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# NEW INFORMATION INEQUALITIES ON DIFFERENCE OF GENERALIZED DIVERGENCES AND ITS APPLICATION

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**Abstract** Divergence measures are useful for comparing two probability distributions. Depending on the nature of the problem, the different divergences are suitable. So it is always desirable to create a new divergence measure.
In this work, new information inequalities, corresponding to difference of two generalized f- divergences, are obtained and characterized. Secondly, we obtain new divergence measure corresponding to new convex function and define the properties. Further, bounds of new divergence in terms of other standard divergences are evaluated. Comparison of this divergence with others is done as well.

**Index terms:** New Convex and normalized function, New divergence measure, Comparison graph of divergences, New information inequalities, Bounds of new divergence.

**Mathematics Subject Classification:** Primary 94A17, Secondary 26D15.

## Introduction

Let $\Gamma_n = \left\{ P = (p_1, p_2, p_3 ..., p_n) : p_i > 0, \sum_{i=1}^{n} p_i = 1 \right\}, n \geq 2$ be the set of all complete finite discrete probability distributions. If we take $p_i \geq 0$ for some $i = 1, 2, 3, ..., n$, then we have to suppose that $0 f(0) = 0 f\left(\frac{0}{0}\right) = 0$.

**Csiszar's f- divergence [1]** is a generalized information divergence measure, which is given by (1.1), i.e.,

$$C_f(P,Q) = \sum_{i=1}^{n} q_i f\left(\frac{p_i}{q_i}\right) \tag{1.1}$$

And
$$E_{f'}(P,Q) = C_{f'}\left(\frac{P^2}{Q}, P\right) - C_{f'}(P,Q) = \sum_{i=1}^{n} (p_i - q_i) f'\left(\frac{p_i}{q_i}\right) \tag{1.2}$$

Similarly **(Jain and Saraswat [5])** introduced a generalized measure of information given by

$$S_f(P,Q) = \sum_{i=1}^{n} q_i f\left(\frac{p_i + q_i}{2q_i}\right). \tag{1.3}$$

Where f: (0,∞) → R (set of real no.) is real, continuous and convex function and $P=(p_1, p_2, p_3..., p_n), Q=(q_1, q_2, q_3..., q_n) \in \Gamma_n$, where $p_i$ and $q_i$ are probability mass functions. Many known divergences can be obtained from these generalized measures by suitably defining the convex function f. Some of those are as follows.

**Chi- square divergence [8]** = $\chi^2(P,Q)=\sum_{i=1}^{n}\frac{(p_i-q_i)^2}{q_i}$. (1.4)

**Relative JS divergence [9]** = $F(P,Q)=\sum_{i=1}^{n} p_i \log\left(\frac{2p_i}{p_i+q_i}\right)$. (1.5)

**Relative J- divergence [3]** = $J_R(P,Q)=\sum_{i=1}^{n}(p_i-q_i)\log\left(\frac{p_i+q_i}{2q_i}\right)$. (1.6)

**Relative AG divergence [10]** = $G(P,Q)=\sum_{i=1}^{n}\left(\frac{p_i+q_i}{2}\right)\log\left(\frac{p_i+q_i}{2p_i}\right)$. (1.7)

**Triangular discrimination [2]** = $\Delta(P,Q)=\sum_{i=1}^{n}\frac{(p_i-q_i)^2}{p_i+q_i}$. (1.8)

**J- divergence [6,7]** = $J(P,Q)=\sum_{i=1}^{n}(p_i-q_i)\log\left(\frac{p_i}{q_i}\right)$. (1.9)

We can see that $J_R(P,Q)=2\left[F(Q,P)+G(Q,P)\right], \Delta(P,Q)=2\left[1-W(P,Q)\right]$ and

$J(P,Q)=J_R(P,Q)+J_R(Q,P)$, where $W(P,Q)=2\sum_{i=1}^{n}\frac{p_i q_i}{p_i+q_i}$ is Harmonic mean divergence. Divergences from (1.4) to (1.7) are non- symmetric and (1.8), (1.9) are symmetric, with respect to probability distribution P, Q ∈ $\Gamma_n$. (1.4) and (1.9) are also known as Pearson divergence and Jeffreys- Kullback- Leibler divergence, respectively.

Beside these, **Symmetric Chi- square divergence [4]** can be written as the sum of Chi- square divergence and its adjoint, i.e.,

$$\chi^2(P,Q)+\chi^2(Q,P)=\psi(P,Q)=\sum_{i=1}^{n}\frac{(p_i-q_i)^2(p_i+q_i)}{p_i q_i}. \quad (1.10)$$

**New Information Inequalities**

In this section, we introduce new information inequalities on difference of two generalized f- divergences. Such inequalities are for instance needed in order to calculate the relative efficiency of two divergences.

**Theorem 2.1** Let $f_1, f_2 : I \subset R_+ \to R$ be two convex and normalized functions, i.e. $f_1(1)=f_2(1)=0$ and suppose the assumptions:

a. $f_1$ *and* $f_2$ are twice differentiable on (α, β) where $0<\alpha\le 1\le\beta<\infty, \alpha\ne\beta$.
b. There exists the real constants m, M such that m < M and

$$m\le\frac{f_1''(t)}{f_2''(t)}\le M,\ f_2''(t)>0\ \forall t\in(\alpha,\beta), \quad (2.1)$$

If $P,Q\in\Gamma_n$, then we have the inequalities,

$$m\left[E_{f_2'}(P,Q)-S_{f_2}(P,Q)\right]\le E_{f_1'}(P,Q)-S_{f_1}(P,Q)\le M\left[E_{f_2'}(P,Q)-S_{f_2}(P,Q)\right]. \tag{2.2}$$

Where $E_{f'}(P,Q), S_f(P,Q)$ are given by (1.2) and (1.3) respectively.

**Proof:** Let us consider two functions

$$F_m(t)=f_1(t)-mf_2(t)\ , \tag{2.3}$$

and

$$F_M(t)=Mf_2(t)-f_1(t). \tag{2.4}$$

Where "m" and "M" are the minimum and maximum values of the function $\frac{f_1''(t)}{f_2''(t)}, \forall\, t\in(\alpha,\beta)$.

Since $f_1(1)=f_2(1)=0\Rightarrow F_m(1)=F_M(1)=0$, (2.5)

and the functions $f_1(t)$ and $f_2(t)$ are twice differentiable. Then in view of (2.1), we have

$$F_m''(t)=f_1''(t)-mf_2''(t)=f_2''(t)\left(\frac{f_1''(t)}{f_2''(t)}-m\right)\ge 0, \tag{2.6}$$

and

$$F_M''(t)=Mf_2''(t)-f_1''(t)=f_2''(t)\left(M-\frac{f_1''(t)}{f_2''(t)}\right)\ge 0. \tag{2.7}$$

In view (2.5), (2.6) and (2.7), we can say that the functions $F_m(t)$ and $F_M(t)$ are normalized and convex on (α, β).

Now, with the help of linearity property, we have,

$$\begin{aligned}E_{F_m'}(P,Q)-S_{F_m}(P,Q)&=E_{f_1'-mf_2'}(P,Q)-S_{f_1-mf_2}(P,Q)\\&=E_{f_1'}(P,Q)-mE_{f_2'}(P,Q)-S_{f_1}(P,Q)+mS_{f_2}(P,Q),\end{aligned} \tag{2.8}$$

and

$$\begin{aligned}E_{F_M'}(P,Q)-S_{F_M}(P,Q)&=E_{Mf_2'-f_1'}(P,Q)-S_{Mf_2-f_1}(P,Q)\\&=ME_{f_2'}(P,Q)-E_{f_1'}(P,Q)-MS_{f_2}(P,Q)+S_{f_1}(P,Q).\end{aligned} \tag{2.9}$$

Since $E_{f'}(P,Q)\ge S_f(P,Q)$ from [5], therefore (2.8) and (2.9) can be written as the followings.

$$\left[E_{f_1'}(P,Q)-S_{f_1}(P,Q)\right]-m\left[E_{f_2'}(P,Q)-S_{f_2}(P,Q)\right]\ge 0,$$

and

$$M\left[E_{f_2'}(P,Q)-S_{f_2}(P,Q)\right]-\left[E_{f_1'}(P,Q)-S_{f_1}(P,Q)\right]\ge 0.$$

Or

$$\left[E_{f_1'}(P,Q)-S_{f_1}(P,Q)\right]\ge m\left[E_{f_2'}(P,Q)-S_{f_2}(P,Q)\right], \tag{2.10}$$

and

$$M\left[E_{f_2'}(P,Q)-S_{f_2}(P,Q)\right]\ge\left[E_{f_1'}(P,Q)-S_{f_1}(P,Q)\right]. \tag{2.11}$$

(2.10) and (2.11), together give the result (2.2).

**New Divergence Measure and Properties**

In this section, we obtain new divergence measure for new convex function; further define the properties of new convex function and new divergence. Firstly,

Let $f:(0,\infty)\rightarrow R$ be a function defined as

$$f(t)=f_1(t)=\frac{(t^2-1)^2}{t},\forall t\in(0,\infty),\ f_1(1)=0,\ f_1'(t)=\frac{(3t^2+1)(t^2-1)}{t^2}, \tag{3.1}$$

and

$$f_1''(t)=\frac{2(3t^4+1)}{t^3}. \tag{3.2}$$

Properties of function defined by (3.1), are as follows.

a. Since $f_1(1)=0\Rightarrow f_1(t)$ is a normalized function.

b. Since $f_1''(t)\geq 0\forall t\in(0,\infty)\Rightarrow f_1(t)$ is a convex function as well.

c. Since $f_1'(t)<0$ at $(0,1)$ and $f_1'(t)>0$ at $(1,\infty)\Rightarrow f_1(t)$ is monotonically decreasing in $(0,1)$ and monotonically increasing in $(1,\infty)$ and $f_1'(1)=0,\ f_1''(1)=8\neq 0$.

d.

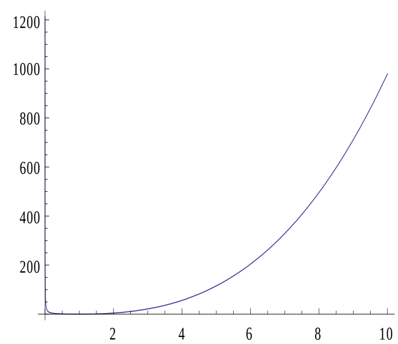


**Figure 3.1**: Convex function $f_1(t)$

Now put $f_1(t)$ and $f_1'(t)$ in (1.3) and (1.2) respectively, we get the following new divergence measure,

$$S^*(P,Q)=E_{f_1'}(P,Q)-S_{f_1}(P,Q)$$
$$=\sum_{i=1}^{n}\frac{(p_i-q_i)^2\left(23p_i^4+23p_i^2q_i^2+42p_i^3q_i+16p_iq_i^3+8q_i^4\right)}{8p_i^2q_i^2(p_i+q_i)}. \tag{3.3}$$

Properties of new divergence measure defined in (3.3), are as follows.

a. $S^*(P,Q)$ is convex and non- negative in the pair of probability distribution $(P,Q)\in\Gamma_n\times\Gamma_n$.

b. $S^*(P,Q)=0\ if\ P=Q or\ p_i=q_i$ (Attains its minimum value).

c. Since $S^*(P,Q) \neq S^*(Q,P) \Rightarrow S^*(P,Q)$ is non- symmetric divergence measure.

## Application of New Information Inequalities

In this section, we obtain bounds of new divergence measure (3.3) by using new inequalities defined in (2.2), in terms of standard divergences.

**Proposition 4.1** Let $\chi^2(P,Q), J_R(P,Q), J(P,Q)$ and $S^*(P,Q)$ be defined as in (1.4), (1.6), (1.9) and (3.3) respectively. For P, Q$\in\Gamma_n$, we have

a. If $0 < \alpha \leq 0.65$, then

$$2.863\left[J(P,Q)+\chi^2(Q,P)-\frac{1}{2}J_R(P,Q)\right] \leq S^*(P,Q)$$

$$\leq 2\max\left[\frac{3\alpha^4+1}{\alpha(1+\alpha)}, \frac{3\beta^4+1}{\beta(1+\beta)}\right]\left[J(P,Q)+\chi^2(Q,P)-\frac{1}{2}J_R(P,Q)\right]. \quad (4.1)$$

b. If $0.65 < \alpha \leq 1$, then

$$2\left(\frac{3\alpha^4+1}{\alpha(1+\alpha)}\right)\left[J(P,Q)+\chi^2(Q,P)-\frac{1}{2}J_R(P,Q)\right] \leq S^*(P,Q)$$

$$\leq 2\left(\frac{3\beta^4+1}{\beta(1+\beta)}\right)\left[J(P,Q)+\chi^2(Q,P)-\frac{1}{2}J_R(P,Q)\right]. \quad (4.2)$$

**Proof**: Let us consider

$$f_2(t) = (t-1)\log t, t \in (0,\infty),\ f_2(1) = 0,\ f_2'(t) = \frac{(t-1)}{t} + \log t \text{ and}$$

$$f_2''(t) = \frac{1+t}{t^2}. \quad (4.3)$$

Since $f_2''(t) > 0 \ \forall \ t > 0$ and $f_2(1) = 0$, so $f_2(t)$ is convex and normalized function respectively. Now, Put $f_2(t)$ in (1.3) and $f_2'(t)$ in (1.2), we get the followings.

$$S_{f_2}(P,Q) = \sum_{i=1}^{n}\left(\frac{p_i - q_i}{2}\right)\log\left(\frac{p_i+q_i}{2\,q_i}\right) = \frac{1}{2}J_R(P,Q). \quad (4.4)$$

$$E_{f_2'}(P,Q) = \sum_{i=1}^{n}(p_i - q_i)\log\left(\frac{p_i}{q_i}\right) + \sum_{i=1}^{n}\frac{(p_i-q_i)^2}{p_i} = J(P,Q) + \chi^2(Q,P). \quad (4.5)$$

Now, let $g(t) = \frac{f_1''(t)}{f_2''(t)} = \frac{2(3t^4+1)}{t(1+t)}$, where $f_1''(t)$ *and* $f_2''(t)$ are given by (3.2) and (4.3) respectively.

And $g'(t) = \frac{2(6t^5+9t^4-2t-1)}{t^2(1+t)^2}, g''(t) = 4\left[3+\frac{1}{t^3}-\frac{4}{(1+t)^3}\right]$.

If $g'(t)=0 \Rightarrow t=0.649793 \approx 0.65$.

It is clear that g (t) is decreasing in (0, 0.65] and increasing in (0.65,∞).

Also g (t) has a minimum value at t=0.65, because $g''(0.65)=23.0035>0$. Now,

a. If $0<\alpha \le 0.65$, then

$$m=\inf_{t\in(\alpha,\beta)} g(t)=g(0.65)=2.863 \tag{4.6}$$

$$M=\sup_{t\in(\alpha,\beta)} g(t)=\max\{g(\alpha),g(\beta)\}=2\max\left[\frac{3\alpha^4+1}{\alpha(1+\alpha)},\frac{3\beta^4+1}{\beta(1+\beta)}\right]. \tag{4.7}$$

b. If $0.65<\alpha \le 1$, then

$$m=\inf_{t\in(\alpha,\beta)} g(t)=g(\alpha)=2\left(\frac{3\alpha^4+1}{\alpha(1+\alpha)}\right). \tag{4.8}$$

$$M=\sup_{t\in(\alpha,\beta)} g(t)=g(\beta)=2\left(\frac{3\beta^4+1}{\beta(1+\beta)}\right). \tag{4.9}$$

The results (4.1) and (4.2) are obtained by using (3.3), (4.4), (4.5), (4.6), (4.7), (4.8) and (4.9) in (2.2).

**Proposition 4.2** Let $\chi^2(P,Q), F(P,Q)$ and $S^*(P,Q)$ be defined as in (1.4), (1.5) and (3.3) respectively. For P, Q∈$\Gamma_n$, we have

a. If $0<\alpha \le 0.58$, then

$$4.618\left[\chi^2(Q,P)-F(Q,P)\right]\le S^*(P,Q)$$

$$\le 2\max\left[\frac{3\alpha^4+1}{\alpha},\frac{3\beta^4+1}{\beta}\right]\left[\chi^2(Q,P)-F(Q,P)\right]. \tag{4.10}$$

b. If $0.58<\alpha \le 1$, then

$$2\left(\frac{3\alpha^4+1}{\alpha}\right)\left[\chi^2(Q,P)-F(Q,P)\right]\le S^*(P,Q)$$

$$\le 2\left(\frac{3\beta^4+1}{\beta}\right)\left[\chi^2(Q,P)-F(Q,P)\right]. \tag{4.11}$$

**Proof**: Let us consider

$$f_2(t)=-\log t, t\in(0,\infty),\ f_2(1)=0,\ f_2'(t)=-\frac{1}{t} \text{ and}$$

$$f_2''(t)=\frac{1}{t^2}. \tag{4.12}$$

Since $f_2''(t)>0\ \forall\ t>0$ and $f_2(1)=0$, so $f_2(t)$ is convex and normalized function respectively. Now, Put $f_2(t)$ in (1.3) and $f_2'(t)$ in (1.2), we get the followings.

$$S_{f_2}(P,Q)=\sum_{i=1}^{n} q_i \log\left(\frac{2q_i}{p_i+q_i}\right)=F(Q,P). \tag{4.13}$$

$$E_{f_2'}(P,Q) = -\sum_{i=1}^{n}\frac{(p_i-q_i)q_i}{p_i} = \sum_{i=1}^{n}\frac{q_i^{\,2}-p_iq_i}{p_i} = \sum_{i=1}^{n}\frac{q_i^{\,2}-2p_iq_i+p_i^{\,2}+p_i(q_i-p_i)}{p_i}$$

$$= \sum_{i=1}^{n}\frac{(p_i-q_i)^2}{p_i} + \sum_{i=1}^{n}(q_i-p_i) = \chi^2(Q,P). \tag{4.14}$$

Now, let $g(t) = \frac{f_1''(t)}{f_2''(t)} = \frac{2(3t^4+1)}{t}$, where $f_1''(t)$ *and* $f_2''(t)$ are given by (3.2) and (4.12) respectively.

And $g'(t) = \frac{2(9t^4-1)}{t^2}, g''(t) = 4\left(9t+\frac{1}{t^3}\right)$.

If $g'(t)=0 \Rightarrow t = \pm 0.5773 \approx 0.58\ (\because t>0)$.

It is clear that g (t) is decreasing in (0, 0.58] and increasing in $(0.58,\infty)$.

Also g (t) has a minimum value at t=0.58, because $g''(0.58) = 41.38 > 0$. Now,

a. If $0 < \alpha \le 0.58$, then

$$m = \inf_{t\in(\alpha,\beta)} g(t) = g(0.58) = 4.618\ . \tag{4.15}$$

$$M = \sup_{t\in(\alpha,\beta)} g(t) = \max\{g(\alpha), g(\beta)\} = 2\max\left[\frac{3\alpha^4+1}{\alpha}, \frac{3\beta^4+1}{\beta}\right]. \tag{4.16}$$

b. If $0.58 < \alpha \le 1$, then

$$m = \inf_{t\in(\alpha,\beta)} g(t) = g(\alpha) = 2\left(\frac{3\alpha^4+1}{\alpha}\right). \tag{4.17}$$

$$M = \sup_{t\in(\alpha,\beta)} g(t) = g(\beta) = 2\left(\frac{3\beta^4+1}{\beta}\right). \tag{4.18}$$

The results (4.10) and (4.11) are obtained by using (3.3), (4.13), (4.14), (4.15), (4.16), (4.17) and (4.18) in (2.2).

**Proposition 4.3** Let $G(P,Q), J(P,Q)$ and $S^*(P,Q)$ be defined as in (1.7), (1.9) and (3.3) respectively. For P, Q$\in\Gamma_n$, we have

a. If $0 < \alpha \le 0.76$, then

$$6.928\left[J(P,Q)-G(Q,P)\right] \le S^*(P,Q)$$

$$\le 2\max\left[\frac{3\alpha^4+1}{\alpha^2}, \frac{3\beta^4+1}{\beta^2}\right]\left[J(P,Q)-G(Q,P)\right]. \tag{4.19}$$

b. If $0.76 < \alpha \le 1$ , then

$$2\left(\frac{3\alpha^4+1}{\alpha^2}\right)\left[J(P,Q)-G(Q,P)\right] \le S^*(P,Q)$$

$$\le 2\left(\frac{3\beta^4+1}{\beta^2}\right)\left[J(P,Q)-G(Q,P)\right]. \tag{4.20}$$

**Proof**: Let us consider

$f_2(t) = t\log t, t \in (0,\infty)$, $f_2(1) = 0$, $f_2'(t) = 1 + \log t$ and

$$f_2''(t) = \frac{1}{t}. \tag{4.21}$$

Since $f_2''(t) > 0\ \forall\ t > 0$ and $f_2(1) = 0$, so $f_2(t)$ is convex and normalized function respectively. Now, Put $f_2(t)$ in (1.3) and $f_2'(t)$ in (1.2), we get the followings.

$$S_{f_2}(P,Q) = \sum_{i=1}^{n} \left(\frac{p_i + q_i}{2}\right) \log\left(\frac{p_i + q_i}{2q_i}\right) = G(Q,P). \tag{4.22}$$

$$E_{f_2'}(P,Q) = \sum_{i=1}^{n} (p_i - q_i) \log\left(\frac{p_i}{q_i}\right) = J(P,Q). \tag{4.23}$$

Now, let $g(t) = \frac{f_1''(t)}{f_2''(t)} = \frac{2(3t^4+1)}{t^2}$, where $f_1''(t)$ *and* $f_2''(t)$ are given by (3.2) and (4.21) respectively.

And $g'(t) = \frac{4(3t^4-1)}{t^3}, g''(t) = 12\left[1 + \frac{1}{t^4}\right]$.

If $g'(t) = 0 \Rightarrow t \pm 0.7598 \approx 0.76\ (\because t > 0)$.

It is clear that g (t) is decreasing in (0, 0.76] and increasing in (0.76,∞).

Also g (t) has a minimum value at t=0.76, because $g''(0.76) = 47.96 > 0$. Now,

a. If $0 < \alpha \le 0.76$, then

$$m = \inf_{t\in(\alpha,\beta)} g(t) = g(0.76) = 6.928\ . \tag{4.24}$$

$$M = \sup_{t\in(\alpha,\beta)} g(t) = \max\{g(\alpha), g(\beta)\} = 2\max\left[\frac{3\alpha^4+1}{\alpha^2}, \frac{3\beta^4+1}{\beta^2}\right]. \tag{4.25}$$

b. If $0.76 < \alpha \le 1$, then

$$m = \inf_{t\in(\alpha,\beta)} g(t) = g(\alpha) = 2\left(\frac{3\alpha^4+1}{\alpha^2}\right). \tag{4.26}$$

$$M = \sup_{t\in(\alpha,\beta)} g(t) = g(\beta) = 2\left(\frac{3\beta^4+1}{\beta^2}\right). \tag{4.27}$$

The results (4.19) and (4.20) are obtained by using (3.3), (4.22), (4.23), (4.24), (4.25), (4.26) and (4.27) in (2.2).

**Proposition 4.4** Let $\chi^2(P,Q), \Delta(P,Q)$ and $S^*(P,Q)$ be defined as in (1.4), (1.8) and (3.3) respectively. For P, Q∈$\Gamma_n$, we have

$$(3\alpha^4+1)\left[\chi^2(Q,P) + U(P,Q) - \frac{1}{2}\Delta(P,Q)\right] \le S^*(P,Q)$$

$$\le (3\beta^4+1)\left[\chi^2(Q,P) + U(P,Q) - \frac{1}{2}\Delta(P,Q)\right]. \tag{4.28}$$

**Proof**: Let us consider

$$f_2(t)=\frac{(t-1)^2}{t},t\in(0,\infty),\ f_2(1)=0,\ f_2'(t)=\frac{t^2-1}{t^2}\text{ and}$$

$$f_2''(t)=\frac{2}{t^3}. \tag{4.29}$$

Since $f_2''(t)>0\ \forall\ t>0$ and $f_2(1)=0$, so $f_2(t)$ is convex and normalized function respectively. Now, Put $f_2(t)$ in (1.3) and $f_2'(t)$ in (1.2), we get the followings.

$$S_{f_2}(P,Q)=\frac{1}{2}\sum_{i=1}^{n}\frac{(p_i-q_i)^2}{p_i+q_i}=\frac{1}{2}\Delta(P,Q). \tag{4.30}$$

$$E_{f_2'}(P,Q)=\sum_{i=1}^{n}\frac{(p_i-q_i)^2(p_i+q_i)}{p_i^2}=\sum_{i=1}^{n}\frac{(p_i-q_i)^2}{p_i}+\sum_{i=1}^{n}\frac{(p_i-q_i)^2q_i}{p_i^2}$$

$$=\chi^2(Q,P)+U(P,Q). \tag{4.31}$$

where $U(P,Q)=\sum_{i=1}^{n}\frac{(p_i-q_i)^2q_i}{p_i^2}$.

Now, let $g(t)=\frac{f_1''(t)}{f_2''(t)}=(3t^4+1)\ and\ g'(t)=12t^3>0\ \forall t>0$, where $f_1''(t)\ and\ f_2''(t)$ are given by (3.2) and (4.29), respectively.

It is clear that g (t) is increasing in $(0,\infty)$, so

$$m=\inf_{t\in(\alpha,\beta)}g(t)=g(\alpha)=3\alpha^4+1. \tag{4.32}$$

$$M=\sup_{t\in(\alpha,\beta)}g(t)=g(\beta)=3\beta^4+1. \tag{4.33}$$

The result (4.28) is obtained by using (3.3), (4.30), (4.31), (4.32) and (4.33) in (2.2).

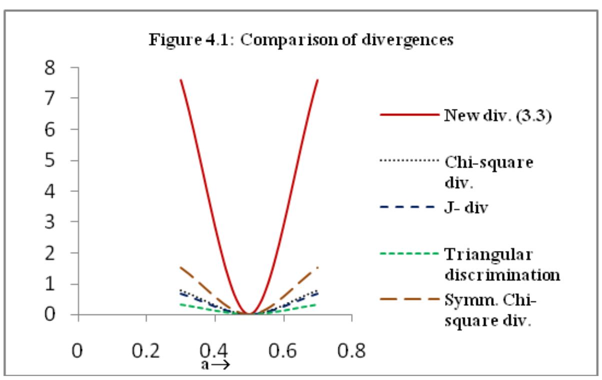


**Figure 4.1** shows the behavior of $S^*(P,Q),\chi^2(P,Q),J(P,Q),\Delta(P,Q)\ and\ \psi(P,Q)$. We have considered $p_i=(a,1-a),\ q_i=(1-a,a)$, where $a\in(0,1)$. It is clear from figure that the new divergence $S^*(P,Q)$ has a steeper slope than $\chi^2(P,Q),J(P,Q),\Delta(P,Q)\ and\ \psi(P,Q)$.

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