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Research Article

Leibnitz-Maclaurin method for solving temperature distribution in straight fins with temperature-dependent thermal conductivity

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Abstract The Leibnitz-Macluarin Method (LMM) via successive differential coefficients has been employed to proffer series solution of the nonlinear equation arising in the convective straight fins with temperature-dependent thermal conductivity problem. Solutions are presented for the dimensionless temperature distribution and fin efficiency of the nonlinear equation. Parametric analyses indicated two dominant non-dimensional parameters describing the thermal conductivity and thermo-geometrical property of fins. The results revealed that increase in the thermal conductivity increases the wall temperature, while increase in the thermo-geometrical parameter reduces the wall temperature, and that fin efficiency is dependent on both the thermal conductivity and thermo-geometrical property. The LMM results compared with previous numerical, HAM, DTM and available analytical results demonstrated excellent agreements.

Keywords Leibnitz-Maclaurin Method (LMM), Success Differential Coefficients (SDC), Straight Fins

Introduction

Fins are highly conductive surfaces that extend the area of a given system and are used to increase or enhance the rate of heat transfer from a heated surface to a convecting fluid. Ghoshdastidar [1] stated that the extended surfaces could be attached to the base of the material by pressing, soldering, or welding or in some cases they may be integral parts of the base material obtained by a casting or extruding process. Fatoorehchi and Abolghasemi[2] simply regarded fins as echo-friendly and economic means of convective heat transfer enhancement. The goal of such systems sometimes is to promote heat loss from a hot surface, and so, they are better known as heat sinks. Fins have served thermal management of electronic systems over the years [3] and [4], and they are very much still in use in the engineering industry. Examples of the use of finned surfaces are bound such as those used on car radiators and heating units, heat exchangers, air-cooled engines, electrical transformers, motors, electronic transistors, and computer systems. There are several types of fins of different profile shapes, viz. rectangular, cylindrical, trapezoidal, conical, triangular, etc., which are well discussed in Rong-Hua [5]; Middleman [6]; Kraus et al. [7]; Bejan and Kraus [8].

It must be said here that to understand the heat transfer physics via finned surfaces, simple mathematical models are derived considering the type of the fin vis-a-vis the material properties of the fin. The resulting mathematical models are either partial or ordinary differential equations for the temperature distribution; depending also on whether the heat transfer scenarios are steady or unsteady states. The derivations of the mathematical models are based on simplifying assumptions such as (a) heat flow in the extended surface is steady or unsteady, (b) the temperature of the surrounding fluid is uniform and constant, (c) uniform convective heat transfer coefficient, (d) thermal conductivity of the fin material is constant or temperature-dependent, and (e) one-dimensional heat conduction in the fin. However, most heat exchangers may not satisfy only one of these assumptions.

Heat transfer mechanisms include heat conduction, thermal radiation, and mass transfer, and these are extensively discussed in many standard texts such as in Holman [9]. The purpose of the fin is to increase the product of the surface area and the heat transfer coefficient [10]. Knowing the efficiency of the fin is very useful to the heat exchanger design or in the estimation of heat exchanger performance [11].

Fins with variable thermal conductivity are more realistic and have been paid attention to by many thermal engineers and researchers. Linearly temperature-dependent thermal conductivity for a straight longitudinal fin has been studied by Arslanturk [12]. A very similar problem was solved by Joneidi et al. [13] through the Differential Transform Method (DTM). Rajabi [14] used Homotopy Perturbation Method (HPM) to evaluate the efficiency of straight fins with temperature-dependent thermal conductivity. Coşkun and Atay [15] employed Variational Iteration Method (VIM) to solving heat transfer problem in radiating extended surfaces, which are common for enhancing heat transfer between primary surface and the environment. It is known that especially for large temperature differences, variable thermal conductivity has a long effect on the performance of such a surface. Aziz and Bouaziz [16] used the least square method for predicting the performance of a longitudinal fin with temperature-dependent internal heat generation and thermal conductivity. Ganji et al. [17] explored the HPM for solving most engineering problems especially heat transfer equations in non-linear form.

On the hand, Liao [18] applied Homotopy Analysis Method (HAM) in tackling the solutions of non-linear problems. Saravanakumar et al. [19] used HAM to evaluate nonlinear boundary value problem of fin efficiency of convective straight fins with temperature-dependent thermal conductivity.

It is pertinent to state here that the techniques of DTM, HPM, HAM and VIM all render their results in series form. Recently, Mebine [20] used Successive Differential Coefficients (SDC) technique, a power series method for solving MHD velocity slip boundary layer flow over a plane plaque. It was observed that the solution obtained revealed its simplicity, capability, effectiveness and high accuracy when compared to numerical experimentation. It is the objective of this work, therefore, to apply the SDC technique which is a token of power series solution called Leibnitz-Maclaurin Method (LMM) for the problem of fin efficiency for convective straight fin with temperature-dependent thermal conductivity. For effectiveness and accuracy of the LMM, the results so obtained are compared with the DTM applied by Joneidi et al.[13] and HAM applied by Saravanakumar et al.[19] coupled with an exact analytical and numerical solutions. The sections that follow hereafter are Mathematical Formulations, Leibnitz-Maclaurin Method of Solution, Analyses of Results, and Concluding Remarks.

Mathematical Formulations

Consider a straight fin with a temperature-dependent thermal conductivity, with an arbitrary constant cross sectional area A_C; perimeter P and length b. Figure 1 depicts the physical model of the fin. The fin is attached to a base surface of temperature T_b and extends into a fluid of temperature T_a, and its tip is insulated.

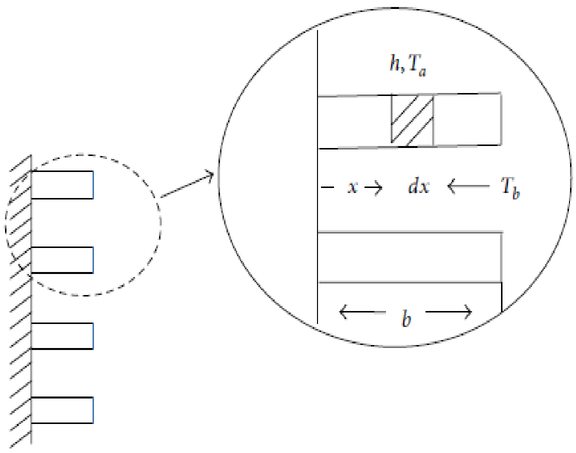

Figure 1: Sketch of physical model of straight fins adapted from Saravanakumar et al. [19]

The one-dimensional energy balance equation is given as follows:

$$A_C \frac{d}{dx}\left[k(T)\frac{dT}{dx}\right] - Ph(T_b - T_a) = 0, \quad (1)$$

where T is temperature, $k(T)$ is temperature-dependent thermal conductivity of the fin material, P is the fin perimeter, and h is the heat transfer coefficient. The thermal conductivity of the material is assumed as follows:

$$k(T) = k_a[1 + \lambda(T - T_a)], \tag{2}$$

where k_a is the thermal conductivity at the ambient fluid of the fin temperature and λ is the parameter describing the variation of the thermal conductivity. As in, Mokheimer [21], the following dimensionless parameters are introduced:

$$\theta = \frac{T - T_a}{T_b - T_a}, \xi = \frac{x}{b}, \beta = \lambda(T_b - T_a), \psi = \left(\frac{hPb^2}{k_a A_C}\right)^{1/2}. \tag{3}$$

Equation (1) is now rendered non-dimensional as

$$\frac{d^2\theta}{d\xi^2} + \beta\theta\frac{d^2\theta}{d\xi^2} + \beta\left(\frac{d\theta}{d\xi}\right)^2 - \psi^2\theta = 0, \tag{4}$$

with the associated boundary conditions stated as follows:

$$\frac{d\theta}{d\xi} = 0 \quad \text{when} \quad \xi = 0$$
$$\theta = 1 \quad \text{when} \quad \xi = 1, \tag{5}$$

where θ is the dimensionless temperature, ξ is the non-dimensional coordinate, β is the non-dimensional parameter describing thermal conductivity, and ψ is the thermogeometric fin parameter.

The heat transfer rate from the fin is found by using Newton's law of cooling which states that "for a body cooling in a draft, that is forced convection, the rate of heat loss is proportional to the difference in temperature between the body and the surrounding."

Consider

$$Q = \int_0^b P(T - T_a)dx. \tag{6}$$

The ratio of the fin heat transfer rate to the heat transfer rate of the fin if the entire fin was at the base temperature is commonly called as the fin efficiency:

$$\eta = \frac{Q}{Q_{ideal}} = \frac{\int_0^b P(T - T_a)dx}{Pb(T_b - T_a)} = \int_{\xi=0}^1 \theta(\xi)d\xi. \tag{7}$$

In other words, the fin efficiency is simply the parameter that indicates the effectiveness of a fin in transferring a given quantity of heat.

The equation (4) together with the equation (5) is solved with the LMM via the SDC for the analyses of the problem of fin efficiency (7) of convective straight fins with temperature-dependent thermal conductivity.

Leibnitz-Maclaurin Method of Solution

For two differentiable and continuous functions, say u and v, which are functions of say x, Leibnitz concise formula for the nth differential coefficient of their product is:

$$D^n(uv) = \sum_{r=0}^{n} {}^nC_r D^{n-r}v \cdot D^r u, \tag{8}$$

where ${}^nC_r = \frac{n!}{(n-r)!r!}, D = \frac{d}{dx}, D^2 = \frac{d^2}{dx^2}, D^3 = \frac{d^3}{dx^3}, \ldots, D^n = \frac{d^n}{dx^n}$.

The Leibnitz's formula is applied to the differential equations (4, 5) in obtaining a recurrence relation between successive differential coefficients. This forms a step towards finding a power series solution of the problem at hand.

In the application of Leibnitz's formula, the solution of the problem is written in terms of Maclaurin series which is a special case of Taylor series, such that

$$\theta(\xi) = \theta(0) + \frac{\xi}{1!}\theta'(0) + \frac{\xi^2}{2!}\theta''(0) + \ldots = \sum_{r=0}^{n} \frac{\xi^r}{r!}\theta^{(r)}(0), \tag{9}$$

where $\theta(0) = \theta(\xi)\big|_{\xi=0}, \theta'(0) = \frac{d\theta}{d\xi}\big|_{\xi=0}, \theta''(0) = \frac{d^2\theta}{d\xi^2}\Big|_{\xi=0}, \cdots, f^{(r)}(0) = \frac{d^r\theta}{d\xi^r}\big|_{\xi=0}$.

From the boundary conditions, it is observed that the value of $\theta(0)$ is unknown. To apply the Leibnitz formula to finding the SDC at $\xi = 0$, let $\theta(0) = \alpha$, be an undetermined coefficient that would be computed with the help of condition (5b). From the physical point of view, $\theta(0)$ is the quantity of interest, and it is the rate of heat at the wall of the fin. The recurrence relation for the SDC by the application of equation (8) to the equation (4) is now written as

$$\theta^{n+2}(0) = \psi^2 \sum_{r=0}^{n} {}^nC_r\, D^{n-r}(1+\beta\,\theta(0))^{-1} \cdot D^r\theta(0) - \beta \sum_{r=0}^{n} D^{n-r}(1+\beta\,\theta(0))^{-1} \cdot D^r\left(\frac{d\theta(0)}{d\xi}\right)^2 \text{ for } n \geq 0. \tag{10}$$

The first nine SDC from equation (10) with the use of condition (5a) are stated as follows:

$$\begin{aligned}
&\theta(0) = \alpha,\\
&\theta'(0) = 0,\\
&\theta''(0) = \frac{\alpha\psi^2}{(1+\alpha\beta)},\\
&\theta'''(0) = 0,\\
&\theta^{iv}(0) = \frac{\alpha\psi^4}{(1+\alpha\beta)^2} - \frac{3\alpha^2\psi^4\beta}{(1+\alpha\beta)^3},\\
&\theta^{v}(0) = 0,\\
&\theta^{vi}(0) = \frac{\alpha\psi^6}{(1+\alpha\beta)^3} - \frac{18\alpha^2\psi^6\beta}{(1+\alpha\beta)^4} + \frac{45\alpha^3\psi^6\beta^2}{(1+\alpha\beta)^5},\\
&\theta^{vii}(0) = 0,\\
&\theta^{viii}(0) = \frac{\alpha\psi^8}{(1+\alpha\beta)^4} - \frac{66\alpha^2\psi^8\beta}{(1+\alpha\beta)^5} + \frac{669\alpha^3\psi^8\beta^2}{(1+\alpha\beta)^6} - \frac{1440\alpha^4\psi^8\beta^3}{(1+\alpha\beta)^7}.
\end{aligned} \tag{11}$$

From the equation (11) one can readily see that all the odd terms are zero. For want of more accuracy, as many SDC as possible may be computed, but with much more difficulty in the computations. One advantage of SDC is that in some physical problems only few terms may be computed and it converges to the required result. Of course, with the aid of Symbolic Computation Software such as Maple, Mathematica and Matlab, as many terms as possible and as desired could be computed!

It is important to note that the Taylor series and the Maclaurin series only represent the function $\theta(\xi)$ in their intervals of convergence.

In the absence of the thermal conductivity parameter β, (i.e. $\beta = 0$) equation (4) together with the boundary conditions (5) gives an exact solution:

$$\theta(\xi) = \frac{\cosh(\psi\xi)}{\cosh(\psi)}. \tag{12}$$

Consequent upon $\beta = 0$, the series solution (9) together with SDC (10) now becomes

$$\theta(\xi)=\alpha+\frac{1}{2}\alpha\psi^2\xi^2+\frac{1}{24}\alpha\psi^4\xi^4+\frac{1}{720}\alpha\psi^6\xi^6+\frac{1}{40320}\alpha\psi^8\xi^8+\cdots \tag{13}$$

To determine α from equation (13), use is made of the boundary condition $\theta=1$ when $\xi=1$ (equation (5)).

The efficiency of the fin according to equation (7) is now obtained as

$$\eta=\frac{1}{362880}\frac{\alpha\psi^8}{(1+\alpha\beta)^4}-\frac{11}{60480}\frac{\alpha^2\psi^8\beta}{(1+\alpha\beta)^5}+\frac{233}{120960}\frac{\alpha^3\psi^8\beta^2}{(1+\alpha\beta)^6}-\frac{1}{252}\frac{\alpha^4\psi^8\beta^3}{(1+\alpha\beta)^7}$$
$$+\frac{1}{5040}\frac{\alpha\psi^6}{(1+\alpha\beta)^3}-\frac{1}{280}\frac{\alpha^2\psi^6\beta}{(1+\alpha\beta)^4}+\frac{1}{112}\frac{\alpha^3\psi^6\beta^2}{(1+\alpha\beta)^5}+\frac{1}{120}\frac{\alpha\psi^4}{(1+\alpha\beta)^2}-\frac{1}{40}\frac{\alpha^2\psi^4\beta}{(1+\alpha\beta)^3}$$
$$+\frac{1}{6}\frac{\alpha\psi^2}{(1+\alpha\beta)}+\alpha. \tag{14}$$

Analyses of Results

Figure 2 depicts two scenarios-solid lines and field plots (arrows). The solid lines are plots of the non-dimensional temperature θ versus the non-dimensional coordinate ξ for various values of ψ when $\beta=0$ (linear case), while the field plots are for the particular case of $\psi=0\cdot50$ when $\beta=0$ (linear case). From the Figure 2, it is seen that when ψ increases (i.e., the fin length b increases or the cross-sectional area of the fin A_c decreases), the dimensionless temperature θ decreases at the wall. This implies that the temperature of the wall is directly affected by the increase or decrease of the thermo-geometric fin parameter ψ.

The field plot or direction field, on the other hand, for $\psi=0\cdot50$ when $\beta=0$ (linear case) demonstrates the visualization of the solution for $\alpha=0\cdot8868189600\,4$ which is the rate of heat at the wall in the absence of thermal conductivity parameter β. Therefore, it is important to say that the solution (9) to the problem posed by the equation (4), where $\theta'(\xi)=f(\xi,\theta)$ has a simple geometrical interpretation in case f is real-valued, and θ is defined on a set S of real numbers. Then for each ξ in an interval I and θ in S there is a number $f(\xi,\theta)$, which may be thought of as the slope of a straight line through the point (ξ,θ). A solution of $\theta'(\xi)=f(\xi,\theta)$ on f on I is a function ϕ whose graph (the set of points $(\xi,\phi(\xi))$, ξ in I) is a curve whose tangent at $(\xi,\phi(\xi))$ has the slope $\phi'(\xi)$, which is the same as the region slope $f(\xi,\phi(\xi))$ at this point. Thus, geometrically we are given a set of directions, and the differential equation (4) is the problem of finding curves having these directions as tangents. The set of directions $|f(\xi,\theta)|$ is called a direction field. Figure 2 shows such a field for

$$\theta'(\xi)=\frac{\xi\alpha\psi^2}{(1+\alpha\beta)}+\frac{1}{6}\xi^3\left(\frac{\alpha\psi^4}{(1+\alpha\beta)^2}-\frac{3\alpha^2\psi^4\beta}{(1+\alpha\beta)^3}\right)+\frac{1}{120}\xi^5\left(\frac{\alpha\psi^6}{(1+\alpha\beta)^3}-\frac{18\alpha^2\psi^6\beta}{(1+\alpha\beta)^4}+\frac{45\alpha^3\psi^6\beta^2}{(1+\alpha\beta)^5}\right)$$
$$+\frac{1}{5040}\xi^7\left(\frac{\alpha\psi^8}{(1+\alpha\beta)^4}-\frac{66\alpha^2\psi^8\beta}{(1+\alpha\beta)^5}+\frac{669\alpha^3\psi^8\beta^2}{(1+\alpha\beta)^6}-\frac{1440\alpha^4\psi^8\beta^3}{(1+\alpha\beta)^7}\right)$$

for $\psi=0\cdot50$ when $\beta=0$ and the solid lines is the solution (12) of the equation (4) for $\beta=0$ for respective values of $\psi=0\cdot50,\ 0\cdot55,\ 0\cdot60,\ 0\cdot65$.

By the reason of the exact linear solution (12) for $\beta=0$, the solid lines in Figure 2 depict the dependence on ψ of the exact linear solution of the dimensionless temperature with respect to the dimensionless coordinate ξ. Therefore, the exact linear solution (12) for $\beta=0$ is now applied to validate the LMM solution (13) versus numerical solution (NS) for

$\xi = 0$ as shown in Table 1. It is clearly observed that the LMM solution exactly represents the NS, demonstrating the authenticity of the LMM solution.

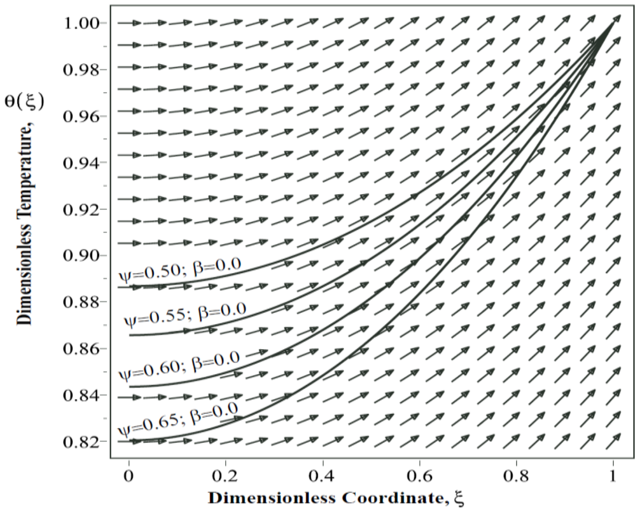

Figure 2: Dimensionless temperature θ versus dimensionless coordinate ξ for various values of ψ when $\beta = 0$ and field plots for $\psi = 0 \cdot 50$; $\beta = 0$

Table 1: LMM (13) compared with numerical and exact solution (12) for values of α for various values of ψ when $\beta = 0$

ψ	$\theta(0) = \alpha$, $\beta = 0$		
	NS	**Exact**	**LMM Present**
0.50	0.886819	0.886819	0.886819
0.55	0.865725	0.865725	0.865725
0.60	0.843551	0.843551	0.843551
0.65	0.820484	0.820484	0.820484

Table 2 gives a comparison of the LMM, HAM (Saravanakumar et al. [19]), DTM (Joneidi et al. [13]) and the exact linear solution (12) for $0.00 \leq \xi \leq 1.00$ for $\psi = 0.5$ and $\psi = 1.0$, respectively, at $\beta = 0$. The LMM agrees exactly with the HAM, DTM and the exact linear solution (12).

For the non-linear case of $\beta \neq 0$, Table 3 compares the LMM with the results of HAM, DTM and NS. The results agree excellently well.

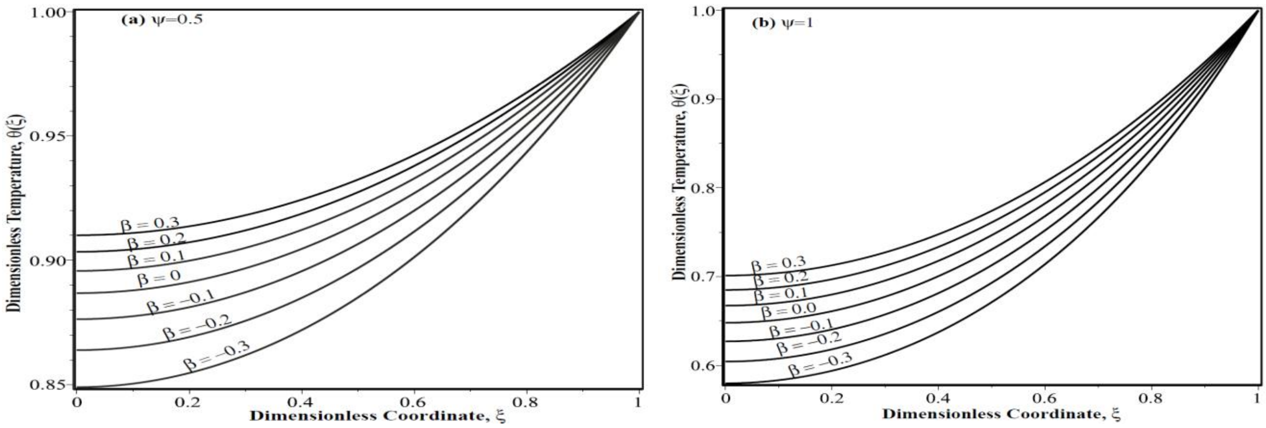

Figure 3: Dimensionless temperature θ versus dimensionless coordinate ξ for various values of β: (a) $\psi = 0.5$, (b) $\psi = 1$

Table 2: Comparisons of LMM, HAM, DTM and Exact linear solution (12)

ξ	(a) $\beta = 0, \psi = 0.5$ (linear case)				(b) $\beta = 0, \psi = 1.0$ (linear case)			
	HAM	**DTM**	**Exact**	**LMM Present**	**HAM**	**DTM**	**Exact**	**LMM Present**
0.00	0.88681	0.88681	0.88681	0.886819	0.64805	0.64805	0.64805	0.648054
0.05	0.88709	0.88709	0.88709	0.887096	0.64886	0.64886	0.64886	0.648865
0.10	0.88792	0.88792	0.88792	0.887928	0.65129	0.65129	0.65129	0.651298
0.15	0.88931	0.88931	0.88931	0.889314	0.65535	0.65535	0.65535	0.655359
0.20	0.89125	0.89125	0.89125	0.891257	0.66105	0.66105	0.66105	0.661059
0.25	0.89375	0.89375	0.89375	0.893756	0.66841	0.66841	0.66841	0.668412
0.30	0.89681	0.89681	0.89681	0.896814	0.67743	0.67743	0.67743	0.677436
0.35	0.90043	0.90043	0.90043	0.900433	0.68815	0.68815	0.68815	0.688155
0.40	0.90461	0.90461	0.90461	0.904614	0.70059	0.70059	0.70059	0.700594
0.45	0.90936	0.90936	0.90936	0.909361	0.71478	0.71478	0.71478	0.714785
0.50	0.91467	0.91467	0.91467	0.914677	0.73076	0.73076	0.73076	0.730763
0.55	0.92056	0.92056	0.92056	0.920564	0.74856	0.74856	0.74856	0.748569
0.60	0.92702	0.92702	0.92702	0.927026	0.76824	0.76824	0.76824	0.768246
0.65	0.93406	0.93406	0.93406	0.934068	0.78984	0.78984	0.78984	0.789844
0.70	0.94169	0.94169	0.94169	0.941693	0.81341	0.81341	0.81341	0.813418
0.75	0.94990	0.94990	0.94990	0.949907	0.83902	0.83902	0.83902	0.839025
0.80	0.95871	0.95871	0.95871	0.958715	0.86673	0.86673	0.86673	0.866731
0.85	0.96812	0.96812	0.96812	0.968123	0.89660	0.89660	0.89660	0.896603
0.90	0.97813	0.97813	0.97813	0.978135	0.92871	0.92871	0.92871	0.928718
0.95	0.98875	0.98875	0.98875	0.988758	0.96315	0.96315	0.96315	0.963155
1.00	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000

Table 3: Comparisons of LMM, HAM, DTM and Numerical solution (Non-linear)

ξ	$\beta = 0.4, \psi = 1.0$ (non-linear case)				$\beta = 0.2, \psi = 0.5$ (non-linear case)			
	HAM	**DTM**	**NS**	**LMM Present**	**HAM**	**DTM**	**NS**	**LMM Present**
0.00	0.71604	0.71604	0.71604	0.716044	0.90344	0.90344	0.90344	0.903447
0.05	0.71674	0.71674	0.71674	0.716740	0.90344	0.90344	0.90368	0.903686
0.10	0.71883	0.71883	0.71883	0.718828	0.90440	0.90440	0.90440	0.904404
0.15	0.72231	0.72231	0.72231	0.722309	0.90559	0.90559	0.90559	0.905599
0.20	0.72718	0.72718	0.72718	0.727186	0.90727	0.90727	0.90727	0.907276
0.25	0.73346	0.73346	0.73346	0.733462	0.90942	0.90942	0.90942	0.909429
0.30	0.74114	0.74114	0.74114	0.741140	0.91206	0.91206	0.91206	0.912063
0.35	0.75022	0.75022	0.75022	0.750226	0.91517	0.91517	0.91517	0.915178
0.40	0.76072	0.76072	0.76072	0.760725	0.91877	0.91877	0.91877	0.918774
0.45	0.77264	0.77264	0.77264	0.772644	0.92285	0.92285	0.92285	0.922853
0.50	0.78599	0.78599	0.78599	0.785990	0.92741	0.92741	0.92741	0.927416
0.55	0.80077	0.80077	0.80077	0.800770	0.93246	0.93246	0.93246	0.932464
0.60	0.81699	0.81699	0.81699	0.816995	0.93799	0.93799	0.93799	0.937998
0.65	0.83467	0.83467	0.83467	0.834672	0.94402	0.94402	0.94402	0.944021
0.70	0.85381	0.85381	0.85381	0.853811	0.95053	0.95053	0.95053	0.950533
0.75	0.87442	0.87442	0.87442	0.874425	0.95753	0.95753	0.95753	0.957537
0.80	0.89652	0.89652	0.89652	0.896523	0.96503	0.96503	0.96503	0.965034
0.85	0.92011	0.92011	0.92011	0.920117	0.97302	0.97302	0.97302	0.973026
0.90	0.94522	0.94522	0.94522	0.945219	0.98151	0.98151	0.98151	0.981517
0.95	0.97184	0.97184	0.97184	0.971843	0.99050	0.99050	0.99050	0.990507
1.00	1.00000	0.99999	1.00000	1.000000	0.99999	0.99999	0.99999	1.000000

temperature distribution of the fin for different values of $\psi$and β. Figure 3(a) and (b) indicate that the wall temperature α increases with increase in the thermal conductivity β with the wall temperatures in Figure 3(a) significantly greater than those in Figure 3(b). Physically, thermal conductivity enhances the wall temperatures, while increase in the thermo-geometric parameter ψ de-enhances the wall temperatures. The de-enhancing nature of the thermo-geometric parameter ψ is equally observed in Figure 2 and Tables 1-3.
The fin efficiency result (14) depends on the thermal conductivity of the fin β and thermo-geometric parameter ψ. In order to obtain optimal fin efficiency, optimal or critical value of the wall temperature α , should be calculated from the result, equation (14). For the purpose of brevity, graphical illustrations of the fin efficiency are not presented.

Concluding Remarks

The problem of convective straight fins with temperature-dependent thermal conductivity has been tackled by the use of Leibnitz-Maclaurin Method via Successive Differential Coefficients. The provided comparisons ascertain the high accuracy and excellent performance of the LMM, which houses the DTM, HPM and HAM. The approach has several merits such as, fast convergence to the available analytical solution, high accuracy, simplicity, algorithmic nature, and not requiring any linearization, discretization, or perturbation. For further validation of the LMM, the problem is extendable to incorporate other heat transfer mechanisms such as nucleate boiling and radiation in a unified temperature distribution model in straight fins with temperature-dependent thermal conductivity and heat transfer coefficient. The variability of thermal conductivity and heat transfer coefficient would be physically meaningful and would serve as a practical departure from previously investigated constant scenarios of thermal conductivity and heat transfer coefficient.

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