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Research Article

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Unsteady MHD mixed convective oscillatory flow through aporous medium filled in a vertical channel with heat and mass transfer

Magaji AS

Department of Mathematical Sciences, Kaduna State University - Nigeria

Abstract The unsteady MHD fixed convective oscillatory flow through a porous medium filled in a vertical channel with heat and mass transfer was analyzed. The two parallel stationary walls of the channel are distance d apart. The analytical solution of the governing equations is obtained. The consequence of the flow parameters on velocity, temperature and concentration field are demonstrated through graphs. The skin friction, Nusselt number and Sherwood numbers were obtained and tabulated. It was found out that the results of this problem have an excellent agreement with analytical prediction.

Keywords MHD, mixed convective, porous medium, oscillatory flow.

Introduction

In recent years, the flows of fluid through porous media are of principal interest because these are quite prevalent in nature. Such flows have attracted the attention of a number of scholars due to their application in many branches of science and technology, in the field of agriculture engineering to study the underground water resources, seepage of water in river-beds, in petroleum technology to study the movement of natural gas, oil and water through oil reservoirs, in chemical engineering for filtration and purification processes. The convection problem in porous medium has also important applications in geothermal reservoirs and geothermal energy extractions.

Magnetohydrodynamic (MHD) is going through a period of great enlargement and differentiation of subject matter. The interest in magnetohydrodynamic (MHD) convection with heat transfer is renewed due to its importance in the design of MHD generators, and accelerators in geophysics, in the design of underground water and energy storage systems, soil sciences, astrophysics and so on. Several scholars have shown their interest in MHD flows because of their varied applications. Singh (2011) [1] obtained an exact solution of an oscillatory MHD flow in a channel filled with porous medium. Rahman and Sarkar (2004) [2] analyzed the unsteady MHD flow of a dusty visco-elastic Oldroyd fluid under time varying body force through a rectangular channel. Attia and Ewis (2010) investigated an unsteady MHD Couette flow with heat transfer of a visco-elastic fluid under exponential decaying pressure gradient [3]. Very recently, Choudhary and Das (2012) [4] extended the problem of Makinde and Mhone (2005) [5] by taking into account the visco-elastic fluid. Choudhary and Das (2012) carried along all the mistakes of Makinde and Mhone (2005) which have examined forced convection in such channels [4-5].

Hence, the aim of the present study is to analyze the very important physical problem of unsteady flow, incompressible and finitely electrically conducting fluid flow through a porous medium bounded by two vertical plates in the presence of heat radiation and uniform transverse magnetic field, and heat and mass transfer.

Mathematical Formulation

An oscillatory flow of an unsteady fluid, incompressible and electrically conducting fluid in a vertical channel filled with porous medium is considered. The two parallel stationary walls of the channel are distance d apart. Choose a Cartesian coordinate system ($X*$,$Y*$) where $X*$-axis lies along the centre line of the channel and $Y*$-axis is perpendicular to the parallel plates. A magnetic field, B0, of uniform strength is applied along Y*-axis. The magnetic Reynolds number is assumed small enough so that the induced magnetic field is negligible. Hall effect, electrical and polarization effects are also neglected. All physical quantities are independent of $x*$ for this problem of fully developed laminar flow. Under the usual Boussinesq approximation the flow is governed by the following equations

Momentum equation;

$$\frac{\partial u^*}{\partial t^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \vartheta_1 \frac{\partial^2 u^*}{\partial y^{*2}} - \vartheta_1 \frac{u^*}{k^*} - \frac{\sigma B_0^2 u^*}{\rho} + g\beta T^* + gB^* C^* \quad (1)$$

Energy equation;

$$\frac{\partial T^*}{\partial t^*} = \frac{k}{\rho c_p}\frac{\partial^2 T^*}{\partial y^{*2}} - \frac{1}{\rho c_p}\frac{\partial q^*}{\partial y^*} \quad (2)$$

Concentration equation;

$$\frac{\partial c^*}{\partial t^*} = D\frac{\partial^2 c^*}{\partial y^{*2}} \quad (3)$$

where in momentum equation (1) term on the L. H. S. is the inertial force and on the R. H. S. the terms respectively represent imposed pressure gradient, viscous force, viscoelastic term, pressure drop across the porous matrix, Lorentz force due to magnetic field B_0 and the buoyancy force due to temperature difference of the plates. In energy equation (2) term on the L. H. S. is the heat due to convection and on the R. H. S. the terms respectively represent conduction heat and radiation heat.

The boundary conditions of the problem are

$$u^* = 0, T^* = T_o e^{i\omega^* t^*}, C^* = C_o e^{i\omega^* t^*} \;\; At \;\; y^* = \frac{d}{2} \quad (4)$$

$$u^* = 0, T^* = 0, C^* = 0 \;\; At \;\; y^* = -\frac{d}{2} \quad (5)$$

where$u*$ is the axial velocity, $T*$ is the temperature, $t*$ is the time, $p*$ is the pressure, $\rho$is the density, is the kinematic viscosity, σ is the electric conductivity, c_p is the specific heat at constant pressure, $K*$ is the permeability of the porous medium, $\omega*$ is the frequency of oscillations. Here '*' stands for the dimensional quantities.

Now introducing the following non-dimensional quantities

$$x = \frac{x^*}{d}, y = \frac{y^*}{d}, u = \frac{u^*}{U}, t = \frac{t^* U}{d}, \omega = \frac{\omega^* d}{U}, P = \frac{p^*}{\rho U^2}, T^* = T - T_o, C^* = C - C_o, S_c = \frac{\vartheta_1}{D}$$

From equation (1) to (4),we obtain equation in dimensionless form as

$$Re\frac{\partial u}{\partial t} = -Re\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - \left(\left(\frac{1}{\sqrt{Da}}\right)^2 + \left(B_o d\sqrt{\frac{\sigma}{\vartheta_1 \rho}}\right)^2\right)u + G_r T + G_c C \quad (6)$$

$$Pe\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial y^2} + N^2 T \quad (7)$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} \quad (8)$$

With boundary conditions at

$$u = 0, T = e^{i\omega t} \;\; At \;\; y = \frac{1}{2} \quad (9)$$

$$u = 0, T = 0, \;\; At \;\; y = -\frac{1}{2} \quad (10)$$

Where Gr (Grashof number) $= \frac{\beta T_{od^2}}{\vartheta_1 U}$,

Pe(Peclet number) $= \frac{\rho c_p U d}{k}$

Re(Reynold number) $= \frac{Ud}{\vartheta_1}$

H(Hartmann number) $= B_o d\sqrt{\frac{\sigma}{\mu}}$

Da (Darcy number) $= \frac{k^*}{d^2}$

s(Porous medium shape factor parameter) $= \frac{1}{\sqrt{Da}}$

N (Radiation parameter) $= 2\alpha\frac{d}{\sqrt{k}}$

The imposed pressure gradient oscillating with time is of the oscillatory internal flow in the channel is of the form

$$-\frac{\partial p}{\partial x} = \lambda e^{i\omega t} \quad (11)$$

Method of Solution

To solve equations (6),(7) and (8) under the boundary condition (9) and (10). We should assume for purely flow

$$u(u,t) = u_o(y)e^{i\omega t}, T(y,t) = \theta_o(y)e^{i\omega t}, C(y,t) = C_o(y)e^{i\omega t} \text{ and } -\frac{\partial p}{\partial x} = \lambda e^{i\omega t} \quad (12)$$

Substituting (12) into (6), (7) and (8), we obtain

$$u_o''(y) - Fu_o(y) = -Re\lambda - Gr\theta_o(y) - GcC_o(y), \quad (13)$$

$$\theta_o''(y) - A\theta_o(y) = 0 \quad (14)$$

$$C_o''(y) - D_j C_o(y) = 0 \quad (15)$$

$Where\ F = S^2 + H^2 + i\omega Re, A = N^2 + i\omega Pe, and\ D_j = i\omega S_c$

The transformed boundary condition becomes

$$u_0 = 0,\ \theta_o = 1,\ C_o = 1\ At\ y = \frac{1}{2} \quad (16)$$

$$u_0 = 0,\ \theta_o = 0,\ C_o = 0\ At\ y = -\frac{1}{2} \quad (17)$$

The ordinary differential equations (13), (14) and (15) are solved under the boundary conditions (16) and (17) and the solutions for the mean velocity, mean temperature and concentration fields are obtained respectively as follows

$$u_0(y) = C_5e^{m5y} + C_6e^{m6y} + k_1 + k_2e^{m1y} + k_3e^{m2y} + k_4e^{m3y} + k_5e^{m4y} \quad (18)$$

$$\theta_0(y) = C_1e^{m1y} + C_2e^{m2y} \quad (19)$$

$$C_0(y) = C_3e^{m3y} + C_4e^{m4y} \quad (20)$$

Therefore the solution for the velocity, temperature and concentration fields are

$$u(y,t) = (C_5e^{m5y} + C_6e^{m6y} + k_1 + k_2e^{m1y} + k_3e^{m2y} + k_4e^{m3y} + k_5e^{m4y})e^{i\omega t} \quad (21)$$

$$\theta(y,t) = (C_1e^{m1y} + C_2e^{m2y})e^{i\omega t} \quad (22)$$

$$C(y,t) = (C_3e^{m3y} + C_4e^{m4y})e^{i\omega t} \quad (23)$$

The Skin Friction

From the velocity field in equation (21) we can obtain the skin friction at the left wall as

$$\tau_L = \frac{\partial u}{\partial y}\Big|_{y=-\frac{1}{2}}$$

$$\tau_L = \frac{\partial u}{\partial y} = \left(m_5C_5e^{-\frac{1}{2}m5} + m_6C_6e^{-\frac{1}{2}m6} + Q'(y)\right)e^{i\omega t} \quad (24)$$

Where $Q(y) = k_1 + k_2e^{m1y} + k_3e^{m2y} + k_4e^{m3y} + k_5e^{m4y}$

And the skin friction at the right wall is

$$\tau_R = \frac{\partial u}{\partial y}\Big|_{y=\frac{1}{2}}$$

$$\tau_R = \frac{\partial u}{\partial y} = \left(m_5C_5e^{\frac{1}{2}m5} + m_6C_6e^{\frac{1}{2}m6} + Q'(y)\right)e^{i\omega t} \quad (25)$$

The Nusselt Number

From the temperature field given in equation (22) the heat transfer coefficient Nu (Nusselt number) at the left wall is

$$Nu_L = \frac{\partial\theta}{\partial y}\Big|_{y=-\frac{1}{2}}$$

$$Nu_L = \frac{\partial\theta}{\partial y} = \left(m_1C_1e^{-\frac{1}{2}m1} + m_2C_2e^{-\frac{1}{2}m2}\right)e^{i\omega t} \quad (26)$$

Heat transfer coefficient Nu (Nusselt number) at the right wall is

$$Nu_R = \frac{\partial\theta}{\partial y}\Big|_{y=\frac{1}{2}}$$

$$Nu_R = \frac{\partial\theta}{\partial y} = \left(m_1C_1e^{\frac{1}{2}m1} + m_2C_2e^{\frac{1}{2}m2}\right)e^{i\omega t} \quad (27)$$

The Sherwood Number

From the concentration field given in equation (23) the Sh (shawood number) at the left wall is $Sh_L = \frac{\partial C}{\partial y}\Big|_{y=-\frac{1}{2}}$

$$Sh_L = \frac{\partial C}{\partial y} = \left(m_3C_3e^{-\frac{1}{2}m3} + m_4C_4e^{-\frac{1}{2}m4}\right)e^{i\omega t} \quad (28)$$

And the Sherwood number at the right wall is

$$Sh_R = \frac{\partial C}{\partial y}\Big|_{y=\frac{1}{2}}$$

$$Sh_R = \frac{\partial C}{\partial y} = \left(m_3C_3e^{\frac{1}{2}m3} + m_4C_4e^{\frac{1}{2}m4}\right)e^{i\omega t} \quad (29)$$

Result and Disscussion

The problem of an oscillatory mixed convection MHD flow in a vertical channel filled with porous medium in the presence of a transverse uniform magnetic field and heat radiation is solved analytically.

The pressure gradient in the channel also oscillates with time. The velocity field, temperature and species concentration are evaluated numerically and graphically using MATLAB for the different values of Grashof number Gr, the Reynolds number Re, porous medium shape factor parameter s, the Hartmann number H, the Peclet number Pe, radiation parameter N, the pressure gradient, and the frequency of oscillations. For numerical validation, only the real parts of the results are considered. The effects of these parameters are displayed through figures.

The variations in the velocity field due to the increase of the Grashof number for heat transfer G_r and Grashof number for mass transfer G_c in figure 1 and figure 2 respectively. It is clear from the figure that the velocity decrease meaning that the flow retards as the Grashof number increases for heat transferG_r and Grashof number for mass transfer G_c.

Figure 3 depicts the variation of Reynolds number Re on velocity profile u.It can be depicted that the velocity increases with increase in Reynold numberRe.

The effect of Hartmann number Hon velocity profile is shown in figure 4. It is clear that the velocity decreases with increase in Hartmann number. This is true since the role of a magnetic field in a flow field is to suppress turbulence.

Figure 5 shows the variation of frequency oscillation ω on velocity. It is evident that the velocity oscillates with increase in ω.

Figure 6 shows the effect of Schmidt number Sc on species concentration. It can be seen that the species concentration increases with increase in Schmidt number.

Figures 7 and 8 demonstrate respectively the variation radiation parameter N and Peclect number Pe. It is demonstrated that increase in both the radiation parameter N and Peclectnumber Pe decrease the velocity profile of the flow field.

Table 1 shows the variation of skin friction, Nusselt number and Sherwood number at different time t with the flow parameters kept constant.

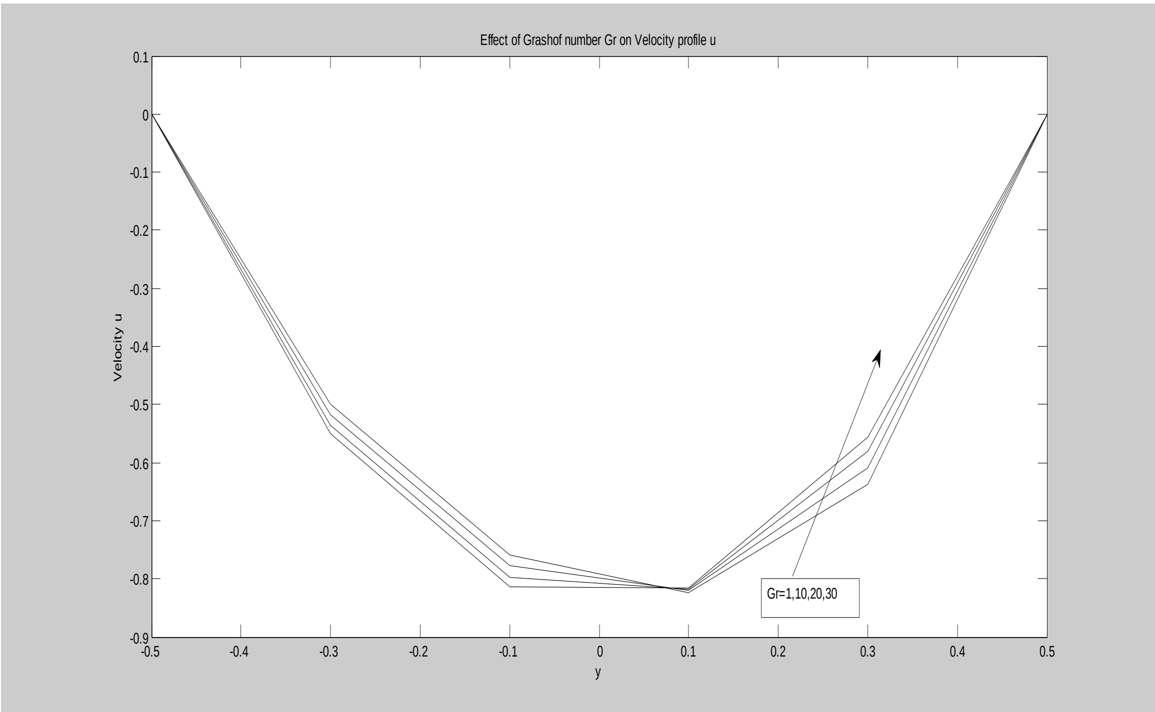

Figure 1: Variation of velocity u with different values of Grashof number Gr.

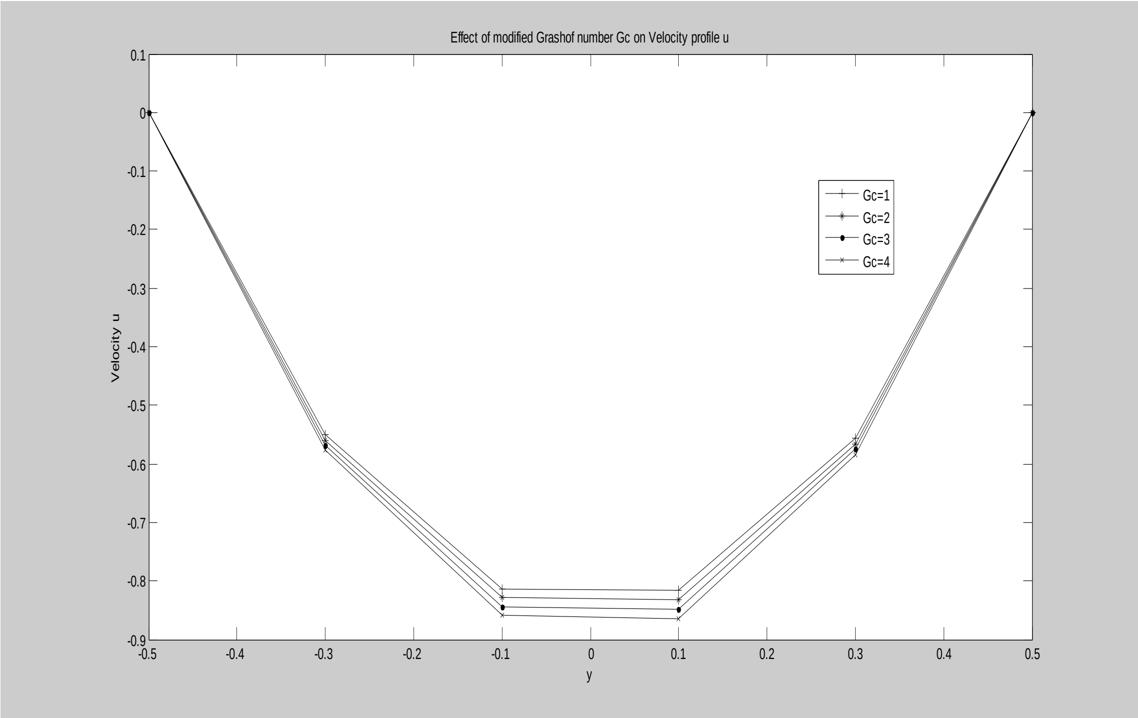

Figure 2: Variation of velocity u with different values of Grashof number Gc.

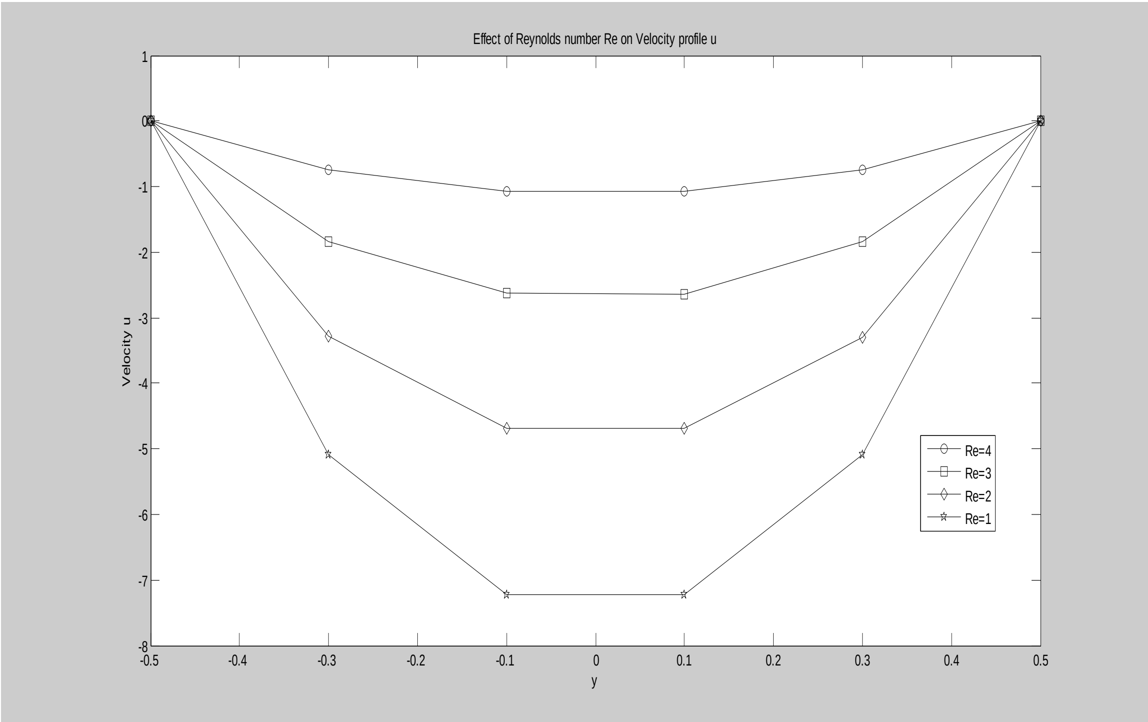

Figure 3: Variation of velocity u with different values of Reynolds number Re.

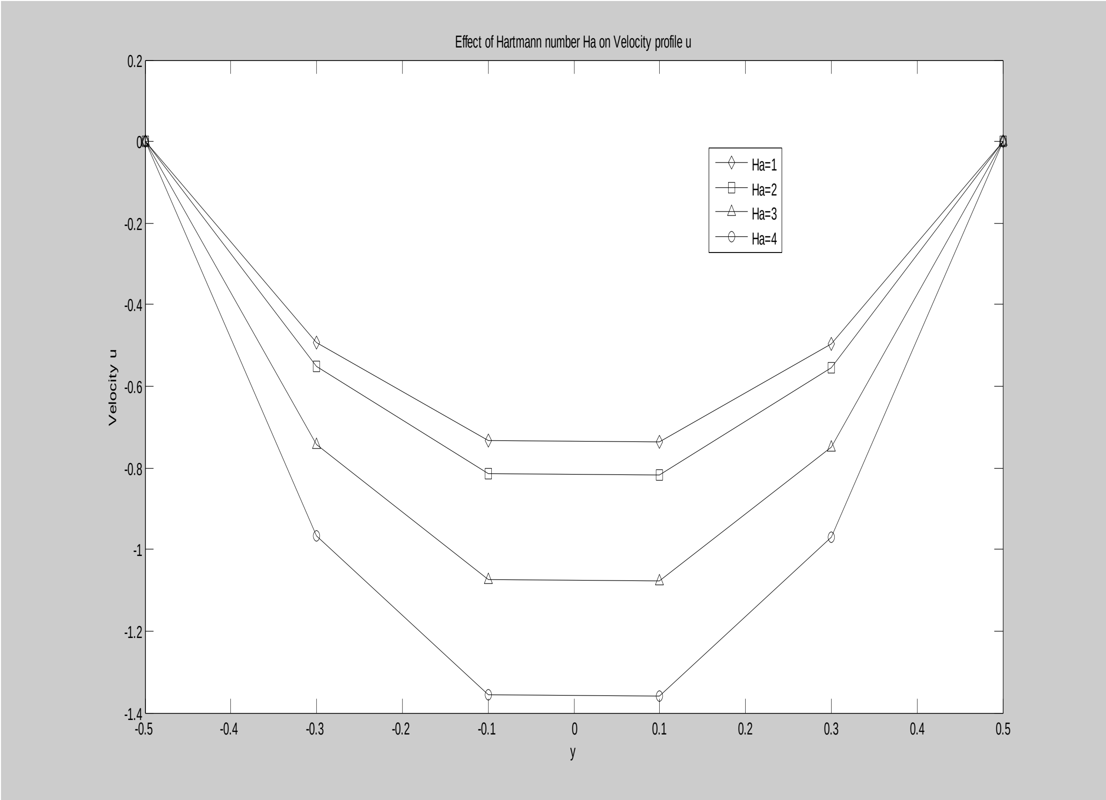

Figure 4: Variation of velocity u with different values of Hartmann number Ha.

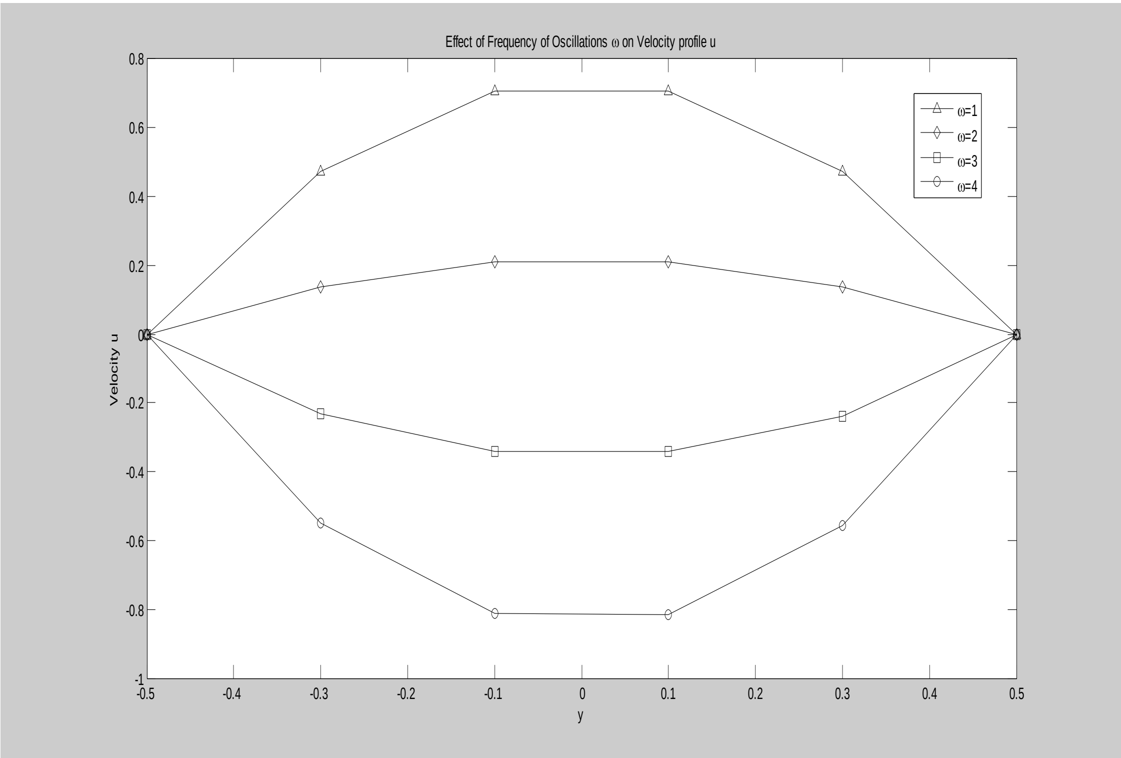

Figure 5: Variation of velocity u with different values of frequency of oscillation ω.

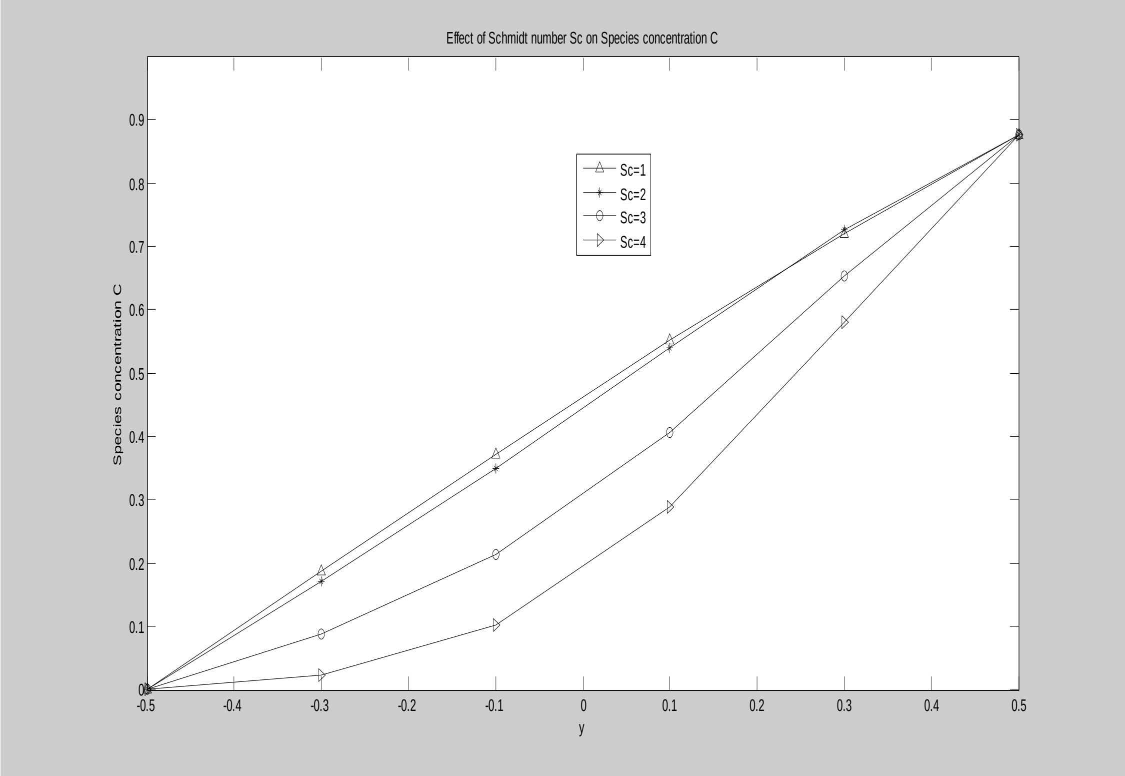

Figure 6: Variation of Species Concentration C with different values of Schmidt number Sc.

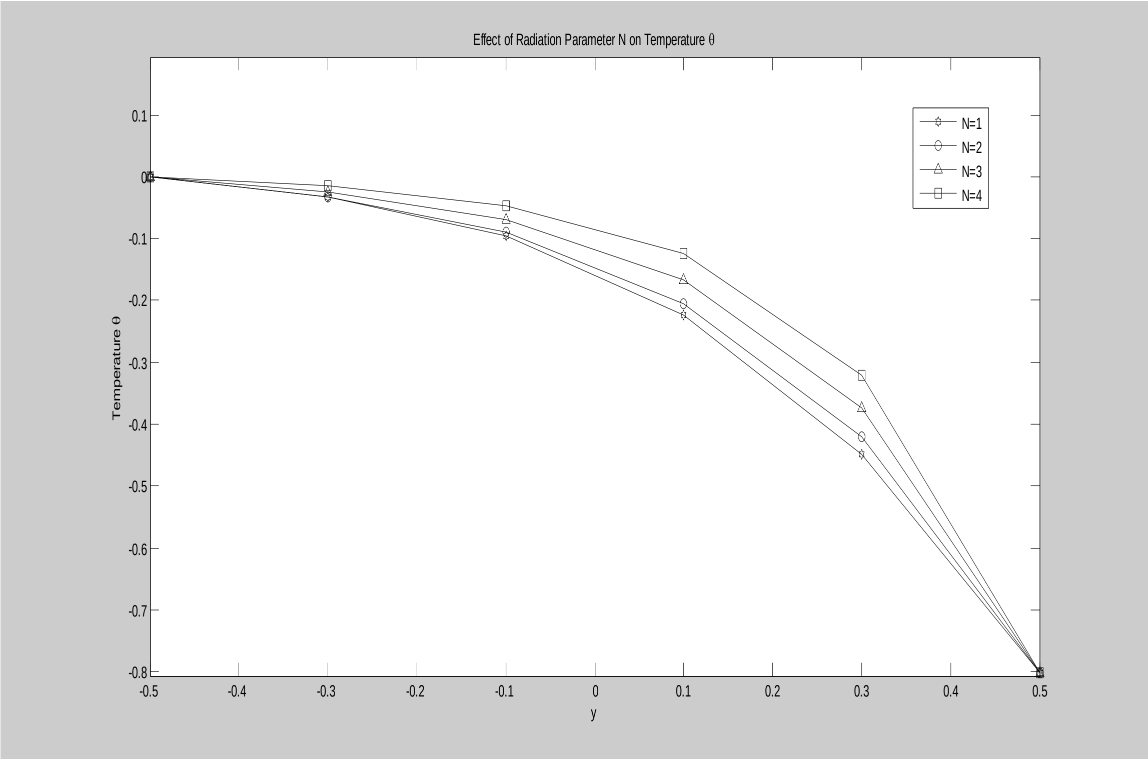

Figure 7: Variation of Temperature θ with different values of N.

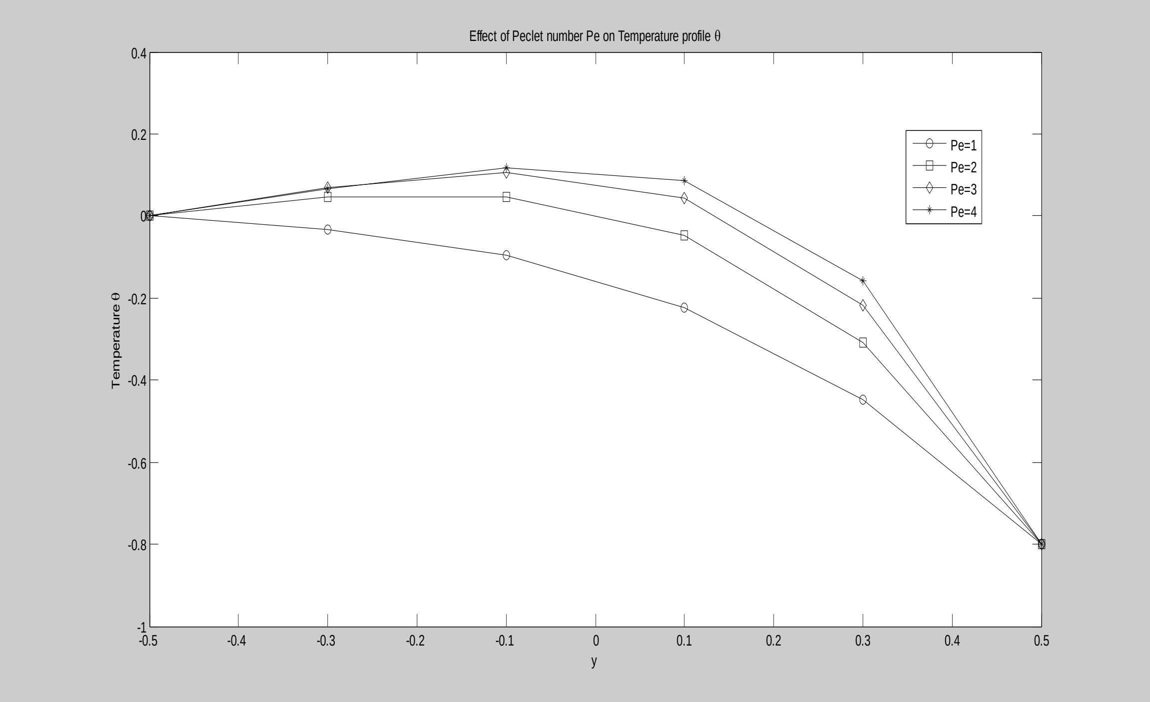

Figure 8: Variation of Temperature θ with different values of Peclet number Pe.

Table 1: Variation of Skin friction, Nusselt number and Sherwood number at different time t

t	τ_L	τ_R	Nu_L	Nu_R	Sh_L	Sh_R
0.0	−0.6385	−10.8738	0.9417	1.2928	0.1682	15.5540
0.1	−1.5105	−10.8373	0.9249	2.4254	0.8184	20.2600
0.3	−1.6216	−10.8238	0.4862	3.3996	0.8193	21.8382
0.5	−9.7702	−10.8137	0.2505	4.3116	0.8227	21.3828
0.7	−10.3714	−10.8038	0.1352	5.6083	0.8714	22.5509
0.9	−11.9400	−10.7737	0.1133	5.6772	0.8832	22.6911
1.0	−12.5069	−10.7638	0.9332	6.2958	0.8890	22.6986

Summary and Conclusion

The unsteady MHD fixed convective oscillatory flow through a porous medium filled in a vertical channel with heat and mass transfer was analyzed. The two parallel stationary walls of the channel are distance d apart. The analytical solution of the governing equations is obtained. The skin friction, Nusselt number and Sherwood numbers were obtained and tabulated.

It was found out that high Hartmann number H retards the velocity of the flow field. While the other flow parameter accelerates the velocity.

Also, both the radiation parameter N and Peclet number Pe increase the temperature distribution of the flow field.

The results of this problem have an excellent agreement with analytical prediction, meaning the flow is purely oscillatory.

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