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Thermal instability in a horizontal layer of Ferrofluid in Brinkman porous medium

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Abstract The onset of Rayleigh-Bénard convection in a horizontal layer of ferrofluid in Brinkman porous medium is investigated by using Galerkin weighted residuals method. Linear stability theory based upon normal mode analysis is employed to find expressions for Rayleigh number and critical Rayleigh number. The boundaries are considered to be free-free. 'Principle of Exchange of Stabilities' hold and the oscillatory modes are not allowed. The effects of magnetic parameters and Brinkman Darcy number on the stationary convection are investigated both analytically and graphically.

Keywords Ferrofluid, Brinkman Darcy number, Magnetic thermal Rayleigh number, Galerkin method, Prandtl number.

Nomenclature

a	wave number
$\mathbf{B}$	magnetic induction
Da	Darcy Number
$\tilde{D}a$	Brinkman Darcy number
d	depth of fluid layer
$\mathbf{g}$	acceleration due to gravity
$\mathbf{H}$	magnetic field
k	thermal conductivity
k_1	medium permeability
K_1	pyomagnetic coefficient
M	magnetization
M_1	buoyancy magnetization
M_3	magnetic parameter
N	magnetic thermal Rayleigh number
n	growth rate of disturbances
p	pressure
P_r	Prandtl number
$\mathbf{q}$	fluid velocity
R	Rayleigh number

R_c	critical Rayleigh number
t	time
T	temperature
Ta	average temperature
u, v, w	fluid velocity components
(x, y, z)	space co-ordinates
Greek symbols	
α	thermal expansion coefficient
β	uniform temperature gradient
μ_o	magnetic permeability
μ	viscosity
ρ	density of the fluid
ρc	heat capacity of fluid
κ	thermal diffusivity
φ_1'	perturbed magnetic potential
ε	porosity
ω	dimensionless frequency of oscillation
χ	magnetic susceptibility
Superscripts	
'	non dimensional variables
' '	perturbed quantities
Subscripts	
0	lower boundary
1	upper boundary
H	horizontal plane

1. Introduction

Thermal instability in a porous medium has gained momentum recently due to its various applications in the engineering and technology, in geophysics, food processing, oil reservoir modeling, building of thermal insulations and nuclear reactors. A detailed account of the thermal instability of a Newtonian fluid, under varying assumptions of hydrodynamics and hydromagnetics has been given by Chandrasekhar [1]. Lapwood [2] has studied the convective flow in a porous medium using linearized stability theory. The Rayleigh instability of a thermal boundary layer in flow through a porous medium has been considered by Wooding [3]. The investigation in porous media has been started with the simple Darcy model and gradually was extended to Darcy-Brinkman model. A good account of convection problems in a porous medium is given by Vafai and Hadim [4], Ingham and Pop [5] and Nield and Bejan [6].

Ferrofluids is suspensions of magnetic nanoparticles which exhibit a specific feature of the magnetic control of their physical parameters and flows appearing in such fluids. This magnetic control can be achieved by means of moderate magnetic fields with strength of the order of 10nm. This sort of magnetic control also enables the design of a wide variety of technical applications such as the use of magnetic forces for basic research in fluid dynamics. One of the major applications of ferrofluid is its use in medical fields such as the transport of drugs to an injured site and the removal of tumors from the body. Ferromagnetic fluid has wide ranges of applications in instrumentation, lubrication, printing, vacuum technology, vibration damping, metals recovery, acoustics and medicine, its commercial usage includes vacuum feed through for semiconductor manufacturing in liquid-cooled loudspeakers and computer disk drives etc. Owing the applications of the ferrofluid its study is important to the researchers. A detailed account on the subject is given in monograph has been given by Rosensweig [7]. This monograph reviews several applications of heat transfer through ferrofluid. One such phenomenon is enhanced convective cooling having a temperature-dependent magnetic moment due to magnetization of the fluid. This magnetization, in general, is a function of the magnetic field, temperature, salinity and density of the fluid. In our analysis, we assume that the magnetization is aligned with the magnetic field. Convective instability of a ferromagnetic fluid for a fluid layer heated from below in the presence of uniform vertical magnetic field has been considered by Finlayson [8]. He explained the concept of thermo-mechanical interaction in ferromagnetic fluids. Thermoconvective stability of ferromagnetic fluids was interestingly continued by Lalas & Carmi [9], Shliomis [10], Stiles & Kagan [11], Blennerhassett et al. [12], Venkatasubramanian & Kaloni [13], Sunil et al. [14-15] Gupta and Gupta [16] and Zebib (1996).While Mahajan [17] studied the Linear and nonlinear convective instability of a ferromagnetic fluid for a fluid layer heated from below under various assumptions.

In this paper an attempt as study the linear convective instability of a ferromagnetic fluid in Brinkman Darcy porous medium for a fluid layer heated from below by Galerkin weighted. Stability is discussed analytically as well as graphically.

2. Mathematical Formulation of the Problem

Consider an infinite, horizontal layer of an electrically non-conducting incompressible ferromagnetic fluid of thickness'd', in a porous medium of porosity ε and medium permeability k_1. Let the fluid layer is bounded by planes z = 0 and z = d and is acted upon by gravity force **g** (0, 0, -g), a uniform magnetic field $\mathbf{H} = H_0^{ext}\hat{k}$ acts outside the fluid layer. The layer is heated from below such that a uniform temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ is maintained, where T denote the temperature. The temperature T at z = 0 taken to be T_0 and T_1 at z = d, ($T_0 > T_1$) as shown in Fig.1.

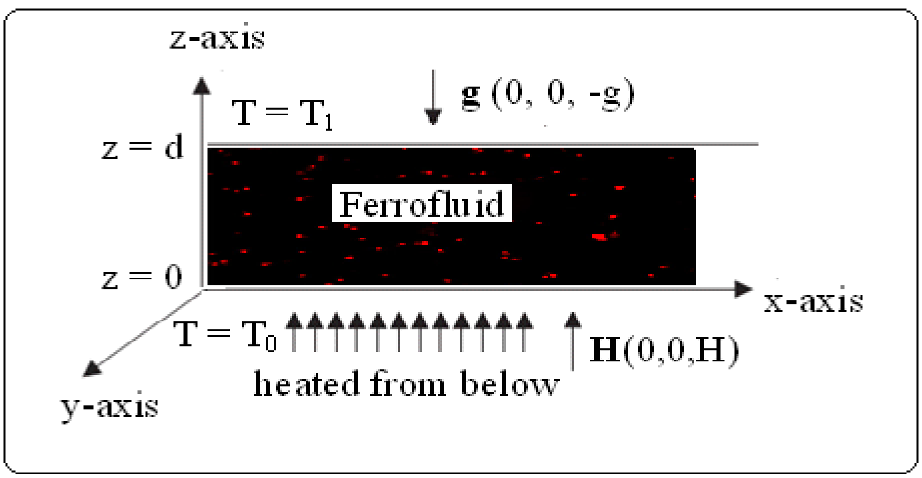

Figure 1: Geometrical configuration of the problem

The mathematical governing equations under Boussinesq approximation for the above model (Finlayson [8], Resenweig [7] and Mahajan [17]) are

$$\nabla.\mathbf{q} = 0, \tag{1}$$

$$\frac{\rho_0}{\varepsilon}\frac{d\mathbf{q}}{dt} = -\nabla p + \rho_0\mathbf{g} + \tilde{\mu}\nabla^2\mathbf{q} - \frac{\mu}{k_1}\nabla^2\mathbf{q} + \mu_0(\mathbf{M}.\nabla)\mathbf{H}\ , \tag{2}$$

where $\frac{d}{dt} = \frac{\partial}{\partial t} + \frac{1}{\varepsilon}(\mathbf{q}.\nabla)$ stands for convection derivative, $\mathbf{q}(u, v, w)$ is the velocity vector, ρ_0 is reference density, p is the hydrostatic pressure, k_1 is medium permeability of fluid, μ is viscosity and $\tilde{\mu}$ is effective viscosity, μ_0 is magnetic permeability, **H** magnetic field, M is magnetization.

$$(\rho_\circ C_0)_m \frac{dT}{dt} + (\rho_\circ C_0)_f \, q.\nabla T = k\nabla^2 T \,, \tag{3}$$

where $(\rho_0 C_0)_m$ is heat capacity of fluid in porous medium, $(\rho_0 C_0)_f$ is heat capacity of fluid and k is thermal conductivity.

Maxwell's equations, in magnetostatic limit:

$$\nabla.\mathbf{B} = 0,\ \nabla \times \mathbf{H} = 0,\quad B = \mu_0(\mathbf{H} + \mathbf{M}). \tag{4}$$

The magnetization has the relationship

$$\mathbf{M} = \frac{\mathbf{H}}{H}[M_0 + \chi(H - H_0) - K_1(T - T_1)]. \tag{5}$$

Where **B** is magnetic induction, K_1 is thermal conductivity, $H = |\mathbf{H}|$, $M = |\mathbf{M}|$ and $M_0 = M(H_0, T_a)$.

The magnetic susceptibility and pyomagnetic coefficient are defined by $\chi = \left(\frac{\partial M}{\partial H}\right)_{H_\circ, T_a}$ and $K_1 = \left(\frac{\partial M}{\partial T}\right)_{H_\circ, T_a}$ respectively.

The density equation of state is taken as

$$\rho = \rho_\circ[1 - \alpha(T - T_a)]. \tag{6}$$

Where T_a is the average temperature given by $T_a = \left(\frac{T_0 + T_1}{2}\right)$,

Since the fluid under consideration is confined between two horizontal planes z = 0 and z = d, on these two planes certain boundary conditions must be satisfied. We take case of free-free surface and assume the temperature is constant on the boundaries. Therefore boundary conditions (Chandrasekhar [1], Nield and Bejan [6]) are

$$w = 0,\quad T = T_0,\ H = 0 \ \text{at}\quad z = 0, \text{and}\quad w = 0,\quad T = T_1,\ H = 0\ \text{at}\quad z = d. \tag{7}$$

2.1. Basic Solutions

The basic state is assumed to be a quiescent state and is given by

$$q(u, v, w) = q_b(u, v, w) = 0\ ,\ p = p_b(z)\ , T = T_b(z) = -\beta z + T_a\,,\ H_b = \left[H_\circ + \frac{K_1(T_b - T_a)}{1+\chi}\right]\hat{k}\,,$$

$$M_b = \left[M_\circ - \frac{K_2(T_b - T_a)}{1+\chi}\right]\hat{k},\ H_\circ + M_\circ = H_\circ ext\,. \tag{8}$$

2.2. The Perturbation Equations

We shall analyze the stability of the basic state by introducing the following perturbations:

$$q = q_b + q',\ p = p_b(z) + \delta p, T = T_b(z) + \theta\ ,\ H = H_b(z) + H'\ M = M_b(z) + M'. \tag{9}$$

where $q'(u, v, w)$, δp, θ, $H'(H'_1, H'_2, H'_3)$ and $M'(M'_1, M'_2, M'_3)$ are perturbations in velocity, pressure, temperature, magnetic field and magnetization. These perturbations are assumed to be small and then the linearized perturbation equations are

$$\nabla.\mathbf{q}' = 0, \tag{10}$$

$$\frac{\rho_\circ}{\varepsilon}\frac{\partial \mathbf{q}'}{\partial t} = -\nabla \delta p - \frac{\mu}{k_1}\nabla^2 \mathbf{q}' + \tilde{\mu}\nabla^2 \mathbf{q} + \rho_0 \alpha g \theta \hat{k} - \frac{\mu_\circ K_1 \beta}{1+\chi}\left((1+\chi)\frac{\partial \varphi_1'}{\partial z}\hat{k} - K_1\theta\hat{k}\right), \tag{11}$$

$$\sigma\frac{\partial \theta}{\partial t} = \kappa\nabla^2\theta + \beta w\,, \tag{12}$$

$$\left(1+\frac{M_0}{H_0}\right)\nabla^2\phi_1' - \left(\frac{M_0}{H_0} - \chi\right)\frac{\partial^2 \phi_1'}{\partial z^2} = K_1\frac{\partial \theta}{\partial z}\,. \tag{13}$$

where $\mathbf{H}' = \nabla\varphi_1'$ and φ_1' is the perturbed magnetic potential, $\sigma = \frac{(\rho_0 c_0)_m}{(\rho_0 c_0)_f}$ and $\kappa = \frac{k}{(\rho_0 c_0)_f}$ is thermal diffusivity of the fluid.

The boundary conditions

$$w = 0, \quad T = T_0, D\varphi_1 = 0 \quad \text{at} \quad z = 0 \text{ and } w = 0, \quad T = T_1, D\varphi_1 = 0 \text{ at } \quad z = d. \tag{14}$$

We introduce non-dimensional variables as

$$(x'', y'', z'') = \left(\frac{x', y', z'}{d}\right), \mathbf{q}'' = \mathbf{q}'\frac{d}{\kappa},\ t' = \frac{\kappa}{d^2}t,\ \delta p' = \frac{k_1}{\mu\kappa}\delta p,\ \theta' = \frac{\theta}{\beta d},\ \varphi_1'' = \frac{(1+\chi)}{K_1 \beta d^2}\varphi_1'.$$

where $\kappa = \frac{k}{(\rho_0 c_0)_f}$ is thermal diffusivity of the fluid.

There after dropping the dashes (") for simplicity.

Equations (10)-(14), in non dimensional form can be written as

$$\nabla.\mathbf{q} = 0\,, \tag{15}$$

$$\frac{1}{Va}\frac{\partial \mathbf{q}}{\partial t} = -\nabla \delta p - \mathbf{q} + \tilde{D}a\nabla^2 \mathbf{q} + R(1+M_1)\theta\hat{k} - RM_1\frac{\partial \varphi_1}{\partial z}\hat{k}\,, \tag{16}$$

$$\sigma\frac{\partial \theta}{\partial t} = \nabla^2\theta + w\,, \tag{17}$$

$$M_3\nabla^2\phi_1 - (M_3 - 1)\frac{\partial^2 \phi_1}{\partial z^2} = \frac{\partial \theta}{\partial z}\,. \tag{18}$$

Where non-dimensional parameters are given as

$\frac{1}{Va} = \frac{\varepsilon Pr}{Da}$ is Vadasz number; $P_r = \frac{\mu}{\rho\kappa}$ is Prandtl number; $Da = \frac{k_1}{d^2}$ is Darcy number; $Da = \frac{\tilde{\mu}k_1}{\mu d^2}$ is is Brinkman Darcy number $R = \frac{\rho_0 g \alpha \beta d^2 k_1}{\mu\kappa}$ is Rayleigh number; $M_1 = \frac{\mu_0 K_1^2 \beta}{\alpha \rho_0 g(1+\chi)}$ measure the ratio of magnetic to gravitational forces, $N = RM_1 = \frac{\mu_0 K_1^2 \beta^2 d^4}{\mu\kappa(1+\chi)}$ is magnetic thermal Rayleigh number; $M_3 = \frac{\left(1+\frac{M_0}{H_0}\right)}{(1+\chi)}$ measure the departure of linearity in the magnetic equation of state and values from one $(M_0 = \chi H_0)$ higher values are possible for the usual equation of state.

The dimensionless boundary conditions are

$$w=0,\quad T=1, D\varphi_1=0 \text{ at } \quad z=0 \text{ and } \quad w=0,\quad T=0, D\varphi_1=0 \text{ at } \quad z=1. \tag{19}$$

Eliminating δp from equation (16), we get

$$\left(\tilde{D}a\nabla^2-1-\frac{1}{Va}\frac{\partial}{\partial t}\right)\nabla^2 w+R(1+M_1)\nabla_H^2\theta-RM_1\nabla_H^2 D\varphi_1=0. \tag{20}$$

Where ∇_H^2, is two-dimensional Laplacian operator on horizontal plane.

3. Normal Mode Analysis

Analyzing the disturbances of normal modes, we assume that the perturbation quantities are of the form

$$[w,\theta,\varphi_1]=[W(z),\Theta(z),\Phi(z)]\exp(ik_x x+ik_y y+nt), \tag{21}$$

where, k_x, k_y are wave numbers in x- and y- directions and n is growth rate of disturbances.

Using equation (21), equations (20) and (17) - (18) become

$$\left(\tilde{D}a(D^2-a^2)-1-\frac{n}{Va}\frac{\partial}{\partial t}\right)(D^2-a^2)W-a^2R(1+M_1)\Theta+a^2RM_1D\Phi=0, \tag{22}$$

$$W+\left(D^2-a^2-\frac{n}{\sigma}\right)\Theta=0, \tag{23}$$

$$D\Theta-(D^2-a^2M_3)\Phi=0\cdot \tag{24}$$

Where $D=\frac{d}{dz}$ and $a^2=k_x^2+k_y^2$ is dimensionless the resultant wave number.

The boundary conditions of the problem in view of normal mode analysis

$$W=0, D^2W=0,\ \Theta=0,\ D\Phi=0 \text{ at } \quad z=0,1\ . \tag{25}$$

4. Method of solution

The Galerkin weighted residuals method is used to obtain an approximate solute on to the system of equations (22) – (24) with the corresponding boundary conditions (25). In this method, the test functions are the same as the base (trial) functions. Accordingly W, Θ and Φ are taken as

$$W=\sum_{p=1}^{n}A_pW_p,\ \Theta=\sum_{p=1}^{n}B_p\Theta_p,\ D\Phi=\sum_{p=1}^{n}C_pD\Phi_p\ . \tag{26}$$

Where A_p, B_p and C_p are unknown coefficients, p =1, 2, 3...N and the base functions W_p, Θ_p and $D\Phi_p$ are assumed in the following form for free-free boundaries are:

$$W_p=\operatorname{Cos}p\pi\ z,\ \Theta_p=\operatorname{Cos}p\,\pi\, z,\ D\Phi_p=\operatorname{Cos}p\pi\, z, \tag{27}$$

Such that W_p, Θ_p and Φ_p satisfy the corresponding boundary conditions. Using expression for W, Θ and $D\Phi$ in equations (24) – (26) and multiplying first equation by W_p second equation by Θ_p and third by $D\Phi_p$ and integrating in the limits from zero to unity, we obtain a set of 3N linear homogeneous equations in 3N unknown A_p, B_p and C_p; p =1,2,3,...N. For existing of non trivial solution, the vanishing of the determinant of coefficients produces the characteristics equation of the system in term of Rayleigh number R.

5. Linear Stability Analysis

We confined our analysis to the one term Galerkin approximation; for one term Galerkin approximation, we take N=1, the appropriate trial function are given as

$$W_p=\cos\pi\ z,\ \Theta_p=\cos\pi\ z,\ D\Phi_p=\cos\pi\ z, \tag{28}$$

which satisfies boundary conditions

$$W=0, D^2W=0,\ \Theta=0,\ D\Phi=0 \text{ at } z=0 \text{ and } W=0, D^2W=0,\ \Theta=0,\ D\Phi=0 \text{ at } z=1. \qquad (29)$$

Substituting solution (29) into equations (22)-(24), integrating each equation from z = 0 to z = 1, by parts, we obtain following matrix equation

$$\begin{bmatrix} J\left(\tilde{D}_a J+1+\frac{n}{Va}\right) & -a^2R(1+M_1) & a^2RM_1 \\ 1 & -\left(J+\frac{n}{\sigma}\right) & 0 \\ 0 & -\pi & \pi^2+\frac{a^2M_3}{\pi} \end{bmatrix}\begin{bmatrix} W_0 \\ \Theta_0 \\ D\Phi_0 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

where $J=\pi^2+a^2$.

The non-trivial solution of the above matrix requires that

$$a^2R\left((1+M_1)(\pi^2+a^2M_3)-M_1\pi\right)=\left(\tilde{D}_a J^2+J+\frac{n}{Va}\right)\left(J+\frac{n}{\sigma}\right)(\pi^2+a^2M_3). \qquad (30)$$

Setting $n=i\omega$, (where ω is real and dimensionaless frequency) in equation (30), we get

$$R=\Delta_1+i\omega\Delta_2. \qquad (31)$$

$$\text{Where } \Delta_1=\frac{\left(\tilde{D}_a J^3+J^2-\frac{\omega^2}{Va}\right)(\pi^2+a^2M_3)}{a^2\left((1+M_1)(\pi^2+a^2M_3)-M_1\pi^2\right)}, \qquad (32)$$

and

$$\Delta_2=\frac{\left(\frac{\tilde{D}_a J^2+J}{\sigma}-\frac{J}{Va}\right)(\pi^2+a^2M_3)}{a^2\left((1+M_1)(\pi^2+a^2M_3)-M_1\pi^2\right)}. \qquad (33)$$

Since R is a physical quantity, so it must be real. Hence, it follow from the equation (31) that either ω = 0 (exchange of stability, steady state) or Δ_2 = 0 (ω # 0 overstability or oscillatory onset).

But Δ_2 # 0, we must have ω = 0, which means that oscillatory modes are not allowed and the Principle of Exchange of Stabilities is satisfied. This is the good agreement of the result as obtained by Finlayson [8].

(a) Stationary Convection

Consider the case of stationary convection i.e., n = 0, from equation (30), we have

$$R=\frac{\left(\tilde{D}a(\pi^2+a^2)^3+(\pi^2+a^2)^2\right)(\pi^2+a^2M_3)}{a^2\left(\pi^2+a^2M_3+a^2M_1M_3\right)}. \qquad (34)$$

This is the good agreement of the result as obtained by Finlayson [8].

In the absence of magnetic parameters M_1=M_3=0, $\tilde{D}a$ =1, the Rayleigh number R for steady onset is given by

$$R=\frac{(\pi^2+a^2)^3}{a^2}. \qquad (35)$$

Consequently critical Rayleigh number is given by $Rc=\frac{27\pi^2}{4}$.

This is exactly the same the result as obtained by Chandrasekhar [1] in the classical Bénard problem.

In order to investigate the effects of magnetization parameter M_3 buoyancy magnetization M_1, and Brinkman Darcy number $\tilde{D}a$ on the stationary convection, we examine the behavior of $\frac{dR}{dM_3}, \frac{dR}{dM_1}$ and $\frac{dR}{d\tilde{D}a}$ analytically.

Equation (34), we have

$$\frac{dR}{dM_3} < 0 ,$$

$$\frac{dR}{dM_1} < 0 \text{ and}$$

$$\frac{dR}{d\tilde{D}a} > 0 .$$

Thus the magnetization parameters M_3 and buoyancy magnetization M_1 have destabilizing effect while Brinkman Darcy number $\tilde{D}a$ has stabilizing effect on the fluid layer.

5. Results and Discussion

An expression for stationary convection is given by equations (35). It is observed that oscillatory modes not allowed for layer of ferrofluid in Brinkman porous medium heated from below. We now discussed the results graphically. The stationary convection curves in (R, a) plane for various values of magnetization M_3 and for fixed values of M_1=1000, $\tilde{D}a$ =0.1 is as shown in Fig. 2. It has been found that the stationary Rayleigh number decreases with increase in the value of magnetization parameter M_3 thus magnetization parameter M_3 have destabilizing effect on the stationary convection.

Figure 3 shows the variation of stationary Rayleigh number with wave number for different values of different value of buoyancy magnetization M_1, with fixed value of $\tilde{D}a$=0.1, M_3 =6 and it has been found that the stationary Rayleigh number decreases with increase in the value of buoyancy magnetization M_1, thus buoyancy magnetization M_1 has destabilizing effect on the stationary convection.

Figure 4 shows the variation of stationary Rayleigh number with wave number for different values of different value of Brinkman Darcy number $\tilde{D}a$ with fixed value of M_1=1000, M_3 =6 and it has been found that the stationary Rayleigh number increases with increase in the value of Brinkman Darcy number $\tilde{D}a$, thus Brinkman Darcy number $\tilde{D}a$ has stabilizing effect on the stationary convection.

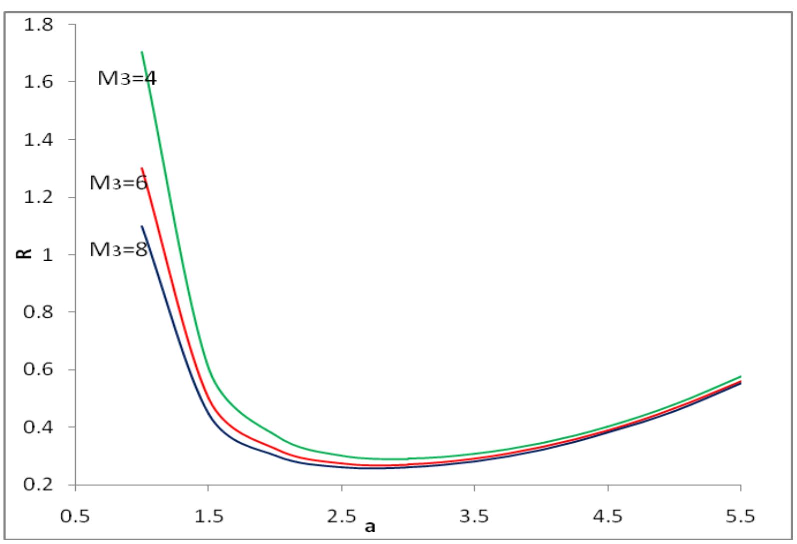

Figure 2: Variation of critical Rayleigh number **R** with wave number **a** for different value of magnetization M_3

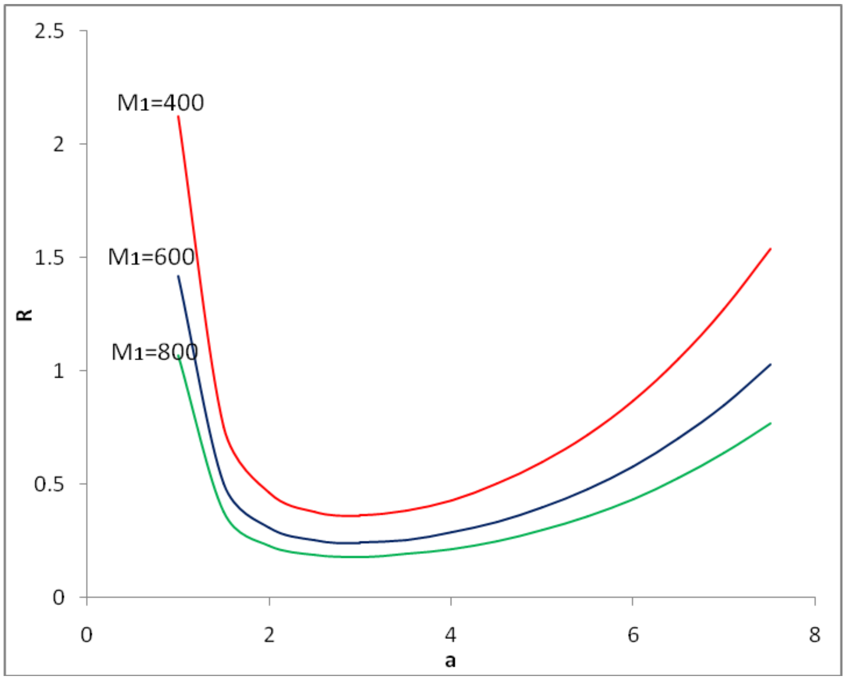

Figure 3: Variation of Rayleigh number **R** with wave number **a** for different value of buoyancy magnetization M_1

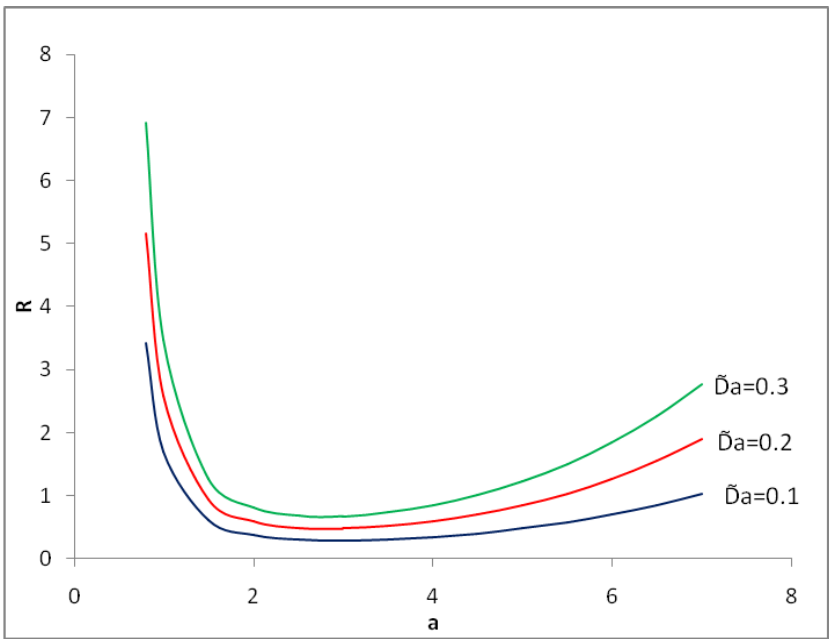

Figure 4: Variation of Rayleigh number **R** with wave number **a** for different value of Brinkman Darcy Number D̃a

6. Conclusions

A linear analysis of thermal instability for ferrofluid in a Brinkman porous medium is investigated. An expression for Rayleigh number for the stationary convection is obtained. We investigated the results both analytically and graphically

The main conclusions are as follows:

1. For the case of stationary convection, the magnetization parameters M_3 and buoyancy magnetization M_1 have destabilizing effect while Brinkman Darcy number $\tilde{D}a$ has stabilizing effect on the fluid layer.
2. The oscillatory modes are not allowed for the ferromagnetic fluid heated from below in a Brickman Darcy porous medium.
3. The 'Principle of exchange of Stabilities' is valid for the problem.
4. In the absence of magnetic parameters (M_1=M_3=0) and Brickman Darcy porous medium ($\tilde{D}a$ =1) obtained result is same as the result obtained by Chandrasekhar [1] in the classical Bénard problem.

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