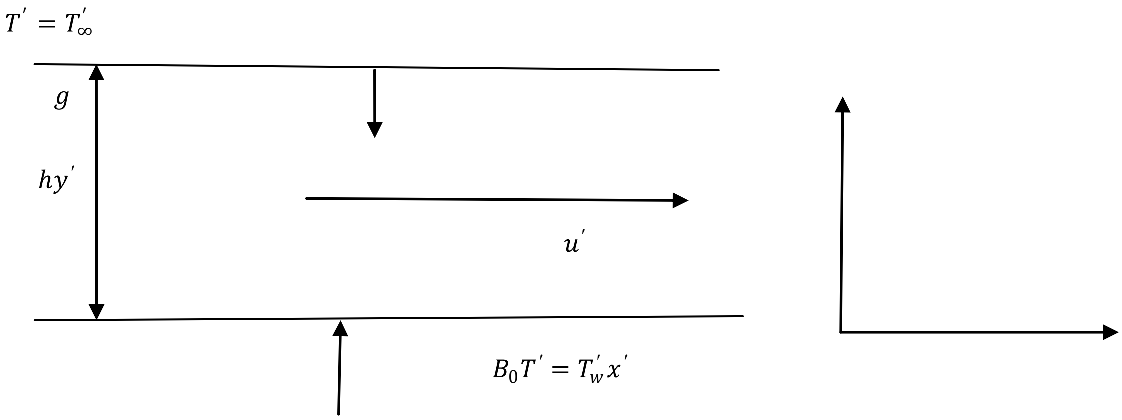

Figure 1: Physical schematic of the flow configuration

A magnetic field of uniform strength B_0 is applied normal to the plate along x' direction and the induced magnetic field is assumed negligible. It is also assumed that the fluid has small electrical conductivity and the electromagnetic force produced is very small. The pressure gradient is also assumed negligible. Since the plate

is considered infinite in $x^{'}$ direction, all the physical quantities will be independent of $x^{'}$. Under these assumptions, the physical variables are functions of $y^{'}$ and $t^{'}$ only.
The time dependent suction is

$$V^{'}(t) = -V_0^{'}(1 + \varepsilon A e^{i\omega t}) \quad (2.2)$$

The basic equations of momentum, energy and species concentration govern such a flow, subject to Boussinesq approximation, are

$$\frac{\partial u^{'}}{\partial t^{'}} - V_0^{'}(1 + \varepsilon A e^{i\omega t})\frac{\partial u^{'}}{\partial y^{'}} - \frac{\partial p}{\partial x^{'}} + \frac{\mu}{\rho(1+\lambda_1)}\frac{\partial^2 u^{'}}{\partial y^{'2}} - \frac{\sigma B_0 u^{'}}{\rho} + g\beta(T^{'} - T_\infty^{'}) + g\beta^{'}(C^{'} - C_\infty^{'}) \quad (2.3)$$

$$\frac{\partial T^{'}}{\partial t^{'}} - V_0^{'}(1 + \varepsilon A e^{i\omega t})\frac{\partial u^{'}}{\partial y^{'}} = \frac{k}{\rho c_p}\frac{\partial^2 T^{'}}{\partial y^{'2}} - \frac{1}{\rho c_p}\frac{\partial q}{\partial y^{'}} \quad (2.4)$$

$$\frac{\partial C^{'}}{\partial t^{'}} - V_0^{'}(1 + \varepsilon A e^{i\omega t})\frac{\partial u^{'}}{\partial y^{'}} = D\frac{\partial^2 C^{'}}{\partial y^{'2}} - K_c^{'}(C^{'} - C_\infty^{'}) \quad (2.5)$$

The q in (2.4) is called the radiative heat flux. It is given by

$$\frac{\partial q}{\partial y^{'}} = 4\alpha^2(T_w^{'} - T^{'}) \quad (2.6)$$

Where, $u^{'}$ is the axial velocity, $T^{'}$ is the fluid temperature, ρ is the fluid density, λ_1 is the ratio of relaxation to retardation times, B_0 is the magnetic field strength, σ is the conductivity of the fluid, g is the acceleration due to gravity, β is the coefficient of volume expansion due to temperature,β' the coefficient of volume expansion due to species concentration,c_p is the specific heat at constant pressure, k is the thermal conductivity, D is the mass diffusion coefficient, K_c is the chemical reaction parameter.
The boundary conditions are given by

$$u^{'} - U_1 = L_1\frac{\partial u^{'}}{\partial y^{'}}, \quad T^{'} = T_w^{'}, \; C^{'} = C_w^{'} \quad at\; y^{'} = 0,\; t^{'} \le 0$$

$$u^{'} - U_2 = L_2\frac{\partial u^{'}}{\partial y^{'}}, \quad T^{'} = T_\infty^{'}, \; C^{'} = C_\infty^{'} \quad at\; y^{'} = h,\; t^{'} \ge 0 \quad (2.7)$$

In order to write the governing equations and the relevant boundary conditions in non – dimensional form, the following dimensionless quantities are introduced

$$x = \frac{x^{'}}{h},\quad y = \frac{y^{'}}{h},\quad u = \frac{u^{'}}{V_0},\quad \theta = \frac{T^{'} - T_w^{'}}{T_\infty^{'} - T_w^{'}},\quad t = \frac{t^{'}V_0}{h},\quad Ha^2 = \frac{\sigma h^2 B_0^2}{\mu},\quad Gr = \frac{\rho h^2 g\beta(T_\infty^{'} - T_w^{'})}{\mu V_0},$$

$$Gc = \frac{\rho h^2 g\beta^{'}(T_\infty^{'} - T_w^{'})}{\mu V_0}\; `Re = \frac{\rho h V_0}{\mu},\quad Pe = \frac{\rho h V_0 c_p}{k},\quad N^2 = \frac{4\alpha^2 h^2}{k},\; K_c = \frac{K_c^{'}\nu}{V_0},\; \alpha_1 = \frac{U_1}{V_0},\; \alpha_2 = \frac{U_2}{V_0},\; h_1 = \frac{L_1 V_0}{\nu},$$

$$h_2 = \frac{L_2 V_0}{\nu}$$

$$Sc = \frac{U}{D},\; Gc = \frac{\rho h^2 \beta^{'}(C_\infty^{'} - C_w^{'})}{\mu V_0} \quad (2.8)$$

The momentum equation (2.2), the energy equation (2.3) and the species concentration equation (2.4) now become

$$Re\frac{\partial u}{\partial t} - Re(1 + \varepsilon A e^{i\omega t}) = \frac{1}{1+\lambda_1}\frac{\partial^2 u}{\partial y^2} - Ha^2 u + Gr\theta + GcC \quad (2.9)$$

$$Pe\frac{\partial \theta}{\partial t} - Pe(1 + \varepsilon A e^{i\omega t}) = \frac{\partial^2 \theta}{\partial y^2} - N^2\theta \quad (2.10)$$

$$Sc\frac{\partial C}{\partial t} - Sc(1 + \varepsilon A e^{i\omega t}) = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} - K_c C \quad (2.11)$$

The boundary conditions in dimensionless form are

$$u = \alpha_1 + h_1\frac{\partial u}{\partial u},\quad \theta = 0,\quad C = 0 \quad at\; y = 0,\; t \le 0$$

$$u = \alpha_2 + h_2\frac{\partial u}{\partial u},\quad \theta = 1,\quad C = 1 \quad at\; y = 0,\; t \ge 0 \quad (2.12)$$

3. Solution of the Problem

To solve equations (2.9) – (2.11) subject to the boundary conditions (2.12), we expand the functions $u(y,t) and\; \theta(y,t)$ as a power series in the perturbative parameterε. Here we assumed small amplitude of oscillations($\varepsilon \ll 1$), thus

$$u(y,t) = u_0(y) + \varepsilon u_1(y)e^{i\omega t} + o(\varepsilon^2) \quad (3.1)$$

$$\theta(y,t) = \theta_0(y) + \varepsilon \theta_1(y)e^{i\omega t} + o(\varepsilon^2) \quad (3.2)$$

$$C(y,t) = C_0(y) + \varepsilon C_1(y)e^{i\omega t} + o(\varepsilon^2) \quad (3.3)$$

The first terms in equations (3.1), (3.2) and (3.3) are called the harmonic terms while the second terms are called the non – harmonic terms.
Substituting equations (3.1), (3.2) and (3.3) into equations (2.9), (2.10) and (2.11), equating the coefficients of the harmonic and non – harmonic terms and neglecting the coefficients of ε^2, we get:

$$au_0''(y) + Reu_0^{'} - Ha^2 u_0(y) = -Gr\theta_0(y) - GcC_0(y) \quad (3.4)$$

$$au_1''(y) + Reu_1^{'} - b^2 u_1(y) = -AReu_0^{'}(y) - Gr\theta_1(y) - GcC_1(y) \quad (3.5)$$

Where, $a = \frac{1}{1+\lambda_1}$ and $b^2 = Ha^2 + i\omega Re$

$\theta_0''(y) + Pe\theta_0'(y) + \theta_0(y) = 0$ (3.6)

$\theta_1''(y) + a_3\theta_1'(y) + a_2\theta_1(y) = -APe\theta_0'(y)$ (3.7)

Where, $a_2 = N^2 - i\omega Pe$ and $a_3 = i\omega Pe$

$C_0''(y) + ScC_0'(y) - KcScC_0(y) = 0$ (3.8)

$C_1''(y) + ScC_1'(y) - a_4C_1(y) = -AScC_0'(y)$ (3.9)

Where, $a_4 = K_cSc + iwSc$

The corresponding boundary conditions become

$u_0 = \alpha_1 + h_1\frac{\partial u_0}{\partial y}, u_1 = h_1\frac{\partial u_1}{\partial y}, \theta_0 = 0, C_0 = 0, \theta_1 = 0, C_1 = 0 \ \ at \ \ y = 0, \ t \le 0$

$u_0 = \alpha_2 + h_2\frac{\partial u_0}{\partial y}, \ u_1 = h_2\frac{\partial u_1}{\partial y}, \theta_0 = 1, C_0 = 1, \theta_1 = 0, C_1 = 0 \ at \ \ y = 1, \ t \ge 0$ (3.10)

We now solved equations (3.4) – (3.9) under the relevant boundary conditions (3.10) for the mean flow and unsteady flow separately.

The mean flows are governed by the equations (3.4), (3.6) and (3.8) where u_0, θ_0, C_0 are respectively called the mean velocity, mean temperature and mean species concentration. The unsteady flows are governed by equations (3.5), (3.7) and (3.9) where u_1, $\theta_1$and C_1 are the unsteady components.

These equations are solved analytically under the relevant boundary conditions (3.10) as follows;

Solving equations (3.6) and (3.8) subject to the corresponding relevant boundary conditions in (3.10), we obtain the mean temperature as

$\theta_0(y) = A_1e^{m_1y} + A_2e^{m_2y}$ (3.11)

$C_0(y) = A_5e^{m_5y} + A_6e^{m_6y}$ (3.12)

Similarly, solving equations (3.7) and (3.9) under the relevant boundary conditions in (3.10), the unsteady temperature becomes

$\theta_1(y) = A_3e^{m_3y} + A_4e^{m_4y} + K_1e^{m_1y} + K_2e^{m_2y}$ (3.13)

$C_1(y) = A_7e^{m_7y} + A_8e^{m_9y} + K_3e^{m_5y} + K_4e^{m_6y}$ (3.14)

Putting equations (3.11), (3.12) and equations (3.13) and (3.14) into equations (3.4) and (3.5) respectively and using the corresponding boundary conditions in (3.10), we obtain the mean velocity $u_0(y)$ and the unsteady velocity component $u_1(y)$ as follows;

$u_0(y) = A_9e^{m_9y} + A_{10}e^{m_{10}y} + K_5e^{m_1y} + K_6e^{m_2y} + K_7e^{m_5y} + K_8e^{m_6y}$ (3.15)

$u_1(y) = A_{11}e^{m_{11}y} + A_{12}e^{m_{12}y} + K_9e^{m_1y} + K_{10}e^{m_2y} + K_{11}e^{m_3y} + K_{12}e^{m_4y} + K_{13}e^{m_5y} + K_{14}e^{m_6y} + K_{15}e^{m_7y} + K_{16}e^{m_8y} + K_{17}e^{m_9y} + K_{18}e^{m_{10}y}$ (3.16)

Therefore, the solutions for the velocity and temperature profiles are

$u(y,t) = A_9e^{m_9y} + A_{10}e^{m_{10}y} + K_5e^{m_1y} + K_6e^{m_2y} + K_7e^{m_5y} + K_8e^{m_6y} + \varepsilon[A_{11}e^{m_{11}y} + A_{12}e^{m_{12}y} +$
$K9em1y+K10em2y+K11em3y+K12em4y+K13em5y+K14em6y+K15em7y+K16em8y+K17em9y+$
$K18em10yei\omega t$ (3.17)

$\theta(y,t) = A_1e^{m_1y} + A_2e^{m_2y} + \varepsilon[A_3e^{m_3y} + A_4e^{m_4y} + K_1e^{m_1y} + K_2e^{m_2y}]e^{i\omega t}$ (3.18)

$C(y,t) = A_5e^{m_5y} + A_6e^{m_6y} + \varepsilon[A_7e^{m_7y} + A_8e^{m_9y} + K_3e^{m_5y} + K_4e^{m_6y}]e^{i\omega t}$ (3.19)

Where, $m_1 = \frac{-Pe+\sqrt{Pe^2-4N^2}}{2}$, $m_2 = \frac{-Pe-\sqrt{Pe^2-4N^2}}{2}$,$A_1 = \frac{1}{e^{m_1}-e^{m_2}}$

$A_2 = \frac{-1}{e^{m_1}-e^{m_2}}$, $m_3 = \frac{-a_3+\sqrt{a_3^2-4a_2^2}}{2}$,$m_4 = \frac{-a_3-\sqrt{a_3^2-4a_2^2}}{2}$

$A_{51} = -AA_1m_1Pe$, $A_{61} = -AA_2m_2Pe$, $K_1 = \frac{A_{51}}{m_1^2+a_3m_1+a_2}$

$K_2 = \frac{A_{61}}{m_2^2+a_3m_2+a_2}$, $a_5 = -(K_1 + K_2)$, $l_1 = -(K_1e^{m_1} + K_2e^{m_2})$

$l_2 = l_1 - a_5e^{m_2}$, $A_3 = \frac{l_2}{e^{m_3}-e^{m_4}}$, $A_4 = a_5 - A_3$, $m_5 = \frac{-Sc+\sqrt{Sc^2+4KcSc}}{2}$

$m_6 = \frac{-Sc-\sqrt{Sc^2+4KcSc}}{2}$, $A_5 = \frac{1}{e^{m_5}-e^{m_6}}$, $A_6 = -A_5$, $m_7 = \frac{-Sc+\sqrt{Sc^2+4a_4}}{2}$

$m_8 = \frac{-Sc-\sqrt{Sc^2+4a_4}}{2}$, $K_3 = \frac{-AA_5m_5Sc}{m_5^2+m_5Sc-a_4}$, $K_4 = \frac{-AA_6m_5Sc}{m_6^2+m_6Sc-a_4}$,

$l_{23} = -(K_3 + K_4)$, $l_{24} = -(K_3e^{m_5} + K_4e^{m_6})$, $A_8 = \frac{l_{23}e^{m_7}-l_{24}}{e^{m_7}-e^{m_8}}$,

$A_7 = l_{23} - A_8$, $m_9 = \frac{-Re+\sqrt{Re^2+4aHa^2}}{2a}$, $m_{10} = \frac{-Re-\sqrt{Re^2+4aHa^2}}{2a}$,

$K_5 = \frac{-A_1Gr}{am_1^2+Rem_1-Ha^2}$, $K_6 = \frac{-A_2Gr}{am_2^2+Rem_2-Ha^2}$, $K_7 = \frac{-A_5Gc}{am_5^2+Rem_5-Ha^2}$,

$K_8 = \frac{-A_6Gc}{am_6^2+Rem_6-Ha^2}$, $l_3 = K_5 + K_6 + K_7 + K_8$,

$l_4 = \alpha_1 + h_1(K_5 m_1 + K_6 m_2 + K_7 m_5 + K_8 m_6)$, $l_5 = l_4 - l_3$,
$l_6 = 1 - h_1 m_9$, $l_7 = 1 - h_1 m_{10}$, $l_8 = K_5 e^{m_1} + K_6 e^{m_2} + K_7 e^{m_5} + K_8 e^{m_6}$,
$l_9 = \alpha_2 + h_2(K_5 m_1 e^{m_1} + K_6 m_2 e^{m_2} + K_7 m_5 e^{m_5} + K_8 m_6 e^{m_6})$,
$l_{10} = l_9 - l_8$, $l_{11} = e^{m_9} - h_2 m_9 e^{m_9}$, $l_{12} = e^{m_{10}} - h_2 m_{10} e^{m_{10}}$,
$A_{10} = \frac{l_5 l_{11} - l_6 l_{10}}{l_7 l_{11} - l_6 l_{12}}$, $A_9 = \frac{l_5 - A_{10} l_7}{l_6}$, $m_{11} = \frac{-Re + \sqrt{Re^2 + 4a}}{2a}$, $m_{12} = \frac{-Re - \sqrt{Re^2 + 4a}}{2a}$
$K_9 = \frac{-ARe K_5 m_1 - Gr K_1}{am_1^2 + Rem_1 - b}$, $K_{10} = \frac{-ARe K_6 m_2 - Gr K_2}{am_2^2 + Rem_2 - b}$, $K_{11} = \frac{-Gr A_3}{am_3^2 + Rem_3 - b}$,
$K_{12} = \frac{-Gr A_4}{am_4^2 + Rem_4 - b}$, $K_{13} = \frac{-ARe K_7 m_5 - Gc K_3}{am_5^2 + Rem_5 - b}$, $K_{14} = \frac{-ARe K_8 m_6 - Gc K_4}{am_6^2 + Rem_6 - b}$,
$K_{15} = \frac{-Gc A_7}{am_7^2 + Rem_7 - b}$, $K_{16} = \frac{-Gc A_8}{am_8^2 + Rem_8 - b}$, $K_{17} = \frac{-ARe A_9 m_9}{am_9^2 + Rem_9 - b}$,
$K_{18} = \frac{-ARe A_{10} m_{10}}{am_{10}^2 + Rem_{10} - b}$, $l_{13} = h_1(K_9 m_1 + K_{10} m_2 + K_{11} m_3 + K_{12} m_4 + K_{13} m_5 + K_{15} m_7 + K_{16} m_8 + K_{17} m_9 +$ $K18m10$,
$l_{14} = h_2(K_9 m_1 e^{m_1} + K_{10} m_2 e^{m_2} + K_{11} m_3 e^{m_3} + K_{12} m_4 e^{m_4} + K_{13} m_5 e^{m_5} + K_{14} m_6 e^{m_6} + K_{15} m_7 e^{m_7} +$ $K16m8em8+K17m9em9+K18m10em10$
$l_{15} = K_9 + K_{10} + K_{11} + K_{12} + K_{13} + K_{15} + K_{16} + K_{17} + K_{18}$,
$l_{16} = 1 - h_1 m_{11}$, $l_{17} = 1 - h_1 m_{12}$, $l_{18} = l_{13} - l_{15}$,
$l_{19} = K_9 m_1 e^{m_1} + K_{10} m_2 e^{m_2} + K_{11} m_3 e^{m_3} + K_{12} m_4 e^{m_4} + K_{13} m_5 e^{m_5} + K_{14} m_6 e^{m_6} + K_{15} m_7 e^{m_7} +$
$K_{16} m_8 e^{m_8} + K_{17} m_9 e^{m_9} + K_{18} m_{10} e^{m_{10}}$,
$l_{20} = e^{m_{11}} - h_2 m_{11} e^{m_{11}}$, $l_{21} = e^{m_{12}} - h_2 m_{12} e^{m_{12}}$, $l_{22} = l_{14} - l_{19}$,
$A_{12} = \frac{l_{18} l_{20} - l_{16} l_{22}}{l_{17} l_{20} - l_{16} l_{21}}$, $A_{11} = \frac{l_{18} - A_{12} l_{17}}{l_{16}}$

4. Analysis and Discussion of Results

In this section, 2-term perturbation series are employed to evaluate the dimensionless velocity, dimensionless temperature and the dimensionless species concentration profiles.

To study the effect of variable suction on unsteady MHD oscillatory flow of Jeffrey fluid in a horizontal channel with heat and mass transfer, the velocity u, temperature θ and the species concentration$C$ profiles are depicted graphically against y for different values of different parameters; Slip flow parameters h_1 *and* h_2 , Suction parameters α_1 *and* α_2, material parameter λ_1, Grashof numbers Gr *and* Gc, Hartman number Ha, Reynolds number Re, radiation parameter N, Schmidt number Sc, chemical reaction parameter K_c and Peclet number Pe.

The velocity of the flow field varies to a great extend with the variation of the flow parameters. The main factors affecting the velocity are Slip flow parameters h_1 *and* h_2 , Suction parameters α_1 *and* α_2, material parameter λ_1, Grashof numbers Gr *and* Gc, Hartman number Ha, Reynolds number Re, radiation parameter N, Schmidt number Sc and chemical reaction parameter K_c. The effects of these parmeters on the velocity flow field are analyzed in figures 2 – 13. These effects are discussed quantitatively.

Figures 2 and 3 demonstrate the effect of slip flow parametersh_1*and* h_2 on velocity u. It is observe in figure 2 that the velocity $u$decreases with increase inh_1 while figure 3 shows that the velocity u increases with increase in h_2.

Figures 4 and 5 show the effect of suction parameters α_1 *and* α_2. It can be seen in figure 4 that the velocity u increases with increase in α_1. While the velocity u decreases with increase in α_2 in figure 5.

Figure 6 shows the effect of Grashof number Gr on velocity u. It is observed that as Gr increases, the velocity increases. To this effect, at higher Grashof number Gr the flow at the boundary is turbulent while at lower Gr the flow at the boundary is laminar.

Figure 7 depicts the effect of Grashof number Gc for mass transfer on velocity u. It is observed that the velocity u increases with increase in Gc.

The effect of Reynolds number Re on velocity u is shown in figure 8. It is shown that the velocity u increases with increasingRe.

Figure 9 depicts the effect of Hartman number Ha on velocity u. It is shown that the velocity u icreases with decrease in Ha. This shows the effect of magnetic field on the fluid flow and this effect suppresses the turbulence flow of the Jeffrey fluid. Physically, when magnetic field is applied to any fluid, then the apparent viscosity of the fluid increases to the point of becoming visco elastic solid. It is of great interest that yield stress of the fluid can be controlled very accurately through variation of the magnetic field intensity. The result is that the ability of the fluid to transmit force can be controlled with the help of electromagnet which give rise to many possible control – based applications, including MHD power generation, electromagnetic casting of metals, MHD ion propulsion etc.

Figure 10 demonstrates the effect of material parameter λ_1 on velocity u. It is evident that the velocity u increases with increase inλ_1. To this effect the ratio of relaxation to retardation enhances the increase flow of velocity when it becomes small.
The effect of the radiation parameter N on velocity u is depicted in figure 11. It is observed that the velocity u increases as the radiation parameter N increases.
Figure 12 illustrates the effect of chemical reaction parameter K_c on velocity u. It is observed that as the chemical reaction parameter decreases, the velocity increases.
Figure 13 demonstrate the effect of Schmidt number Sc on velocity u. It shows that the velocity increases with increase in Sc.
The temperature field suffers a major change in magnitude due to the variation of radiation parameter N and Pecletnumber Pe. The effects of these parameters on the temperature field are discussed in figures 14-15.
Figure 14 depicts the effect of radiation parameter N on temperature θ. It is found out that the temperature increases with increase in N.
The effect of Reynolds number Pe on temperature θ is shown in figure 15. It is observed that the temperature θ increases with increase in Pe.
The presence of foreign mass in the flow field greatly affects the species concentration of the flow field. The factors or parameters responsible for this variations are Schmidt number Sc and chemical reaction parameter K_c.
The effect of Schmidt number Sc on species concentration C is shown in figure 16. It is shown that the species concentration C increases with increase in Sc.
Figure 17 reveals the effect of chemical reaction K_c on species concentration C. It is observed that increase in K_c decreases the species concentration C.

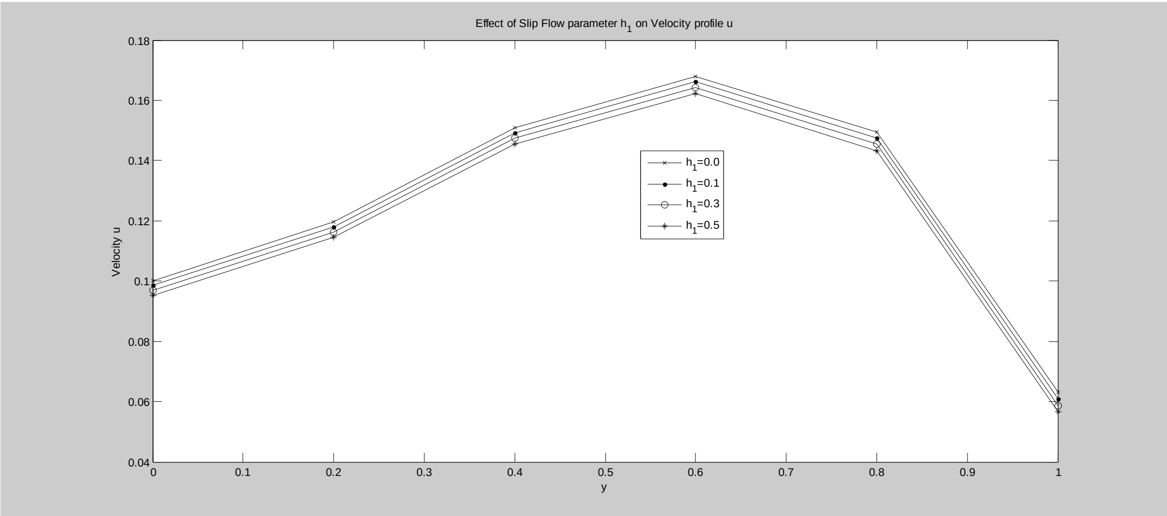

Figure 2: Effect of slip flowh$_1$ parameter on velocity withh$_2$ $= 1, \alpha_1 = 1, \alpha_2 = 1, \lambda_1 = 0.3, Gr = 1, Gc = 1, Ha = 1, N = 1, \omega = 1, Pe = 1, Re = 1, Sc = 1, K_c = 1, \varepsilon = 0.02, A = 0.5\ and t = 0.5.$

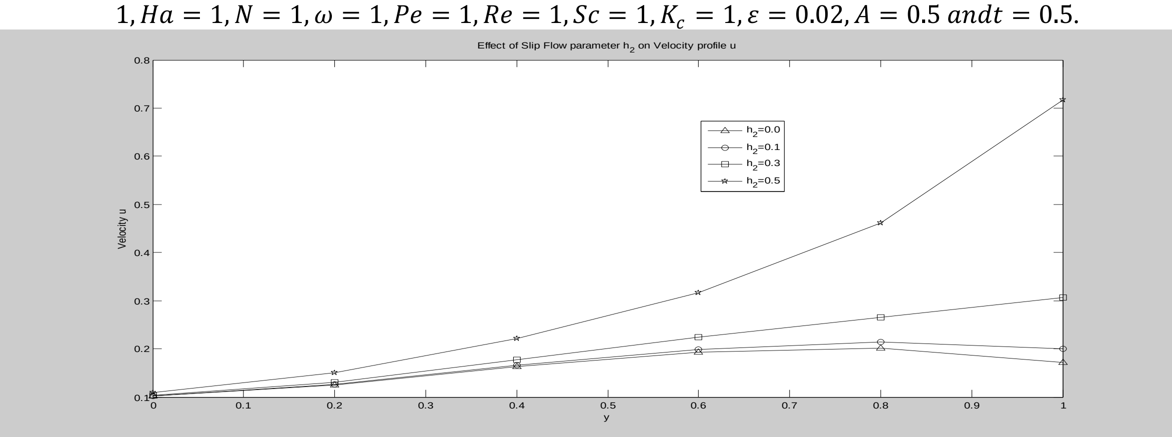

Figure 3: Effect of slip flow h_2 parameter on velocity with $h_1 = 1, \alpha_1 = 1, \alpha_2 = 1, \lambda_1 = 0.3, Gr = 1, Gc = 1, Ha = 1, N = 1, \omega = 1, Pe = 1, Re = 1, Sc = 1, K_c = 1, \varepsilon = 0.02, A = 0.5\ and t = 0.5.$

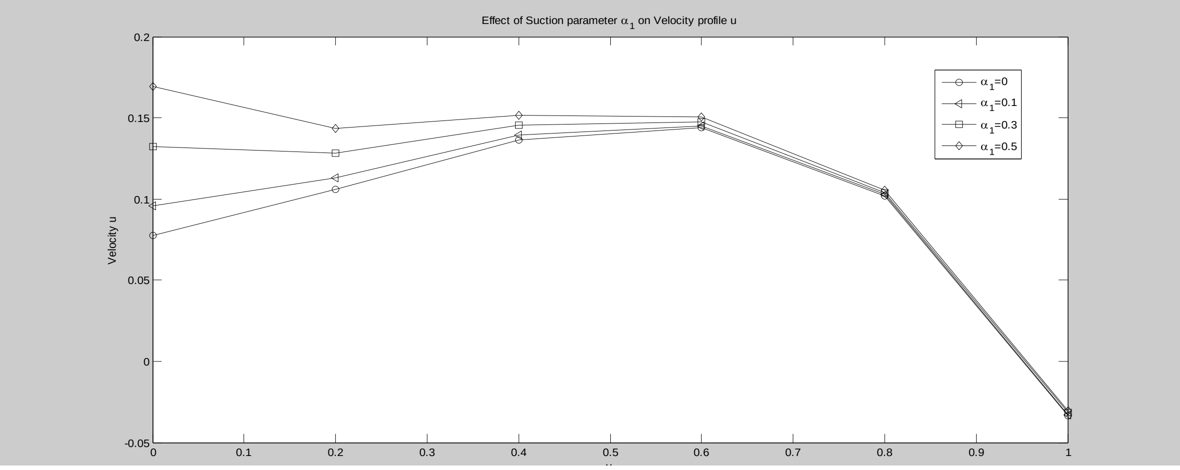

Figure 4: Effect of slip flow α_1 parameter on velocity with $h_1 = 1, h_2 = 1, \alpha_2 = 1, \lambda_1 = 0.3, Gr = 1, Gc = 1, Ha = 1, N = 1, \omega = 1, Pe = 1, Re = 1, Sc = 1, K_c = 1, \varepsilon = 0.02, A = 0.5$ $andt = 0.5$.

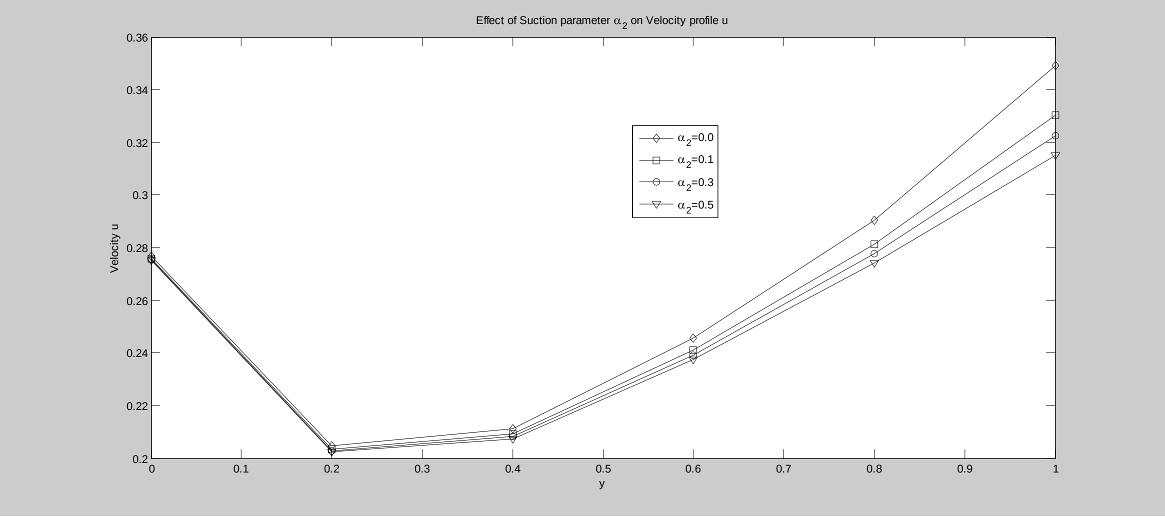

Figure 5: Effect of slip flow α_2 parameter on velocity with $h_1 = 1, h_2 = 1, \alpha_1 = 1, \lambda_1 = 0.3, Gr = 1, Gc = 1, Ha = 1, N = 1, \omega = 1, Pe = 1, Re = 1, Sc = 1, K_c = 1, \varepsilon = 0.02, A = 0.5$ $andt = 0.5$.

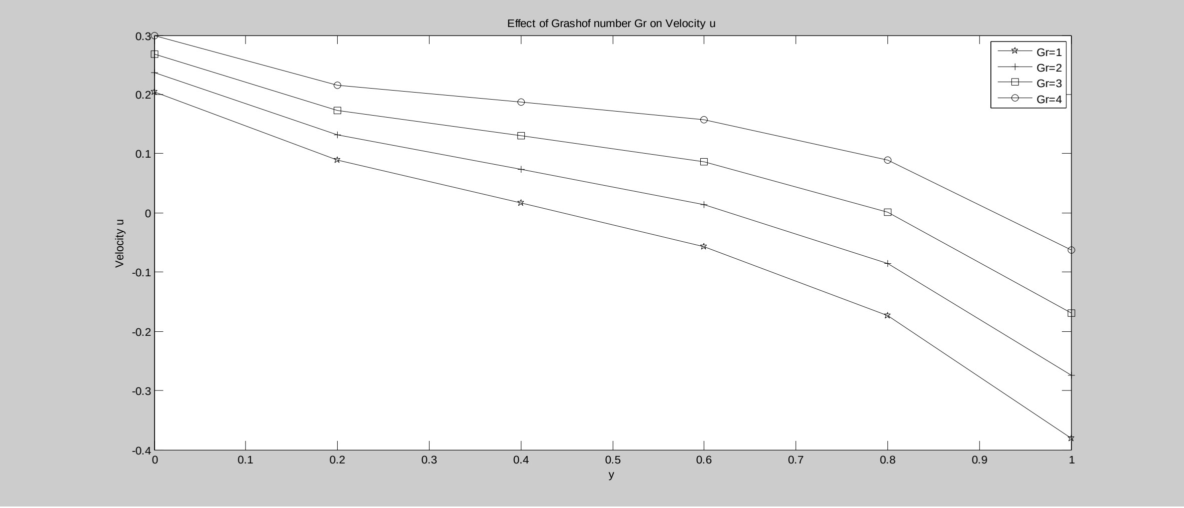

Figure 6: Effect of Grashof number Gr on velocity with $h_1 = 1, h_2 = 1, \alpha_1 = 1, \alpha_2 = 1, \lambda_1 = 0.3, Gc = 1, Ha = 1, N = 1, \omega = 1, Pe = 1, Re = 1, Sc = 1, K_c = 1, \varepsilon = 0.02, A = 0.5$ $andt = 0.5$.

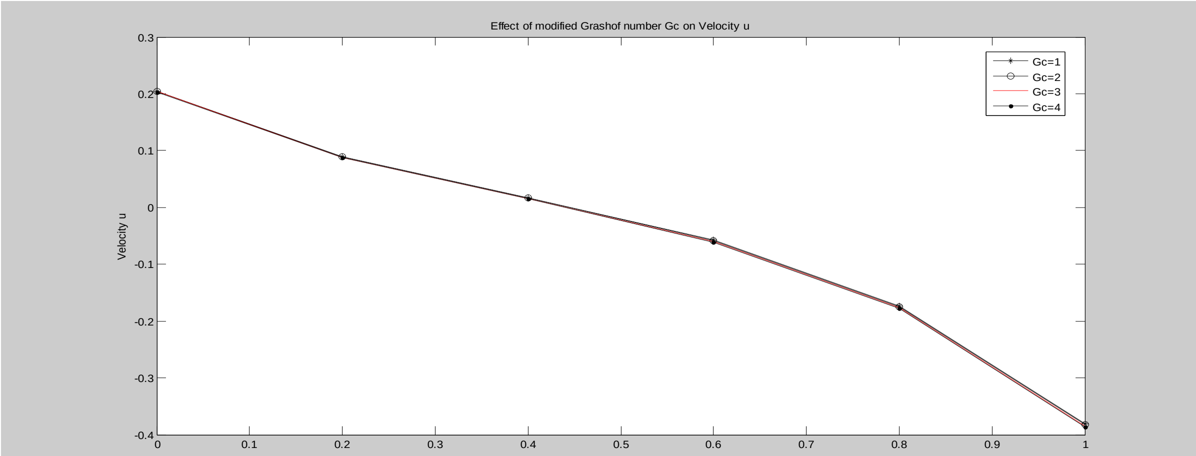

Figure 7: Effect of Grashof number Gc*due to mass transferon velocity with* $h_1 = 1, h_2 = 1, \alpha_1 = 1, \alpha_2 = 1, \lambda_1 = 0.3, Gc = 1, Ha = 1, N = 1, \omega = 1, Pe = 1, Re = 1, Sc = 1, K_c = 1, \varepsilon = 0.02, A = 0.5\ and t = 0.5.$

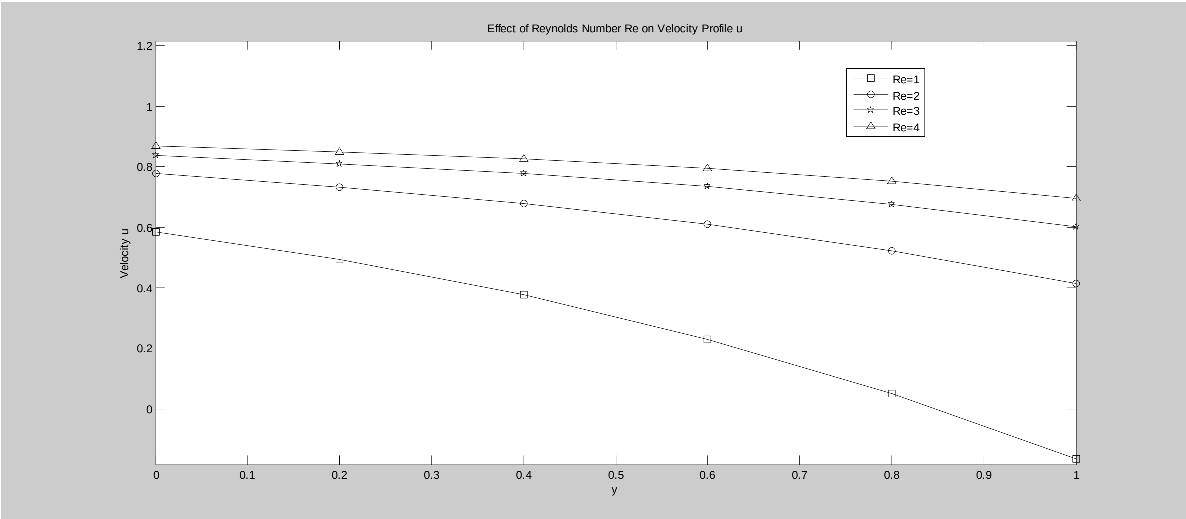

Figure 8: Effect of Reynolds number Re *on velocity with* $h_1 = 1, h_2 = 1, \alpha_1 = 1, \alpha_2 = 1, \lambda_1 = 0.3, Gc = 1, Ha = 1, N = 1, \omega = 1, Pe = 1, Gc = 1, Sc = 1, K_c = 1, \varepsilon = 0.02, A = 0.5\ and t = 0.5.$

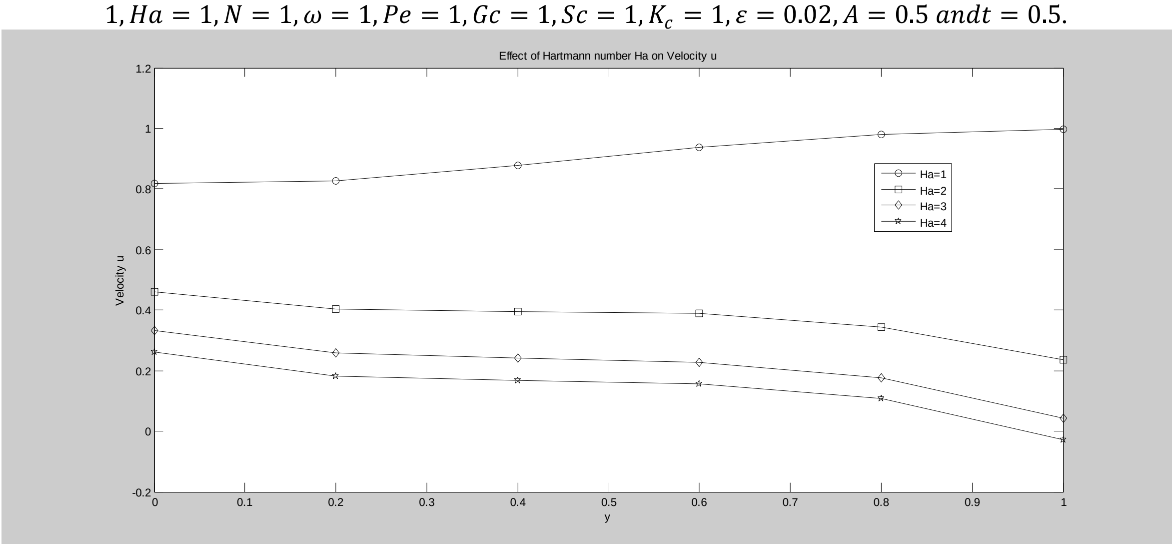

Figure 9: Effect of Hartmann number Ha *on velocity with* $h_1 = 1, h_2 = 1, \alpha_1 = 1, \alpha_2 = 1, \lambda_1 = 0.3, Gc = 1, Re = 1, N = 1, \omega = 1, Pe = 1, Gc = 1, Sc = 1, K_c = 1, \varepsilon = 0.02, A = 0.5\ and t = 0.5.$

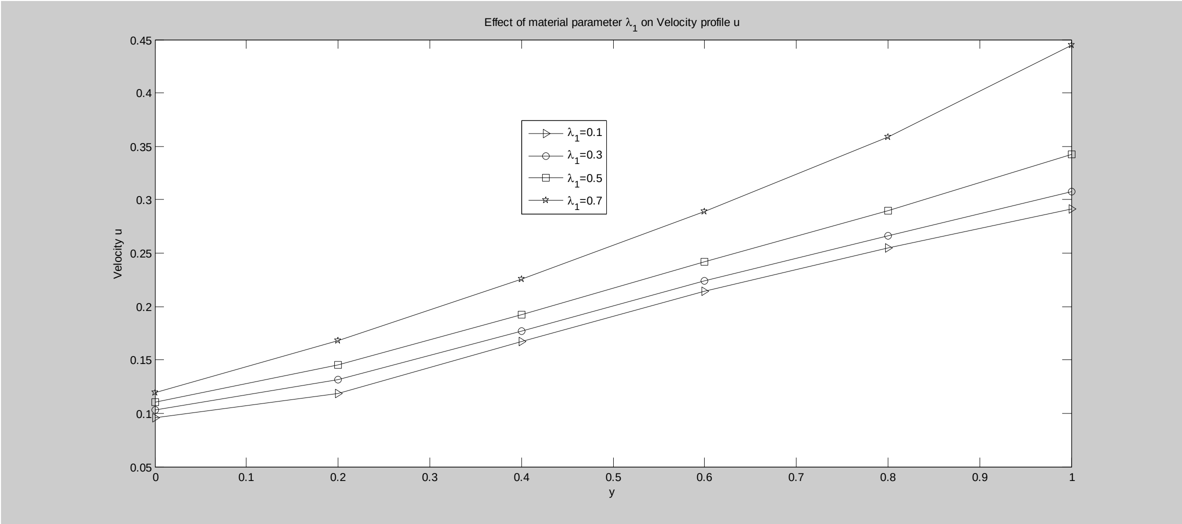

Figure 10: Effect of material parameter λ_1 *on velocity with* $h_1 = 1, h_2 = 1, \alpha_1 = 1, \alpha_2 = 1, Gc = 1, Re = 1, Ha = 1, N = 1, \omega = 1, Pe = 1, Gc = 1, Sc = 1, K_c = 1, \varepsilon = 0.02, A = 0.5\ and t = 0.5.$

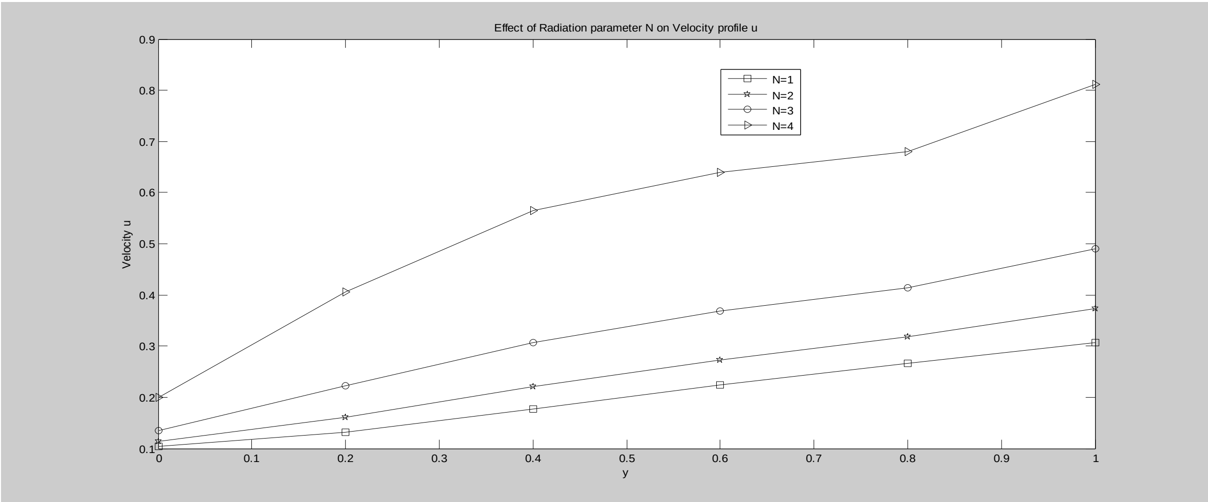

Figure 11: Effect of Radiation Parameter N on velocity with $h_1 = 1, h_2 = 1, \alpha_1 = 1, \alpha_2 = 1, \lambda_1 = 0.3, Gc = 1, Re = 1, \omega = 1, Pe = 1, Gc = 1, Sc = 1, K_c = 1, \varepsilon = 0.02, A = 0.5\ and t = 0.5.$

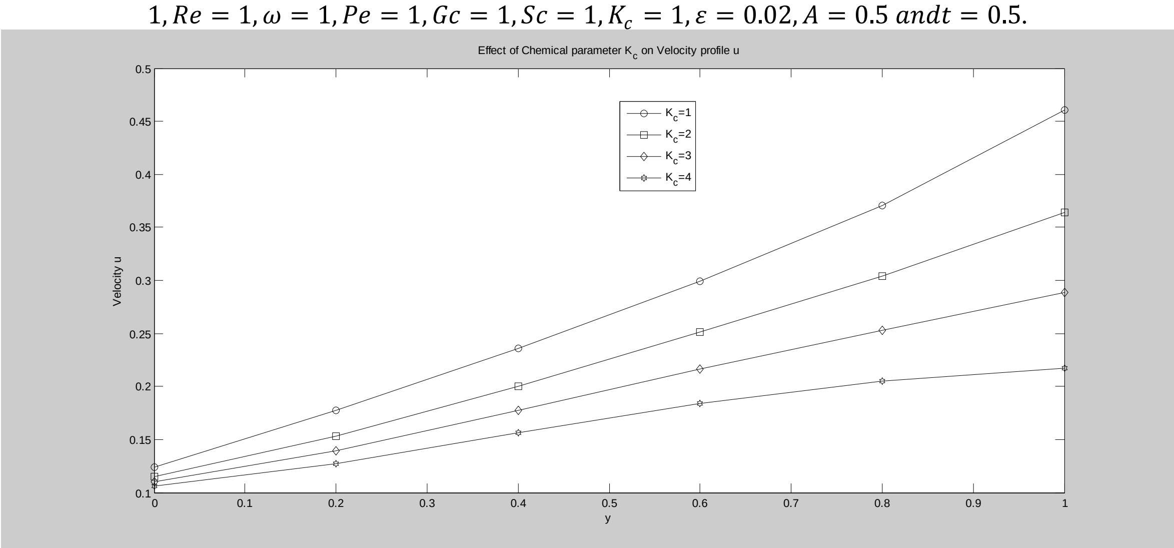

Figure 12: Effect of Chemical Parameter K_c *on velocity with* $h_1 = 1, h_2 = 1, \alpha_1 = 1, \alpha_2 = 1, Gc = 1, Re = 1, Ha = 1, N = 1, \omega = 1, Pe = 1, Sc = 1, Gc = 1, \lambda_1 = 1, \varepsilon = 0.02, A = 0.5\ and t = 0.5.$

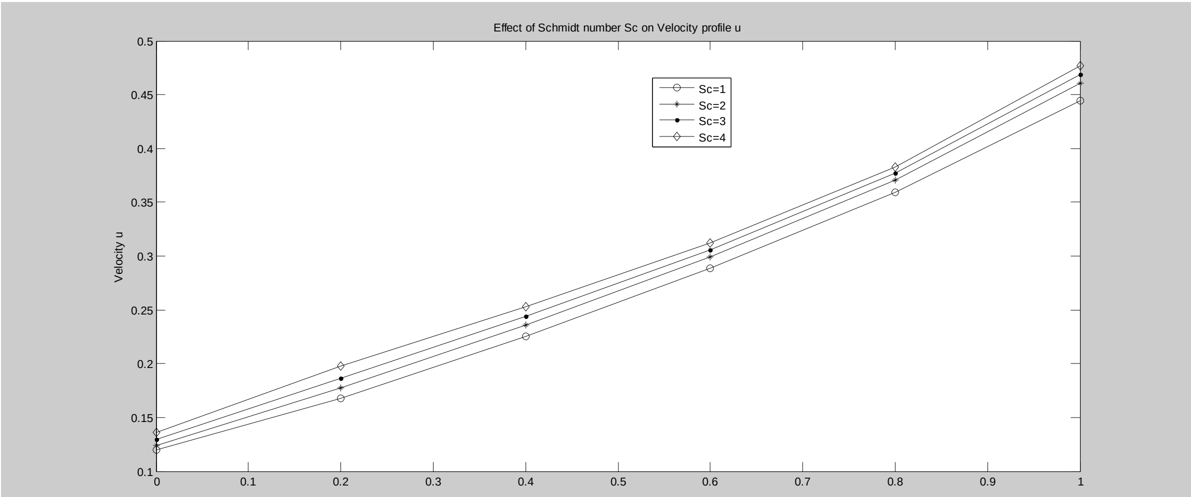

Figure 13: Effect of Chemical Parameter Sc on velocity with $h_1 = 1, h_2 = 1, \alpha_1 = 1, \alpha_2 = 1, Gc = 1, Re = 1, Ha = 1, N = 1, \omega = 1, Pe = 1, Gc = 1, \lambda_1 = 1, K_c = 1, \varepsilon = 0.02, A = 0.5\ and t = 0.5.$

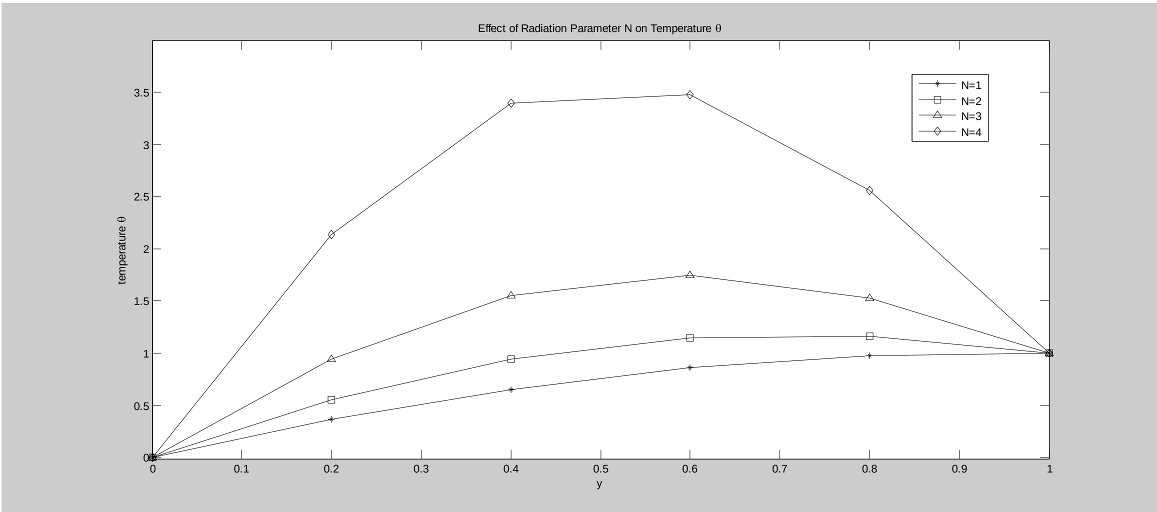

Figure 14: Effect of Radiation parameter N on temperature with $\omega = 1, Pe = 1, \varepsilon = 0.02, A = 0.5\ and t = 0.5.$

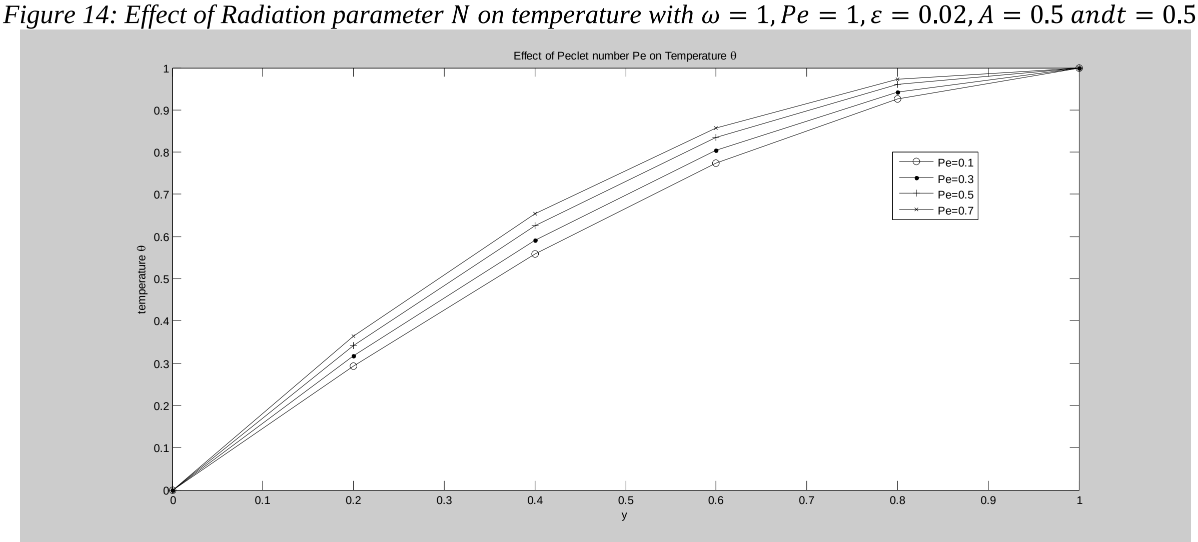

Figure 15: Effect of Peclet number Pe on temperature with $\omega = 1, N = 1, \varepsilon = 0.02, A = 0.5\ and t = 0.5.$

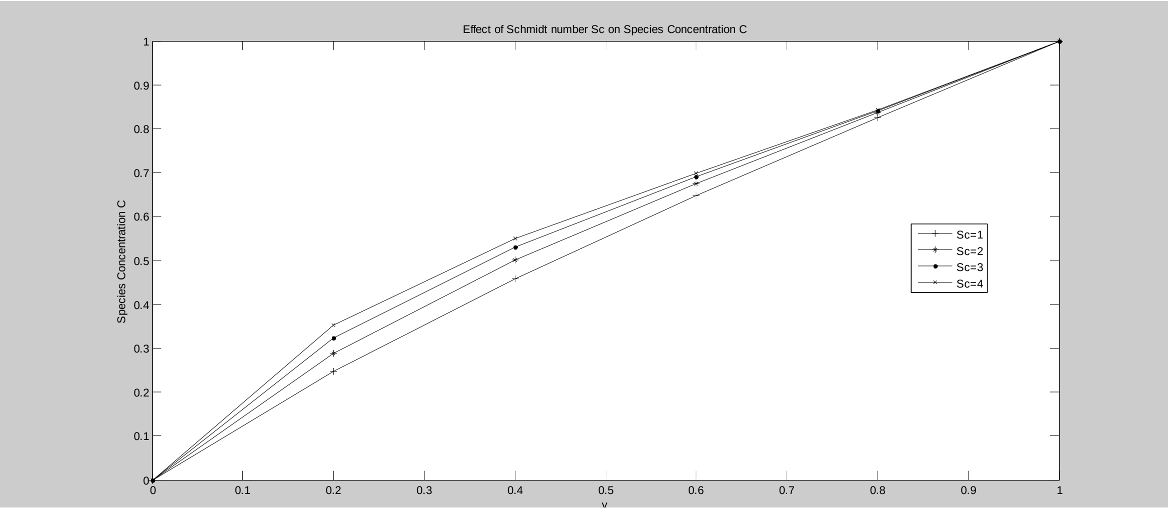

Figure 16: Effect of Schmidt number Sc *on species concentration with* $\omega = 1, K_c = 1, \varepsilon = 0.02, A = 0.5$ $and t = 0.5$.

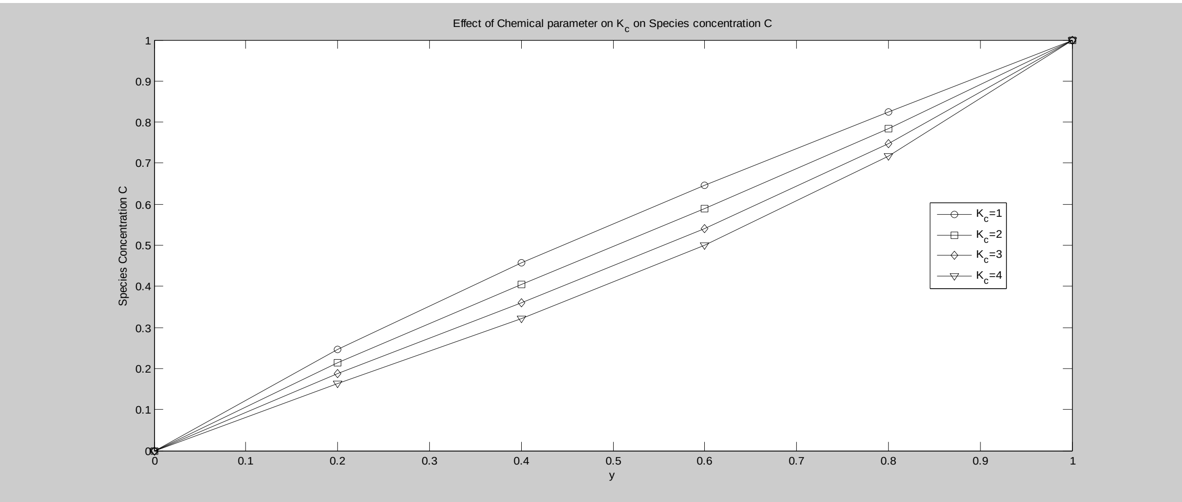

Figure 17: Effect of chemical parameter K_c *on species concentration with* $\omega = 1, Sc = 1, \varepsilon = 0.02, A = 0.5$ $and t = 0.5$.

Summary and Conclusion

In this section, we studied the effect of variable suction on unsteady MHD oscillatory flow of Jeffrey fluid in a horizontal channel with heat and mass transfer.

The governing equations, the momentum, energy and species equations have been written in dimensionless form.

A perturbation method has been employed to evaluate and solved the dimensionless equations for velocity u, the dimensionless temperature θ and the dimensionless species concentration C.

The main findings are summarized below;

i. Increase in slip flow parameter h_2, suction parameter α_1,material parameter λ_1, Grashof number for heat transfer Gr, Grashof number for mass transfer Gc, Radiation parameter N, Reynolds number $Re$and Schmidt number Sc have accelerating effects on velocity of the flow field.

ii. Decrease in slip flow parameter h_1, Hartman number Ha and chemical parameter K_c accelerates the velocity of the flow field.

iii. Increase in the Radiation parameter N, Peclet number Pe and accelerate the magnitude of temperature of the flow field.

iv. Increase in Schmidt number Sc and chemical parameter K_c respectively increase and decrease the species concentration or the concentration boundary layer thickness of the flow field.

It is concluded that these results show that the velocity is more of Jeffrey fluid than the Newtonian fluid.

5. References

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