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## Selection of Automotive Brake Friction Materials using Hybrid Entropy-Topsis Approach

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**Abstract** Technique for order preference by similarity to ideal solution (TOPSIS) approach coupled with entropy method was proposed to select automotive brake friction materials. The brake friction materials modified with lapinus and brass fibre by different weight, were designed, fabricated and characterize for tribological performance on a Krauss type friction tester and the test results were considered as criteria for selection. By entropy, the weight for each criterion was calculated, while TOPSIS was employed to determine the complete ranking of the friction materials. The result shows that the formulation of 10 % lapinus fibre and 5 % brass fiber exhibits the optimal properties.

**Keywords** Brake friction materials, multi-criteria decision-making, Lapinus; Brass, Entropy, TOPSIS

### 1. Introduction

Braking system is one of the most important safety features of an automobile. Safe and repeatable stopping is the ability of brakes, which is related to safety of automobiles and human beings [1]. Key components of any braking system have been considered as the friction material which determines its tribological performance [2]. Friction materials are multi-ingredient composites generally containing phenolic resin as binder in which fibres, fillers property modifiers are distributed to fulfill the diverse and conflicting performance requirements such as resistance to fading, high and stable coefficient of friction, squeal, wear, judder etc. along with good recovery, noise propensity etc. [3-10]. It is sad that friction materials have been developed for a century, the formulations used in current commercial brake friction materials are obtained by a trial and error method, which means that some ingredients are added less on knowledge and more on tradition [11]. Composition of the friction composite material is the main aspect upon which the tribological performance of the friction composite materials largely depends. Compositional variations of different friction materials make the development and performance evaluations of new formulations are an intricate task. The complication in performance evaluation arises more as the same composition the friction composite materials yield different results with different manufacturing conditions [12]. The optimal material is selected for any application from many alternative materials on the basis of different criteria is a Multiple Criteria Decision-Making (MCDM) problem. MCDM methods perform an important role for the decision process for both the small and large problems involving finite number of alternatives and criteria.

In the past, several researchers used different MCDM approaches such as simple additive weighted method, weighted product method, vise kriterijumska optimizacija kompromisno resenje method, technique for order preference by similarity to ideal solution (TOPSIS), elimination and choice translating reality, analytical hierarchy process, grey relation analysis, preference selection index method and entropy to various areas such as engineering, science, management etc [13-22]. Among them, TOPSIS and entropy are two popular ones. TOPSIS was developed by Hwang and Yoon [23] and widely used for tackling ranking problems due to its



simplicity. The entropy method was introduced by Shannon [24] and nowadays it is widely used in engineering and management [22]. Hence this paper used the entropy method for the estimation of weights and TOPSIS to rank the alternatives.

## 2. Materials and Methods

### 2.1. Fabrication of composites

Brake friction materials containing proportions of lapinus and brass fibre are shear mixed with fixed amount of straight phenol-formaldehyde (PF) resin, Kevlar fibre, barite, alumina, vermiculite and graphite amounting to 100% by weight as depicted in Table 1. The fabrication conditions during compression molding are given in Table 2 and briefly reported elsewhere [25-26].

**Table 1:** Details of composite composition and designation

Composition (wt. %)	PF Resin	Kevlar	Vermiculite	Alumina	Graphite	Barite	Lapinus	Brass
FC-1	15	5	5	5	5	50	15	0
FC-2	15	5	5	5	5	50	10	5
FC-3	15	5	5	5	5	50	5	10
FC-4	15	5	5	5	5	50	0	15

**Table 2:** Processing details while fabricating the friction composites

Procedure	Conditions
Sequential mixing	A plough type of shear mixture (feeder and chopper speeds of 300 and 3000 rpm) is used for mixing. Mixing sequence: First powdery ingredients were mixed for 5 min, thereafter fibrous ingredients were added for another 5 min.
Compression moulding	150°C, Compression pressure of 15 MPa, Curing time =10 minutes with five intermittent breathings to expel volatiles.
Post-curing	170°C, 4 h for relieving any residual stresses developed during the compression molding cycle.

### 2.2. Tribo-performance evaluation methodology

The tribo-performance evaluation tests were conducted on a Krauss type friction tester which is computer-controlled and have data acquisition capabilities. In order to evaluate the various tribological properties of the friction materials the standard regulatory test PVW-3212 as per the European norms conforming to Economic Commission for Europe (ECE) R-90 regulations has been adopted. The details of the machine and the protocol behind PVW-3212 standard reported elsewhere [27-29].

### 2.3. Combined entropy and TOPSIS method

Multi-criteria decision making (MCDM) is an emerging branch of operations research and it is exclusively used in decision making problems which involves more than one criterion. Among the various MCDM approaches, entropy and TOPSIS are very popular due to their simplicity, wide applicability and robustness. In case of combined entropy-TOPSIS method the weights of criterion from entropy method are fused with the other steps of TOPSIS as explained below:

*Step 1:* After identifying the relevant goal, criteria and alternatives of the problem, a decision matrix of criteria and alternatives is formulated based on the information available regarding the problem. If the number of alternative is M and the number of criteria are N then the decision matrix having an order of  $M \times N$  is represented as:



$$X_{M \times N} = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1N} \\ x_{21} & x_{22} & \dots & x_{2N} \\ \vdots & \vdots & \dots & \vdots \\ x_{M1} & x_{M2} & \dots & x_{MN} \end{bmatrix} \quad (1)$$

Where, an element  $x_{ij}$  of the performance matrix  $X_{M \times N}$  represents the actual value of the  $i^{\text{th}}$  alternative in term of  $j^{\text{th}}$  criterion.

*Step II:* The decision matrix is normalized to making all the values of the decision matrix comparable. The normalization of the decision matrix is calculated as:

$$X'_{ij} = \frac{x_{ij}}{\left[ \sum_{i=1}^M (x_{ij}^2) \right]^{1/2}} \quad (2)$$

*Step III:* In order to convert the normalized matrix to weighted normalized matrix, the weights of various criteria are determined by the entropy method. First of all the projection value ( $P_{ij}$ ) for each alternative is calculated as:

$$P_{ij} = \frac{x_{ij}}{\sum_{i=1}^M x_{ij}} \quad (3)$$

After the calculation of projection value, entropy of each criterion is calculated as:

$$\Gamma_j = -\alpha \sum_{j=1}^N P_{ij} \ln(P_{ij}) \quad (4)$$

where  $\alpha$  is a constant and calculated as,  $\alpha = \frac{1}{\ln(M)}$

Next the dispersion value ( $\chi_j$ ) of each criterion is calculated as:

$$\chi_j = 1 - \Gamma_j \quad (5)$$

Finally the weight of each criterion is calculated as:

$$\omega_j = \frac{\chi_j}{\sum_{j=1}^N \chi_j} \quad (6)$$

Weighted normalized matrix ( $X''_{ij}$ ) is calculated by multiplying the normalized matrix by its associated weights ( $\omega_j$ ) as:

$$X''_{ij} = X'_{ij} \times \omega_j \quad (7)$$

*Step IV:* The positive ideal solution ( $\rho^+$ ) and the negative ideal solution ( $\rho^-$ ) are determined from the  $X''_{ij}$  as shown:

$$\rho^+ = (X_1^{''+}, X_2^{''+} \dots X_N^{''+}), \text{ and} \\ \rho^- = (X_1^{''-}, X_2^{''-} \dots X_N^{''-}) \quad (8)$$

*Step V:* The Euclidian distances between each of the alternatives and the positive ideal solution and the negative ideal solution are calculated as:



$$\Phi_i^+ = \sqrt{\sum_{j=1}^N (X_j^{''+} - X_{ij}^{''})^2}, \text{ and}$$

$$\Phi_i^- = \sqrt{\sum_{j=1}^N (X_{ij}^{''} - X_j^{''-})^2}, \text{ for } i = 1, 2, \dots, M \quad (9)$$

Step VI: Finally, the overall preference or closeness index ( $\nabla_i$ ) value of the alternatives is calculated. All the alternatives are then arranged in descending order according to the value of their closeness index. The alternative at the top of the list is the most preferred one. The  $\nabla_i$  value of the  $i^{\text{th}}$  alternatives is calculated as:

$$\nabla_i = \frac{\Phi_i^-}{\Phi_i^+ + \Phi_i^-}, \text{ for } i = 1, 2, \dots, M \quad (10)$$

### 3. Result and discussion

#### 3.1. Determination of decision matrix, weight criteria and ranking of the alternatives

The description of eight selected PDAs and their experimental data of the four investigated alternatives are given in Table 3 and Table 4 respectively. The decision matrix from Eq.1 is used for the entropy-TOPSIS analysis. To make matrix compatible, the values of the decision matrix are normalized in the range of 0-1 by using Eq. 2 and the normalized matrix is given in Table 5.

**Table 3:** Description of the different performance defining attributes

Performance defining attributes (PDAs)	Performance implications of PDAs	Description of the individual PDAs
Coefficient of friction ( $\mu$ )	PDA-1 Higher-the-better	It is the average friction coefficient of cold, fade and recovery cycles.
Friction fade-%	PDA-2 Lower-the-better	%-Fade = $\frac{\mu_P - \mu_F}{\mu_P} \times 100$ , $\mu_F$ is the minimum coefficient of friction for the fade cycles taken after 270°C.
Friction recovery-%	PDA-3 Higher-the-better	%-Recovery = $\frac{\mu_R}{\mu_P} \times 100$ , $\mu_R$ is the maximum coefficient of friction for the recovery cycle taken after 100°C.
Stability coefficient (%)	PDA-4 Higher-the-better	It is the ratio of the coefficient of friction to the maximum friction i.e. $\mu_P/\mu_{\max}$ .
Variability coefficient (%)	PDA-5 Lower-the-better	It is the ratio of friction fluctuation to the maximum friction coefficient, i.e. $(\mu_{\max} - \mu_{\min})/\mu_{\max}$ .
Friction Fluctuation ( $\mu_{\max} - \mu_{\min}$ )	PDA-6 Lower-the-better	It is the difference between the maximum and minimum friction coefficient i.e. $\mu_{\max} - \mu_{\min}$ .
Wear (gm)	PDA-7 Lower-the-better	It is the progressive loss of the material from the brake pad surface during working.
Disc temperature rise (DTR) °C	PDA-8 Lower-the-better	It is the maximum temperature rise of the disc due to the friction braking irrespective of all the runs.



**Table 4:** Experimental data of the PDAs

Alternative	PDA-1	PDA-2	PDA-3	PDA-4	PDA-5	PDA-6	PDA-7	PDA-8
FM-0	0.393	22.14	106.11	0.129	0.91	0.70	11.38	576
FM-1	0.400	14.50	113.75	0.122	0.88	0.73	9.53	571
FM-2	0.409	14.67	116.63	0.147	0.86	0.69	10.85	582
FM-3	0.428	16.59	119.63	0.18	0.84	0.65	12.60	598

**Table 5:** Normalized decision matrix

Alternative	PDA-1	PDA-2	PDA-3	PDA-4	PDA-5	PDA-6	PDA-7	PDA-8
FM-0	0.4820	0.6415	0.4648	0.4411	0.5224	0.5051	0.5106	0.4950
FM-1	0.4905	0.4202	0.4983	0.4171	0.5046	0.5275	0.4276	0.4907
FM-2	0.5016	0.4251	0.5109	0.5026	0.4920	0.4986	0.4868	0.5001
FM-3	0.5249	0.4807	0.5240	0.6155	0.4799	0.4669	0.5653	0.5139

The weights of the selected PDAs are calculated by using Eqs. 3-6 and given in Table 6. The relative weight of PDA-2 or fade-% (0.4402) was found to be the maximum followed by PDA-4 or frictional fluctuations (0.3282), PDA-7 or wear (0.1429), PDA-3 or recovery-% (0.0309), PDA-6 or frictional variability (0.0270), PDA-1 or coefficient of friction (0.0154), PDA-5 or frictional stability (0.0116) and PD8-4 or disc temperature rise (0.0038). After the determination of criterion weights, a weighted decision matrix is calculated by using Eq. 7 and given in Table 7. Further the positive ideal solution ( $\rho^+$ ) and the negative ideal solution ( $\rho^-$ ) are determined by using Eq. 8 and shown in Table 7. Then the distance between each alternative from  $\phi_i^+$  and  $\phi_i^-$  is calculated by using Eq. 9. Finally, the overall preference closeness index ( $\Delta_i$ ) of the alternatives is calculated by using Eq. 10. All the alternatives are then arranged in descending order according to the value of their closeness index. The alternative at the top of the list is the most preferred one. The results are shown in Table 8.

**Table 6:** Criteria weights evaluated by entropy method

Criterion	Entropy, $\Gamma_j$	Dispersion value, $\chi_j$	Weight, $w_j$
PDA-1	0.9996	0.0004	0.0154
PDA-2	0.9886	0.0114	0.4402
PDA-3	0.9992	0.0008	0.0309
PDA-4	0.9915	0.0085	0.3282
PDA-5	0.9997	0.0003	0.0116
PDA-6	0.9993	0.0007	0.0270
PDA-7	0.9963	0.0037	0.1429
PDA-8	0.9999	0.0001	0.0038

**Table 7:** Weighted normalized decision matrix

Alternative	PDA-1	PDA-2	PDA-3	PDA-4	PDA-5	PDA-6	PDA-7	PDA-8
FM-0	0.0074	0.2824	0.0144	0.1448	0.0061	0.0136	0.0730	0.0019
FM-1	0.0076	0.1850	0.0154	0.1369	0.0059	0.0142	0.0611	0.0019
FM-2	0.0077	0.1871	0.0158	0.1650	0.0057	0.0135	0.0696	0.0019
FM-3	0.0081	0.2116	0.0162	0.2020	0.0056	0.0126	0.0808	0.0020
$\rho^+$	0.0081	0.1850	0.0162	0.1369	0.0061	0.0142	0.0611	0.0019
$\rho^-$	0.0074	0.2824	0.0144	0.2020	0.0056	0.0126	0.0808	0.0020



**Table 8:** Closeness index and ranking of the alternatives

Alternative	$\phi_i^+$	$\phi_i^-$	$\Delta_i$	Ranking
FM-0	0.0985	0.0577	0.3696	4
FM-1	0.0010	0.1188	0.9919	1
FM-2	0.0294	0.1029	0.7774	2
FM-3	0.0731	0.0708	0.4923	3

### Conclusions

In the present study, ranking of automotive brake friction material with varying proportion of lapinus and brass fibre was carried out successfully. The tribological results obtained from Krauss type tester were considered as performance defining attributes for ranking of friction materials. The weight of PDAs as evaluated with entropy is: coefficient of friction (0.0154), fade performance (0.4402), recovery performance (0.0309), friction stability (0.0116), friction variability (0.0270), frictional fluctuations (0.3282), wear (0.1429) and disc temperature rise (0.0038). Further, TOPSIS is used to rank the alternatives; the order of alternatives could be obtained as FM-1>FM-2>FM-3>FM-0. The alternative with lapinus: brass 10:5 wt.% exhibits the optimal properties. The TOPSIS method strengthened by entropy is an effective tool for the ranking or selection of friction materials and should be helpful in the optimal friction formulation selection without performing long and costly laboratory experiments.

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